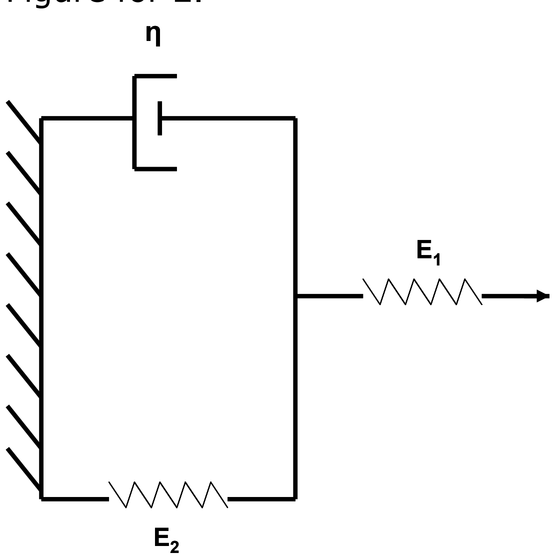
Problem Set #9
Due Friday, November 21th

Spring-Dashpot

1. Describe the difference between a Creep Test and a Stress Relaxation Test. Use graphs of each to explain your answer. (You can state what part dominates)

2.
   a) The simpliest spring-dashpot models are the Maxwell and Voigt elements discussed in class. A better model is the "standard linear solid" which is shown below. Derive a constitutive equation (i.e. one that takes into account the parallel and series properties) for the standard linear solid, which shows how the overall stress, stress rate, strain and strain rate are related by the three parameters E_1, E_2 and η
   b) For the standard linear solid discussed in part a), determine the overall $\varepsilon(t)$ in terms of E_1, E_2 and η_1 when in a state of constant stress.
   c) The retardation time, τ, defined as $\tau = \eta/E_1$ is often used to replace viscosity and Young's modulus(in general, this equation is seen as $\tau = \eta/k$) . If immediately after applying stress, the strain is 0.002, after 1000 seconds the strain is 0.004 and after a very long time the strain tends to be 0.006, what is the retardation time τ?

Figure for 2.



3. In a Kelvin-Voigt model, the creep response of a material is modeled by the following expression:

$$\varepsilon(t) = \frac{\sigma_0}{k}\left(1 - \exp\left(-t/\tau_c\right)\right)$$

where σ_0 is the constant stress applied to the material, k is the spring modulus and τ_c, the retardation time, is defined as $\tau_c = \eta/k$, where η is the viscosity of the dashpot.

In a creep test, a material with k = 600 MPa is initially loaded with a stress σ_0. Half an hour after the initial loading, the strain in the material is measured to be 0.111, and after another hour, it is found to be 0.264. What will be the strain in the material 3 hours after the initial loading? How much time did it take for the strain to reach a value of 0.001?

4. A review of the rubber elasticity that wasn't covered—

a) starting with the expression for the change in Helmholtz free energy in an ideal rubber, derive the rubber elasticity equations of stress versus extension ratio for a biaxial stress (i.e., find $\sigma_x(\lambda_x,\lambda_y)$ and $\sigma_y(\lambda_x,\lambda_y)$.)

b) If E (the elastic modulus in uniaxial deformation) is 10.0 MPa for a rubber, what are the stresses σ_x and σ_y required to deform the rubber to λ_x =2 and λ_y =1/2? Is σ_x an overestimate or an underestimate, compared to experimental data?