Solutions Problem Set #10
Due December 5th

For the following problems, state whether the statement is True or False and briefly explain your answer:

1. The modulus of toughness is a measure of the area under a full stress-strain curve, and the energy at break is a measure of the area under a full load-deformation curve.
    a. TRUE
2. The modulus of resilience is a measure of the area under the elastic portion of a stress-strain curve, and the energy at yield is a measure of the area under the elastic portion of a load-deformation curve.
    a. TRUE
3. The sound heard at fracture of a tension specimen is caused by the plastic energy stored in the bar.
    a. FALSE – the sound you hear is caused by stored elastic energy, which is quickly released upon fracture.

Choose the correct answer for the following statements:

4. The tensile strength of a material refers to the: Answer is C
    a. Yield point loading
    b. Fracture load
    c. Fracture stress
    d. Modulus of toughness
    e. Stress calculated from the maximum load and original cross-sectional area
5. One common relative measure of ductility is the: Answer is B
    a. Rockwell hardness
    b. Percent reduction in area
    c. Modulus of elasticity
    d. Toughness
    e. Ultimate strength

6. For this problem, do the following:

a) Calculate the modulus of resilience of the aluminum alloy 6061-T6, for which the needed properties are given in Appendix H of the Gere textbook.
b) Assume as a very rough approximation that for this same aluminum alloy, the stress-strain curve is linear after yielding, and that the material breaks at the ultimate stress at the percent elongation given in the appendix (17%). Calculate the modulus of toughnes U_{MOT} of the alloy, its plastic work of fracture (U_P) and energy released upon fracture ($U_{elastic}$).

a) From Appendix H (p.899 & 900), the elastic modulus of the 6061-T6 aluminum alloy is 70 GPa and the yield strength is 270 MPa.
→ $U_{MOR} = \sigma_Y^2/2E$ = (270 MPa)^2 / (2*70,000 MPa) = **0.52 MPa or 0.52 * 10^6 J/m^3**

b) The modulus of toughness is the area under the whole curve. From Appendix H (p.900), the ultimate stress is 310 MPa and ε_f is 0.17. Also, $\varepsilon_Y = \sigma_Y/E$ = 270 MPa / 70,000 MPa = 0.00386. From the problem setup, the stress-strain curve is made up of two straight lines. The first part (elastic part) was already calculated in 3(a). The second part is a trapezoid whose area is ((270 MPa + 310 MPa) / 2) * (0.17 - 0.00386) = 48.18 MPa. Thus, the modulus of toughness is the sum of the 2 areas:

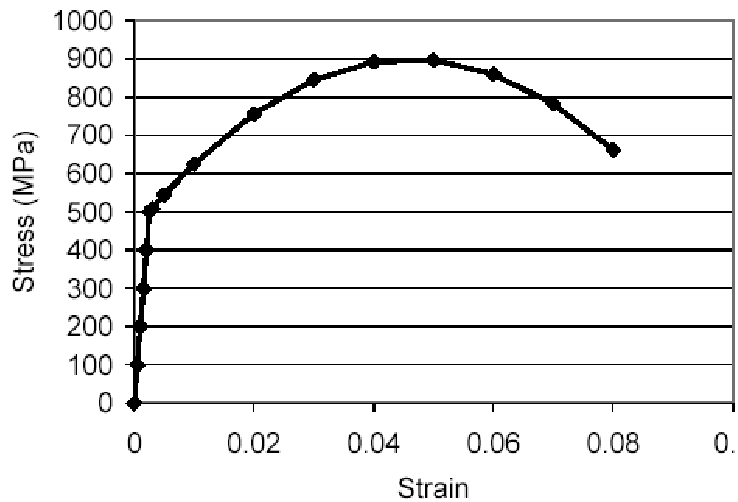
U_{MOT} = 0.52 MPa + 48.18 MPa = **48.70 MPa or 48.70 * 10^6 J/m^3**

From the data given, the residual strain $\varepsilon_P = \varepsilon_f - \sigma_f / E$ = 0.17 - 310 MPa / 70,000 MPa = 0.1656.

This means that $U_{elastic}$ = (0.17 - 0.1656) * 310 MPa / 2 = **0.68 MPa or 0.68 * 10^6 J/m^3**
and that $U_P = U_{MOT} - U_{elastic}$ = 48.70 MPa - 0.68 MPa = **48.02 MPa or 48.02 * 10^6 J/m^3**

7. The following idealized data was obtained from tensile testing of steel. Answer (a) through (c) for the following data: (you can enter the data into an excel spreadsheet if you'd prefer).

Strain	Stress (MPa)
0	0
0.0005	100
0.001	200
0.0015	300
0.002	400
0.0025	500
0.003	509
0.005	544
0.01	625
0.02	756
0.03	845
0.04	892
0.05	897
0.06	860
0.07	782
0.08	662



a) Determine the yield strength (σ_Y) and ultimate tensile strength (σ_{UTS}) of this material. You do not need to do any construction. Eyeballing it should be enough.
b) Calculate the modulus of resilience.
c) Calculate the modulus of toughness (U_{MOT}), the plastic work of fracture (U_P) and energy released upon fracture ($U_{elastic}$). To calculate the area under the curve for the plastic part, you can approximate the area as the sum of the areas of the trapezoids delimited by the data points. This is a standard approximation technique, and will be briefly discussed in the recitation. Show the work you did in obtaining your answers.

(a) From the plotted data, σ_Y is ~500 MPa and σ_{UTS} is ~900 MPa.

(b) From the given data, the elastic modulus (i.e., the slope of the elastic part) is σ/ε = 100 MPa / 0.0005 = 200 / 0.001 = … = 200,000 MPa. Thus, Umor = $\sigma_y^2/2E$= (500 MPa)^2 / (2*200,000 MPa) = **0.625 MPa or 0.625 * 10^6 J/m^3**

(c) The following table shows the area under each trapezoid in the dataset. The sum of each of them gives the total area under the curve.

Strain	Stress (MPa)	Area of trapezoid (MPa)
0	0	
0.0005	100	0.025
0.001	200	0.075
0.0015	300	0.125
0.002	400	0.175
0.0025	500	0.225
0.003	509	0.25225
0.005	544	1.053
0.01	625	2.9225
0.02	756	6.905
0.03	845	8.005
0.04	892	8.685
0.05	897	8.945
0.06	860	8.785
0.07	782	8.21
0.08	662	7.22
U_{MOT} = Total Area =		**61.6 MPa**

(1 pt)

From the given data, the residual strain $\varepsilon_P = \varepsilon_f - \sigma_f / E$ = 0.08 - 662 MPa / 200,000 MPa = 0.0767.

With that known, $U_{elastic} = (\varepsilon_f - \varepsilon_P) * \sigma_f / 2$ = (0.08 - 0.0767) * 662 MPa / 2 = **1.1 MPa** and $U_P = U_{MOT} - U_{elastic}$ = 61.6 MPa - 1.1 MPa = **60.5 MPa**. (0.5 pts each)