RECITATION 12
DECEMBER 2, 2003

I think craze, shear deformation zones and rubber toughening of glassy polymers was covered well in lecture—so I will focus on the more complicated equations dealing with the 3-D deformation. Please ask me if you'd like more clarification and I can cover it in the final recitation (Dec. 9)

PLASTICITY:
Plastic behavior
3-D Plastic Deformation
Review of stress states for 3-D (Von Mises & Tresca)

Plasticity:
At R.T. many materials (esp. metals) have a well defined yield stress, σ_y
$\sigma < \sigma_y$ elastic recoverable deformation
$\sigma > \sigma_y$ plastic irrecoverable

Plastic behavior is important for:
 Material design for strengthening (alloying)
 Work hardening
 Hardness – friction and wear
Some materials fracture before yielding (ceramics (brittle))
BUT, fracture can be suppressed by a hydrostatic stress → even ceramics will yield.

This next stuff—low temperature plasticity (not high temp. creep behavior)

As a review:
We've talked about a couple types of tests:
1. Uniaxial Tensile Test
   a. Destructive to sample
   b. Measures ductility
   c. Shows where yield stress is located (at 0.2% strain)
2. Hardness Test
   a. Indenter pushed into polished surface w/ a known force
   b. Measure the size of indent (you should have done this/will do this in 3.081
   c. Gives hardness, which is related to yield stress
   d. Different versions of indenters (sphere, pyramid, cone)
      i. Advantages
         1. non-destructive, easy to do.
         2. can use this on ceramics

note: for hardness test, it needs to be a polished material, maybe 500um diameter

Equations of plasticity
Assumptions:

1. Plastic flow occurs at constant volume
   a. $\Delta V/V_o = 0$ $\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = 0$ (note: strains are plastic strains in x, y and z direction)
2. Modest hydrostatic pressures to not affect yield stress/strength
   a. "modest"~E/100
3. Neglect work hardening
   a. Assume elastic region got to perfect plastic region
4. Assume Material is Isotropic

Criteria for Yielding Under Multi-Axial Stresses
Yield unaffected by hydrostatic component of stress (mean stress)

$[\sigma_{11}, \sigma_{12}, \ldots \sigma_{33}] = [\sigma_m, \sigma_m, \sigma_m] + [\sigma_{11}\text{-}\sigma_m, \sigma_{12}, \ldots \sigma_{33}\text{-}\sigma_m]$
where $\sigma_m = (\sigma_{11}+\sigma_{12}+\sigma_{33})/3$ ← sigma m is the mean stress component
everything but mean stress is related to the amount of shearing in the material—*called the deviatoric component of* σ.
Yield is governed by the deviatoric part of σ
→ shearing—is what's really controlling yield.

Yield Criteria: Tresca Criterion
Sometimes called the Maximum shear stress criterion

1. get yield when max. shear stress, tau, in component (under general stress state) equals the ma, tau, in a uniaxial tensile test at yield
2. principle stresses $\sigma_1>\sigma_2>\sigma_3$

You could take stress state, rotate and rewrite it. Mohr's circle does this.
For general stress state:
$\tau_{max} = (\sigma_1\text{-}\sigma_3)/2$
In uniaxial test: σ_1 ($\sigma_2=\sigma_3=0$)
$\tau_{max} = (\sigma_1)/2$
so, you'd get yield when $\sigma_1=\sigma_Y$

In general, you get yield when $\sigma_1 - \sigma_3 = \sigma_Y$
Tresca says, you take the difference between the biggest and smallest stress to get yield stress.

Yield Criteria: Von Mises Criterion
You get yield when equivalent sigma = yield stress

σ_{eq} =sqrt(0.5*[($\sigma_{11}-\sigma_{22}$)^2 + ($\sigma_{22}-\sigma_{33}$)^2 + ($\sigma_{33}-\sigma_{11}$)^2] +3σ_{23}^2 + 3σ_{13}^2 +3σ_{12}^2)

How is this related to shearing? We're applying stress. The stress energy for mean + stress energy for shear = multiaxial stress state

Show example of comparison of Tresca and Von Mises for Biaxial stress (sigma22=0)
Tresca: if stress state inside, yield criteria→elastic
If On yield criteria→ plastic

For Von Mises: you have ellipsoid criteria
There is a gap between them that correlates to pure shear.
(it's not too far apart for either criteria—the maximum difference is 13%)

Tresca criterion is more conservative than Von Mises criterion (most used)

Example Problems with Von Mises/Tresca analysis

1. A cylindrical aluminum (σ_Y = 200MPa) pressure vessel has a length of 60 cm, a diameter of 100 mm and a wall thickness of 2 mm. Determine if the pressure vessel will yield under an internal pressure of 6 MPa using (a) the Tresca yield criterion and (b) the von Mises yield criterion.

Solution:

$$\sigma_y = 200\mathbf{MPa}$$
$$\mathbf{l} = 60\mathbf{cm}$$
$$\mathbf{d} = 100\mathbf{cm}$$
$$\mathbf{t} = 2\mathbf{cm}$$
$$\mathbf{p} = 6\mathbf{MPa}$$
$$\sigma_1 = \frac{\mathbf{pr}}{\mathbf{t}} = \frac{(6)(50)}{2} = 150\mathbf{MPa}$$
$$\sigma_2 = \frac{\mathbf{pr}}{2\mathbf{t}} = 75\mathbf{MPa}$$
$$\sigma_3 = 0$$

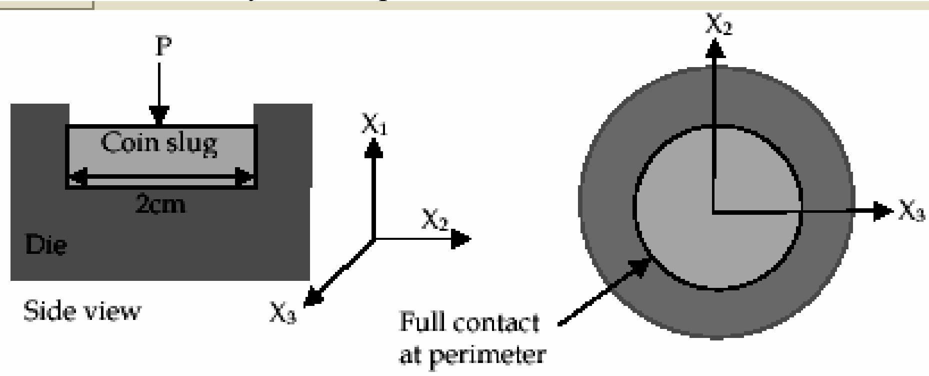
Tresca:

$$\sigma_1 - \sigma_3 = \sigma_y(@\,\mathbf{yield})$$
$$150 < 200 \Rightarrow \mathbf{no\_yield}$$

Von Mises:

$$\sigma_{eq} = \sqrt{\frac{1}{2}\left[(150-72)^2 + (75-0)^2 + (150-0)^2\right]}$$
$$= 130\mathbf{MPa} < 200\mathbf{MPa} \Rightarrow \mathbf{no\_yield}$$

2. When coins are minted, they are stamped out in a die:



a) Use both the Tresca and von Mises yield criteria to determine expressions for the stress in the x_1 direction (as a function of the yield stress and the Poisson's ratio of the coin material) required to initiate yielding of the slug. Assume that the die walls are rigid and frictionless and that the coin material is isotropic.

Solution:

We know that the applied stress will be larger than the "Poisson's stress" in the x2 and x3 direction so we will call:

$$\sigma_1 = \sigma_{x_1} \text{ and } \sigma_1 = \sigma_{x_1} = \sigma_2 = \sigma_3 = \sigma_{x_2} = \sigma_{x_3}$$

Because the coin is restricted in the X_2 and X_3 directions, $\varepsilon_2 = \varepsilon_3 = 0$. Assuming the coin metal is isotropic, we can say:

$$\varepsilon_1 = \frac{1}{E}\sigma_1 - \frac{\nu}{E}\sigma_2 - \frac{\nu}{E}\sigma_3$$

$$\varepsilon_2 = 0 = \frac{1}{E}\sigma_2 - \frac{\nu}{E}\sigma_1 - \frac{\nu}{E}\sigma_3$$

Using the second equation and the fact that $\sigma_2 = \sigma_3$,

$$0 = \sigma_2 - \nu\sigma_1 - \nu\sigma_2$$

$$\Rightarrow \sigma_2 = \left(\frac{\nu}{1-\nu}\right)\sigma_1 = \sigma_3$$

By Tresca:

$$\sigma_1 - \sigma_3 = \sigma_{x_2}$$

$$\sigma_1 - \left(\frac{\nu}{1-\nu}\right)\sigma_1 = \sigma_{x_2}$$

$$\sigma_1\left(1 - \frac{\nu}{1-\nu}\right) = \sigma_{x_2}$$

$$\sigma_1 = \sigma_{x_2}\left(\frac{1-\nu}{1-2\nu}\right)$$

By von Mises:

$$\sigma_{eq} = \sigma_{x_2}$$

$$\sigma_{x_2} = \left\{\frac{1}{2}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]\right\}^{\frac{1}{2}}$$

$$\sigma_{x_2} = \left\{\frac{1}{2}\left[\left(\sigma_1 - \frac{\nu}{1-\nu}\sigma_1\right)^2 + \left(0\right)^2 + \left(\frac{\nu}{1-\nu}\sigma_1 - \sigma_1\right)^2\right]\right\}^{\frac{1}{2}}$$

$$\sigma_{x_2} = \left\{\frac{1}{2}\left[2\left(\sigma_1 - \frac{\nu}{1-\nu}\sigma_1\right)^2\right]\right\}^{\frac{1}{2}}$$

$$\sigma_{x_2} = \sigma_1 - \frac{\nu}{1-\nu}\sigma_1$$

$$\sigma_1 = \sigma_{x_2}\left(\frac{1-\nu}{1-2\nu}\right)$$