FINAL EXAM REVIEW:

Material covered—Mohr's Circle until end.
The exam covers recitations 8-12.
(only 5 recitations!)

TRANSFORMAION OF STRESS & STRAIN

Stress transformation formulas for plane stress at a given location
MOHR's CIRCLE

$$\begin{cases} \sigma_{x'} - \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\ \tau_{x'y'} = -\dfrac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \end{cases}$$

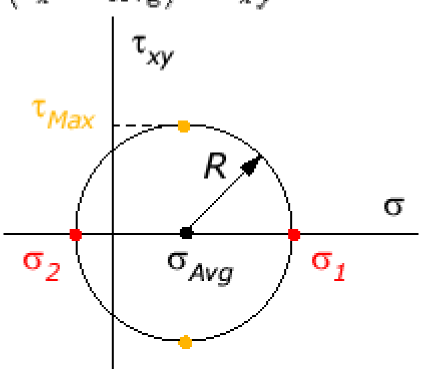
$$\left(\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{x'y'}^{\,2} = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}$$

$$\sigma_{Avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}}$$

, R is equal to the maximum shear stress

$$\left(\sigma_{x'} - \sigma_{Avg}\right)^2 + \tau_{x'y'}^{\,2} = R^2$$

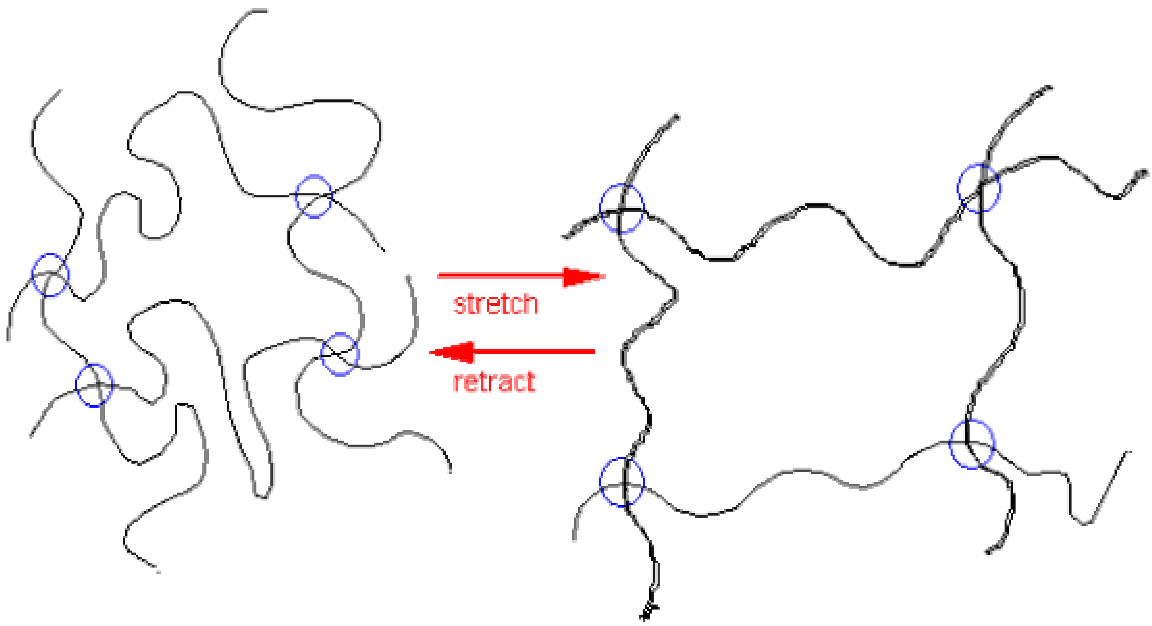


ELASTIC MODULI
Linear elasticity--- modeling bonds between atoms- Lennard Jones Potential
Review all equations from this,
(aka. Write them on your sheets)
remember: re is the distance where F(r)=dU(r)/dr=0

Rubber Elasticity—Freely Jointed Chain Model

➔a random walking polymer at finite T is a Hookean spring

Why is this useful?
Because these equations define the stretching of a single polymer chain.



Loss of entropy upon stretching, means that there is a retractive force for recovery when external stress removed.

This is why a rubber band returns to its original shape.

A simple reminder of polymer statistics.
Suppose the walk has N links:
End to end distance R(N)

From Rubber elasticity :
r = instantaneous chain end-to-end separation distance
(Draw on board--- squiggly lines with beginning and end separated by r)
$<r^2> = na^2$ root mean square end to end distance
a = statistical segment length—local chain stiffness
n = # of a's

Lc = contour length—length of fully extended chain.

Probability of finding a free chain end a radial distance, r, away from a fixed chain end (origin) ~ omega = P(R) = ($4b^3r^2$)/sqrt(pi)*exp($-b^2r^2$)
where b = sqrt (3/($2na^2$))

This is Gaussian form
Macrostate is defined by the length r.
Microstates are the different random walks. So..

$P(R) \sim e^{-3R^2/2Nb^2} \sim \Omega(R)$(# of µstates with length R) (N=n and b=a)

Configurational Entropy (measure of disorder) = S = kb*ln[P(r)]
Helmholtz Free Energy = A or H = -Tkb*ln[P(r)]
Entropic elastic force, linear elasticity (hookean spring) f or F = -dA(r)/dr^2
Entropic chain stiffness = k = dF(r)/dr or second derivative of A.

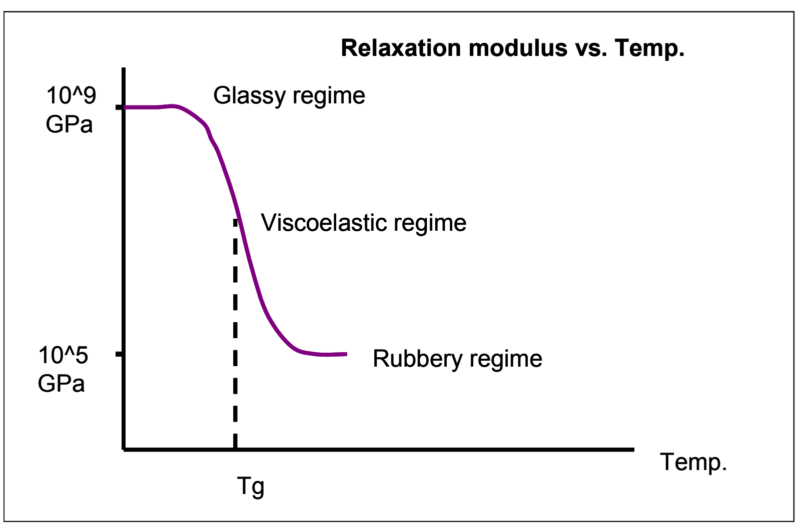
VISCOELASTICITY

Review Spring-Dashpot models.
Behavior between elastic solid and viscous fluid (hence the name viscoelastic)

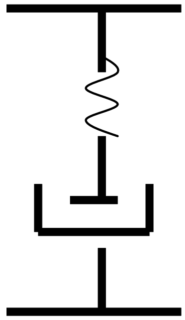
Creep test- Const. stress
If viscoelastic, strain proportional to stress change

Relaxation Test – Const. Strain

Now, temperature effect on amorphous polymers



Spring: E=stress/strain
Dashpot: stress = viscosity($d\varepsilon/dt$) ←strain rate drain
Maxwell model—in series

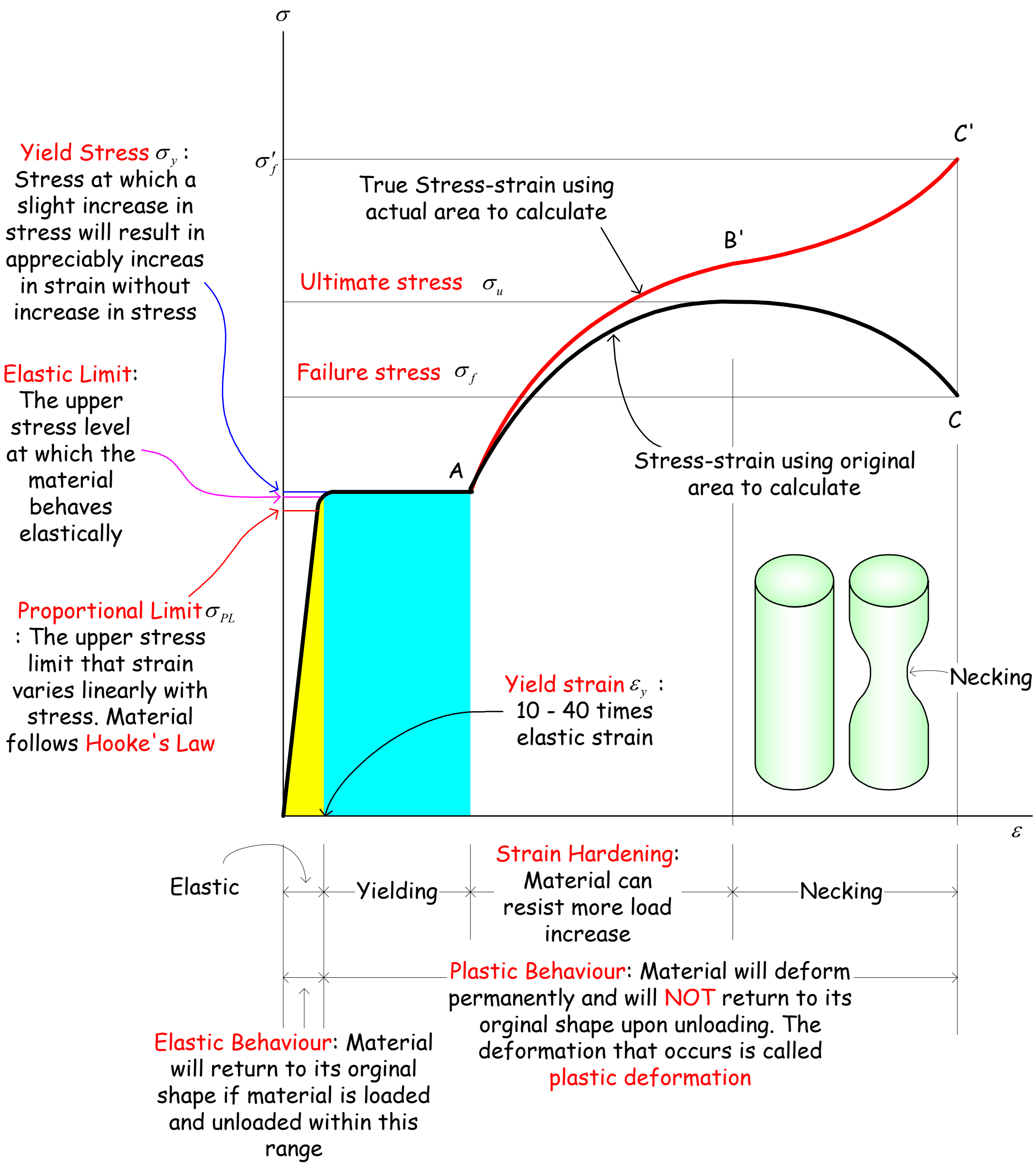


$\sigma s=\sigma d=\sigma total$
$\varepsilon s+\varepsilon d=\varepsilon total$

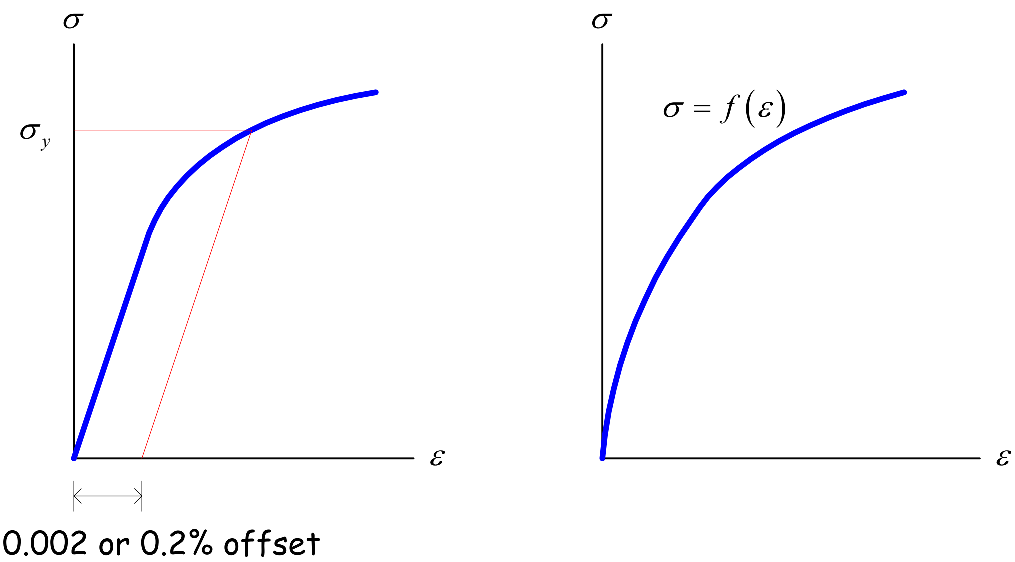
Voigt or Kelvin Model (in parallel)
$\sigma s+\sigma d=\sigma total$
$\varepsilon s=\varepsilon d=\varepsilon total$

Neither give entire model of a polymer—need to sum both models together.

Next, Stress-strain diagrams—now going into plasticity!



- If structure made of ductile materials is overloaded, it will present large deformation before failing
- Some ductile materials do not exhibit a well-defined yield point, we will use offset method to define a yield strength
- Some ductile materials do not have linear relationship between stress and strain, we call them nonlinear materials



- The work done, which equals to the strain energy stored in the element, is

$$dU = \frac{1}{2}(\sigma dxdy)(\varepsilon dz) \text{ or } dU = \frac{1}{2}\sigma\varepsilon dV$$

- The total strain energy stored in a material will be

$$U = \int_V \sigma\varepsilon dV$$

- The strain energy per unit volume or the strain energy density is

$$u = \frac{dU}{dV} = \frac{1}{2}\sigma\varepsilon$$

- If the material is linear elastic ($\sigma = E\varepsilon$, Hooke's Law holds), the strain energy density will be

$$u = \frac{1}{2}\sigma\left(\frac{\sigma}{E}\right) = \frac{1}{2}\frac{\sigma^2}{E}$$

Plasticity—3-D

Yield Criteria: Tresca Criterion

Sometimes called the Maximum shear stress criterion

1. get yield when max. shear stress, tau, in component (under general stress state) equals the ma, tau, in a uniaxial tensile test at yield
2. principle stresses $\sigma_1 > \sigma_2 > \sigma_3$

You could take stress state, rotate and rewrite it. Mohr's circle does this.

For general stress state:

$\tau_{max} = (\sigma_1 - \sigma_3)/2$

In uniaxial test: σ_1 ($\sigma_2 = \sigma_3 = 0$)

$\tau_{max} = (\sigma_1)/2$
so, you'd get yield when $\sigma_1 = \sigma_Y$

In general, you get yield when $\sigma_1 - \sigma_3 = \sigma_Y$
Tresca says, you take the difference between the biggest and smallest stress to get yield stress.

Yield Criteria: Von Mises Criterion
You get yield when equivalent sigma = yield stress

σ_{eq} =sqrt(0.5*[($\sigma_{11} - \sigma_{22}$)^2 + ($\sigma_{22} - \sigma_{33}$)^2 + ($\sigma_{33} - \sigma_{11}$)^2] +3σ_{23}^2 + 3σ_{13}^2 +3σ_{12}^2)

These are looking at effects of Shear on the Material.

Good things to remember--
At R.T. many materials (esp. metals) have a well defined yield stress, σ_y
$\sigma < \sigma_y$ elastic recoverable deformation
$\sigma > \sigma_y$ plastic irrecoverable

Plastic behavior is important for:
- Material design for strengthening (alloying)
- Work hardening
- Hardness – friction and wear

Some materials fracture before yielding (ceramics (brittle))
BUT, fracture can be suppressed by a hydrostatic stress → even ceramics will yield.

Really make sure to understand differences between elastic and plastic states of a system.
Understand necking, fracture, and other deformations- understand why these occur.

Good Luck!