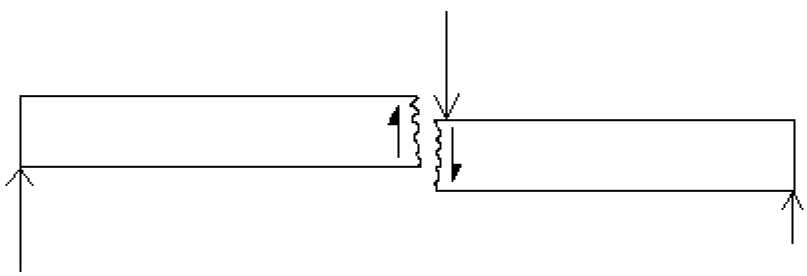
Recitation #5

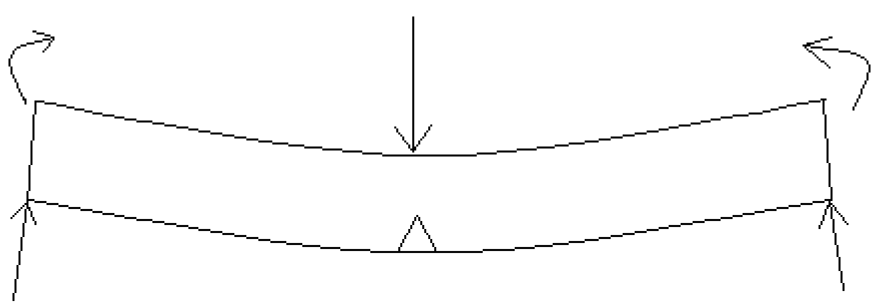
Understanding Shear Force and Bending Moment Diagrams

Shear force and bending moment are examples of interanl forces that are induced in a structure when loads are applied to that structure. Loading tends to cause failure in two main ways:

a) by shearing the beam across its cross-section.

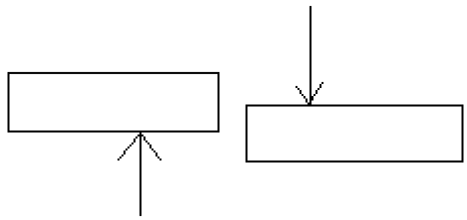


b) by bending the beam to an excessive amount.

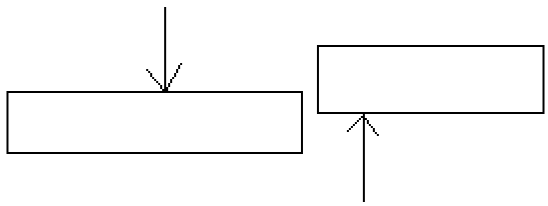


Shear force may be defined as "the algebraic sum of the loads to the left or right of a point (such that the addition of this force restores vertical equilbrium)".

The accepted sign convention is:



+ve shear
sum of forces to the left is upwards (+ve)
sum of forces to the right is downwards (-ve)



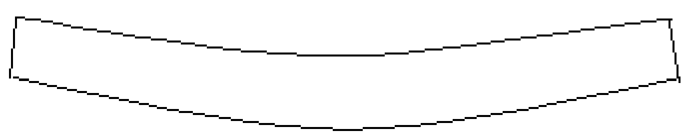
-ve shear
sum of forces to the left is downwards (-ve)
sum of forces to the right is upwards (+ve)

A shear force diagram is one which shows variation in shear force along the length of the beam.

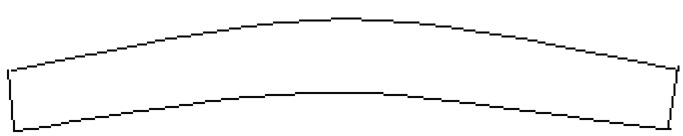
Bending moment may be defined as "the sum of moments about that section of all external forces acting to one side of that section".

Moments, at any point, are calculated by multiplying the magnitude of the external forces (loads or reactions) by the distance between the point at which moment is being determined and the point at which the external forces (loads or reactions) are being applied.

The accepted sign convention is:



+ve bending moment
sum of moments to the left is clockwise
sum of moments to the right is anticlockwise



-ve bending moment
sum of moments to the left is anti-clockwise
sum of moments to the right is clockwise

Note:

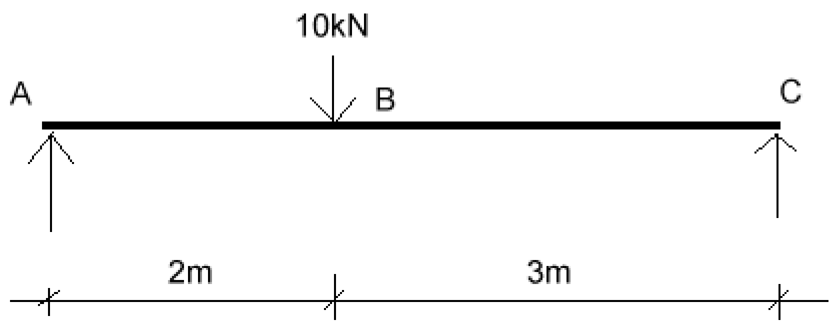
- For beams spanning between two simple pin-jointed supports (i.e. no cantilevers) moment will always be positive and, although the beam sags, moment is drawn above the axis.
- The maximum bending moment occurs at the point of zero shear force.

- Before shear force and bending moments can be calculated the reactions at the supports must be determined. This is usually done by first taking moment s about one of the supports (sum of moments about support = 0) to determine the reaction at the other support and secondly by resolving vertically (sum of vertical reactions = 0) and determining the reaction at the other support.

A bending moment diagram is one which shows variation in bending moment along the length of the beam.

Example 1

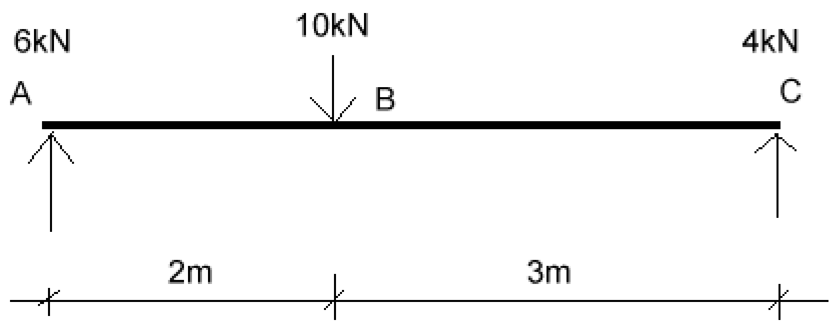
Draw the shear force and bending moment diagrams for the beam shown below



a) determine the reactions at the supports.

Taking moments about A (clockwise moments =anti-clockwise moments)
$10 \text{ x}2 = 5R_C$
$5R_C=20$
$R_C=20/5 =4\text{kN}$
Resolving vertically
$R_A+R_C =10\text{kN}$
Substituting in $R_C=4\text{kN}$
$R_A+4 =10$
$R_A=10\text{-}4=6\text{kN}$

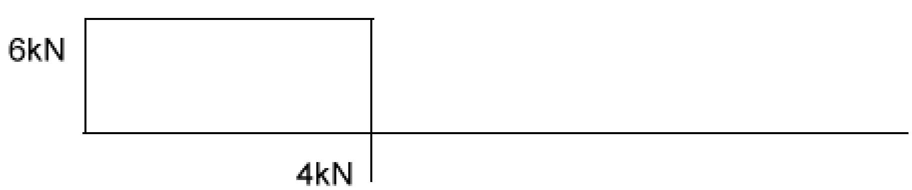
b) draw the shear force diagram



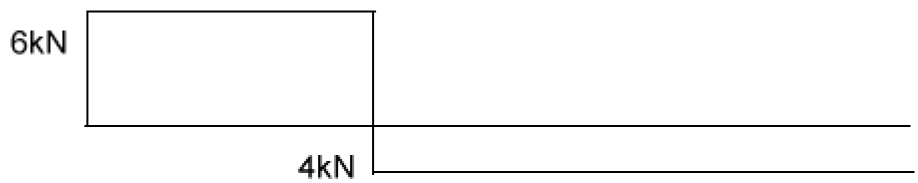
working from left to right:
the reaction at A=+6kN

the shear force remains constant between A and B (i.e. 6kN) and so the shear force diagram is horizontal between these points

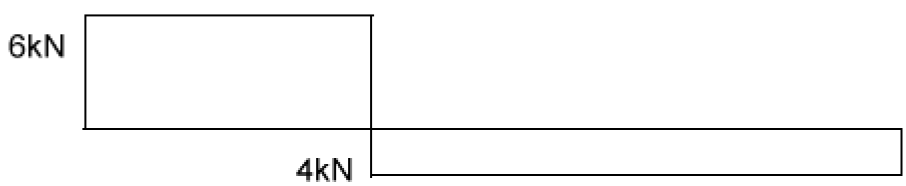
at point B where the 10kN point load is applied the shear force changes from +6kN to +6-10= -4kN



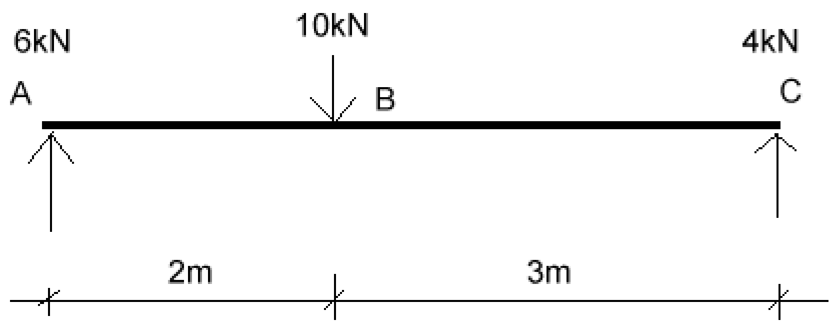
the shear force remains constant between B and C (i.e. -4kN) and so the shear force diagram is horizontal between these points.



at support C the reaction of 4kN brings the value of shear force back to zero and the diagram is complete.

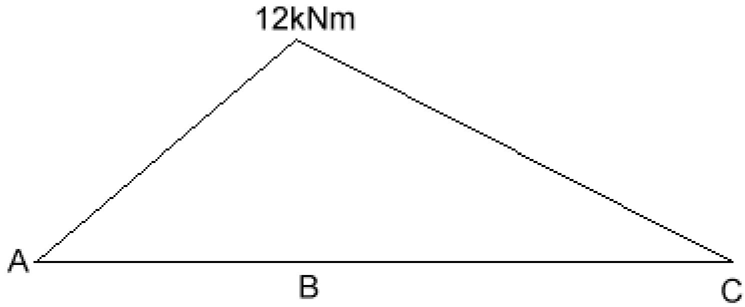


c) draw the bending moment diagram:



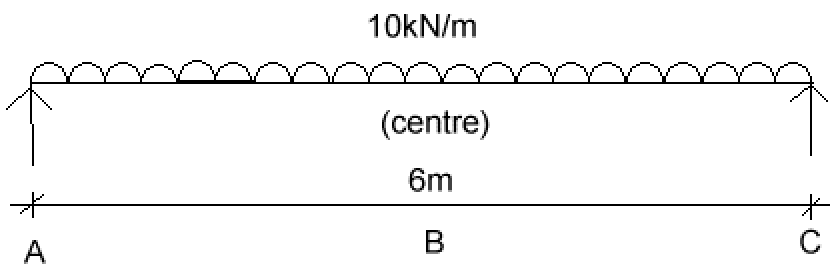
The moments at the supports A and C are zero. The maximum moment occurs at B (point of zero shear force).
Taking moments at B (to the left)
M_B=6 x 2 =12kNm
Bending moment varies uniformly between A and B and between B and C (i.e. the bending moment diagram is a straight line). For example, the bending moment midway between A and B = 6 x1 =6kNm which is half the value of the bending moment at B.

Therefore the bending moment diagram is:



Example 2

Draw the shear force and bending moment diagrams for the beam show below:



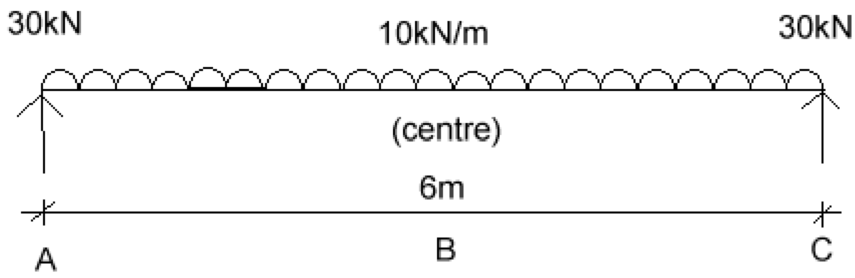
a) determine the reactions at the supports

Taking moments about A (clockwise moments = anti-clockwise moments)
$(10 \times 6) \times 3 = 6R_C$ where $10 \times 6 = 60$kN = total load and 3m =distance from A to where the load is acting.
$6R_C=180$
$R_C=180/6$
$=30$kN

Resolving vertically
$R_A+R_C= 10 \times 6 = 60$kN
Substituting in $R_C=30$kN
$R_A+30=60$
$R_A=60-30=30$kN

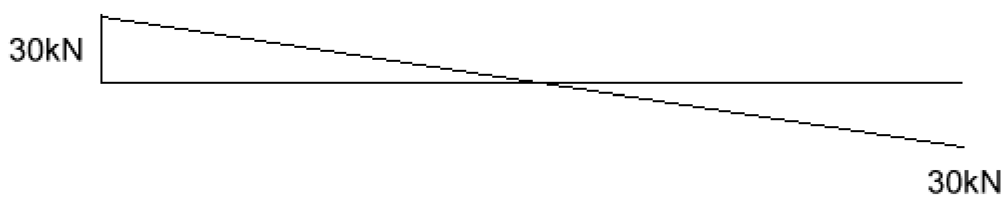
Note In this example, because the loading is applied symmetrically the reactions at each support must be equal and are therefore half the load. It is acceptable to assume this and not necessary to calculate the reactions.

b) draw the shear force diagram

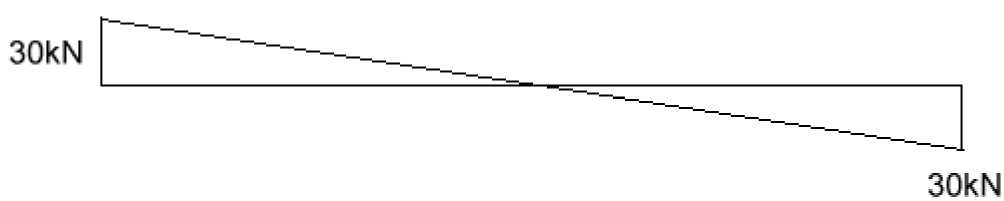


Working from left to right:
the reaction at A = +30kN

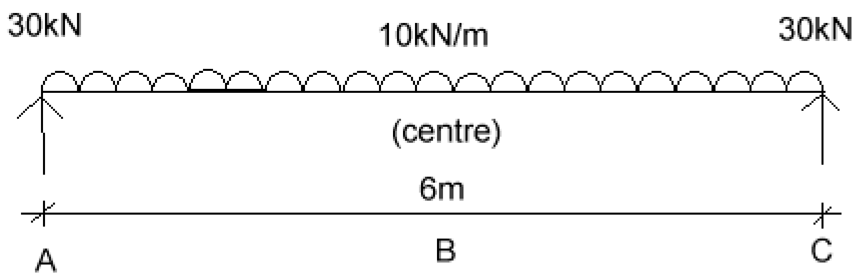
Moving from left to right the shear force reduces by 10kN per metre and so changes from +30kN at A to +30-(6x10)=-30kN at C. The shear force diagram is sloping between A and C and the point aof zero shear force occurs at B, the centre point.



At support C the reaction of 30kN brings the value of shear force back to zero and the diagram is complete.

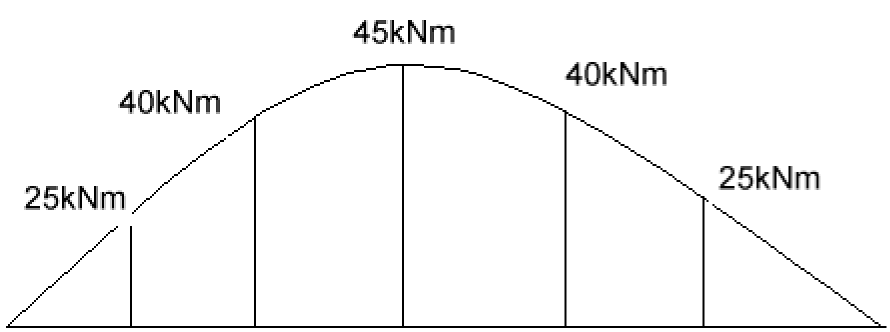


c) draw the bending moment diagram



The moments at the supports A and C are zero. The maximum moment occurs at B (point of zero shear force).
Taking moments about B (to the left)
M_B=(30 x3)-(10x3x[3/2])
= 90 -45 = 45kNm
where 10 x3 =30kN is the value of uniform load contributing to the moment at B
and 3/2 is the distance from B to the point where this load is assumed to act.

Bending moment does not vary uniformly between A and B and between B and C but the bending moment diagram is parabolic (curved). This can be demonstrated by calculating the bending moment at 1m intervals, measured from A:
BM_1=(30 x1)-(10x1 x [1/2])=25kNm
BM_2=(30x2)-(10x2 x [2/2])=40kNm
BM_3=(30x3)-(10x3 x [3/2])=45kNm
BM_4=(30x4)-(10x4 x [4/2])=40kNm
BM_5=(30x5)-(10x5 x [5/2])=25kNm



Note It is usually sufficient to calculate the maximum moment only.

Note: The Bending Moment is Parabolic (CURVED) when you are applying a uniform load, q, to the beam.

SO, Now you can combine these two types of loads, uniform q and your P loads. (don't forget the loads at the pins need to be taken into account).

SO, SUMMARY→Now you know that for your simple beam system:

Use the Free Body Diagram of section at x (you define your system)

SHEAR DIAGRAMS Use $\sum F_y = LoadA + LoadB - V = 0$
Concentrated Load P→ flat line
Uniform Load, q, → some slope on your line (either decreasing or increasing if you have a flat uniform q like the example above)
(we haven't discussed the shear diagram if your q increases by some slope over your beam)

BENDING MOMENT DIAGRAMS
$\sum M = LoadA*distance + LoadB*distance + M = 0$
Solve for M
Concentrated Load P→ inc. or decrease linearly to the applied P.
Uniform Load, q → Parabolic