3.11 Fall 2003
Solutions Problem Set #2

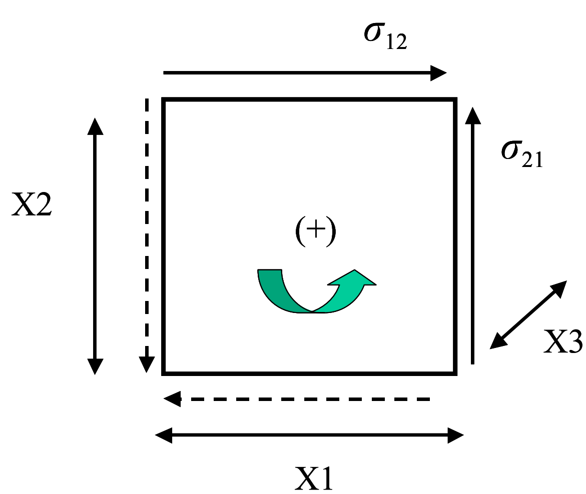
1. Show $\sigma ij = \sigma ji$

$\sigma_{12} = \sigma_{21}$

$\Sigma M_{12} = 0 = F_{12} * d + F_{21} * d$

d is moment arm.

Define system:



$\sigma = F/A$

$F_{12} = \sigma_{12}A \quad F_{21} = \sigma_{21}A$

Because shear stress acts on the plane where it lies, $F_{12} = \sigma_{12}x_2*x_3$

So, we get:

$\Sigma M_{12} = 0 = (\sigma_{12}x_2*x_3)*x_1 - (\sigma_{12}x_1*x_3)x_2$

2.

$$S'_{11} = S'_{22} = S'_{33} = \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)}$$

$$S'_{12} = S'_{13} = S'_{23} = S'_{21} = S'_{31} = S'_{32} = \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)}$$

$$S'_{44} = S'_{55} = S'_{66} = G = \frac{E}{2(1+\upsilon)}$$

$$S'_{ij} = \begin{bmatrix} \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0 \\ \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0 \\ \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} \end{bmatrix}$$

$D_{ij} = (S'_{ij})^T = (S'_{ij})$ because the matrix is symmetric.

$$D_{ij} = \begin{bmatrix} \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0 \\ \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0 \\ \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} \end{bmatrix}$$

$$= \frac{E}{(1-2\upsilon)(1+\upsilon)} \begin{bmatrix} 1-\upsilon & \upsilon & \upsilon & 0 & 0 & 0 \\ \upsilon & 1-\upsilon & \upsilon & 0 & 0 & 0 \\ \upsilon & \upsilon & 1-\upsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-2\upsilon)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\upsilon)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-2\upsilon)}{2} \end{bmatrix}$$

3. 7.2-1

$\sigma x1 = (\sigma x + \sigma y)/2 + (\sigma x-\sigma y)/2*\cos 2\theta + \tau xy \sin 2\theta = 5580psi$

$\sigma x1y1 = - (\sigma x-\sigma y)/2*\sin 2\theta + \tau xy\cos 2\theta = -3450$psi

$\sigma y1 = \sigma x + \sigma y - \sigma x1 = 3220$ psi

4. 7.2-6

$\sigma x1 = (\sigma x + \sigma y)/2 + (\sigma x-\sigma y)/2*\cos 2\theta + \tau xy\sin 2\theta = -1.5$MPa

$\sigma x1y1 = - (\sigma x-\sigma y)/2*\sin 2\theta + \tau xy\cos 2\theta = -17.8$ MPa

$\sigma y1 = \sigma x + \sigma y - \sigma x1 = -19.5$MPa

5. 7.5-3
Plane Stress

a) Normal Strain, εz
$\varepsilon z = -\nu/E (\sigma x + \sigma y)$ (from equation defined in chapter 7, Gere)
$\sigma x = E/ (1-\nu\hat{}2) * (\varepsilon x + \nu\varepsilon y)$
$\sigma y = E/ (1-\nu\hat{}2) * (\varepsilon y + \nu\varepsilon x)$
(From Definitions discussed in class)

Substitute σx and σy into the first equation and simplify:
$\varepsilon z = - \nu/(1-\nu) * (\varepsilon x + \varepsilon y)$

b)
$e = (1-2\nu)/E * (\sigma x + \sigma y)$ (equation defined in chapter)

Substitute σx and σy into equation and simplify.
$e = (1-2\nu)/(1-\nu) * (\varepsilon x + \varepsilon y)$

6.) 7.5-8 Biaxial Stress

$\sigma x=\sigma y= -P/b^2 = -75.0$MPa

Change in Volume
$e = ((1-2\nu)/E) (\sigma x +\sigma y) = -0.00048$
Vo = b^3 = (40mm)^3 = 64x10^3 mm^3
$\Delta V = Vo ((1-2\nu)/E) (\sigma x +\sigma y) = eVo = -30.7$mm^3
Decrease in Volume.

Strain Energy:
$\mu = 1/2E* (\sigma x^2 + \sigma y^2 -2\nu\sigma x\sigma y) = 0.03712$ MPa
($\tau xy = 0$)
$U = \mu Vo = 2.38$J