3.11 Recitation #2
Fall 2003
Tuesday, September 16, 2003

Linear Elasticity Clarification:
Linear Elastic Deformation is instantly recoverable upon unloading. Rubbers (and some collagen/unfolding polymers) do not follow the rules of linear elasticity because as you stretch rubbers, they realign. (Chains slide over each other, and lose orientation)

Static Equilibrium: 3-D examples. (Useful on problem #1 on homework)

$\sigma_{ij} = F_i/A_j$ ← A is in direction of normal area

For equilibrium: $\Sigma F = 0$ for x, y and z directions
$\Sigma M = 0$ for xy, xz and yz directions

It may help to think of matrix in numbers rather than letters. (where σ_{12}, σ_{13}, σ_{23} are shear stresses)

$$\begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix}$$

So, to prove 3-D static equilibrium, show that $\Sigma M_{12} = 0 = F_{12} \times d + F_{21} \times d$. (d is a distance (the moment arm) that you can define on your 3-D system. I suggest having a dx, dy and dz for each axis (or d_1, d_2, d_3).

IMPORTANT NOTE: σ12 is a shear stress (aka σxy) so, the area it acts upon is not that normal to it. Think of σ12 on a box. What side is being changed because of a shearing force.

Additional equations that are valid for 3-D static equilibrium: $d\sigma_{11}/dx_1 + d\sigma_{12}/dx_2 + d\sigma_{13}/dx_3$. Same for σ_2, σ_3. These may prove helpful when solving static equilibrium equations.

Matrix Notation for Generalized Hooke's Law for Anisotropic Materials:

6-independent components of ε
6-independent components of σ
Compliance (S) and Stiffness Matrix (D) are both symmetric (ie. 36 elastic components reduce to 21)
Matrix Notation:

σi, εi are pseudovectors

6-independent σ components:

$\sigma 11 \rightarrow \sigma 1$
$\sigma 22 \rightarrow \sigma 2$
$\sigma 33 \rightarrow \sigma 3$
$\sigma 23$ ($\tau 23$, yz) $\rightarrow \sigma 4$
$\sigma 13$ ($\tau 13$, xz) $\rightarrow \sigma 5$
$\sigma 12$ ($\tau 12$, xy) $\rightarrow \sigma 6$
(To better understand, picture the new 1-6 #'s where they would lie in a 3x3 matrix (the σij matrix above)

6-independent ε components:

$\varepsilon 11 \rightarrow \varepsilon 1$
$\varepsilon 22 \rightarrow \varepsilon 2$
$\varepsilon 33 \rightarrow \varepsilon 3$
$\varepsilon 23$ ($\gamma 23$) $\rightarrow \varepsilon 4$
$\varepsilon 13$ ($\gamma 13$) $\rightarrow \varepsilon 5$
$\varepsilon 12$ ($\gamma 12$) $\rightarrow \varepsilon 6$

So, for Compliance Matrix $\varepsilon i = Sij\sigma j$
Stiffness Matrix $\sigma i = Dij\varepsilon j$
$D=S^{-1}$

Showing that the Matrix is Symmetric: (note: all #'s are subscripts)
$\varepsilon 1 = S11\sigma 1 + S12\sigma 2 + S13\sigma 3 + \ldots. S16\sigma 6$
$Sij=Sji$ $\rightarrow$ 21 independent components

1) S11, S22, S33 $\rightarrow$ Apply a Normal/True $\sigma 1$ only. $\varepsilon 1 = S11\sigma 1$ $\rightarrow$ S11= 1/E1, S22 = 1/E2, S33 = 1/E3. These are related to the Young's Modulus.

2) S44, S55, S66 $\rightarrow$ S66 relates the shear strain, ε, 1-2 plane to shear stress, σ. in same plane. S66=1/G12 (could call this 1/G6) , S44=1/G23, S55=1/G13

3) S12, S13, S23 $\rightarrow$ These are related by the poisson's ratio. For example: if we apply $\sigma 2$ only. $\varepsilon 1=S12\sigma 2$, $\varepsilon 2=S22\sigma 2$ then $\nu 21 = -\varepsilon 1/\varepsilon 2 = (-S12\sigma 2)/(S22\sigma 2)$, $S12 = -\nu 21/E2$ …. $\sigma 2=E2\varepsilon 2$, $1/E2=\varepsilon/\sigma$, $\varepsilon 2=S22\sigma 2$, $S22=\varepsilon 2/\sigma 2 = 1/E2$

4) S14, 15, 16, 24, 25, 26, 34, 35, 36 Physical non-intuitive for most materials. Relates a normal strain to shear stress for most materials. These will be zero.

5) S45, 46, 56 → Relate shear strain in 1 plane to shear stress in a different plane. – for Most materials, these are zero.

Example Problems:

7.2-3 If element in plane stress is subjected to stress $\sigma x = -11{,}100$psi, $\sigma y = -4{,}600$psi, $\tau xy = 3{,}600$psi, $\theta = 50°$.

Determine the stresses acting on an element oriented at an angle 50 degrees from the x-axis, where the angle is positive when counterclockwise. Show these stresses on a sketch of an element oriented at the angle, θ.

See. Page 486 in Gere. Similar problem and extra explanation for pset.

$\sigma x1 = (\sigma x + \sigma y)/2 + (\sigma x - \sigma y)/2 * \cos 2\theta + \tau xy \sin 2\theta = -3740$psi

(b/c of Definition for biaxial stress—Gere Ch. 7)

$\tau x1y1 = - ((\sigma x - \sigma y)/2 * \sin 2\theta + \tau xy \cos 2\theta = 2580$ psi

$\sigma y1 = \sigma x + \sigma y - \sigma x1 = -11{,}960$ psi

7.5-2

A rectangular steel plate with thickness t=10mm is subjected to uniform normal stresses σx and σy. Strain gages A & B, oriented in x and y dir. Are attached to the plate. The gage readings give normal strains εx = 350x10^-6, εy = 85x10^-6 (elongation). E = 200Gpa, ν = 0.30. Determine σx and σy and change in t, thickness of the plate.

For Plane Stress: $\varepsilon x = 1/E (\sigma x - \nu \sigma y)$ for strain in x direction has two components σx/E and -νσy/E ($\nu = -\varepsilon'/\varepsilon$) ← given

$\sigma x = E/ (1-\nu)^2 * (\varepsilon x + \nu \varepsilon y) = 15{,}990$psi

$\sigma y = E/ (1-\nu)^2 * (\varepsilon y + \nu \varepsilon x) = -8{,}700$psi

$\varepsilon' = -\nu/E (\sigma x + \sigma y) = -72.9x10^{-6}$

$\Delta t = \varepsilon' t = -1860x10^{-6}$ mm (decrease in thickness)

Example 3-

A magnesium plate in biaxial stress is subjected to tensile stresses σx and σy. The corresponding strains are εx and εy. Determine the ν and E for the material.

σx=30Mpa
σy = 15Mpa
$\varepsilon x = 550x10^{-6}$
$\varepsilon y = 100x10^{-6}$

$\varepsilon x = \sigma x/E - (\sigma y \nu)/E$
$\varepsilon y = -(\nu \sigma x)/E + \sigma y/E$
$\varepsilon z = -(\nu \sigma x)/E - (\nu \sigma y)/E$ (can ignore z direction)

E $(550x10^{-6})$ = 30 MPa - ν (15MPa)
E $(100x10^{-6})$ = 15 MPa - ν (30 MPa)

Solve simultaneously: $\nu = 0.35$, E = 45 GPa