Problem Set #3 Solutions
3.11 Fall 2003

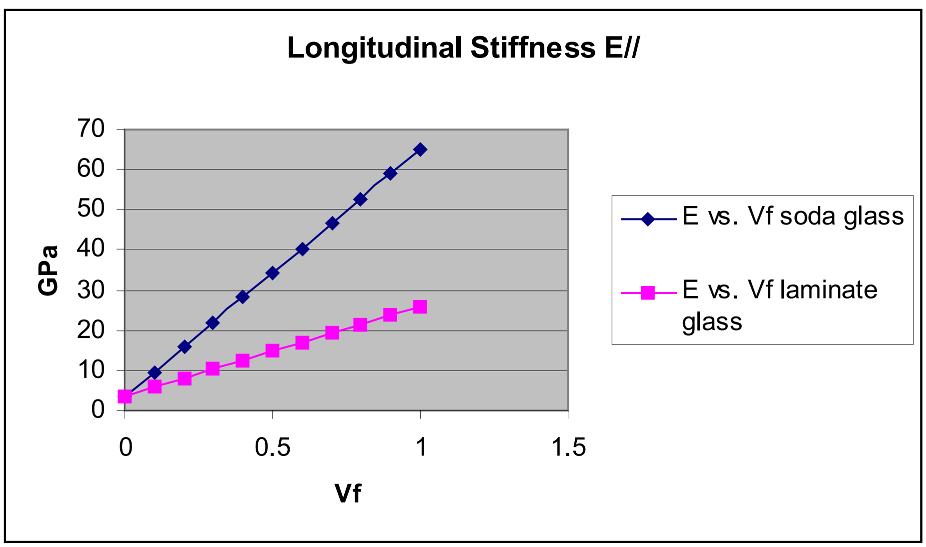
Note: For truss problems, you may verify your answer with MDSolids software, but you should solve the problem manually and show all work.

1. Plot the longitudinal stiffness E// of an E-glass (fiber)/epoxy (matrix) unidirectionally reinforced composite as a function of volume fraction. Necessary values of material properties can be downloaded from here: *http://web.mit.edu/course/3/3.11/www/modules/props.html*

$E// = E_f v_f + E_m v_m$

Can either pick random points to plug in different volumes, or plot in excel or similar program.

Plot E vs. v



2. A metal Matrix composite is made up of randomly dispersed silicon carbide particles (E = 450GPa) in an aluminum matrix (69GPa). Plot the upper and lower bounds for the Young's Modulus of the composite made using these materials, and from this, estimate the Young's Modulus of the composite with 25% volume fraction of the particles.

Solution:

The upper bound is given by the isostrain model such that:

$$E_{UB} = E_p V_p + E_m V_m = (450\,GPa)V_p + (69\,GPa)(1 - V_p)$$

$$E_{UB} = (381 \cdot V_p + 69)\,GPa$$

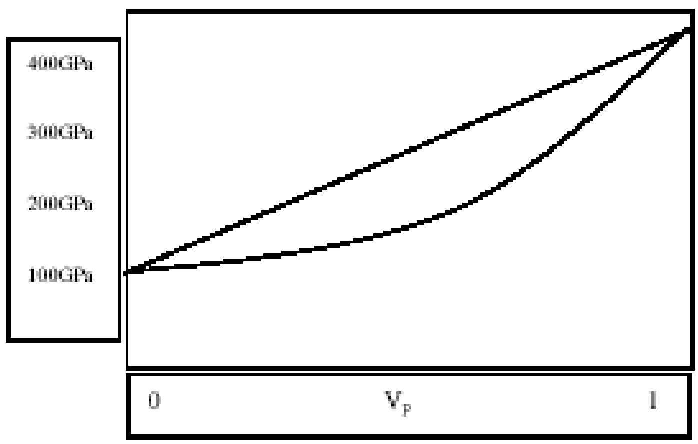
where the subscript p refers to the particle and m to the matrix

The lower bound is given by the isostress model

$$E_{LB} = \frac{E_p E_m}{E_p V_m + E_m V_p} = \frac{(450\,GPa)(69\,GPa)}{(450\,GPa)(1 - V_p) + (69\,GPa)V_p}$$

$$E_{LB} = \frac{10350}{150 - 127V_p}$$

These can be plotted



To estimate the modulus of a 25% volume fraction of particles composite, one should choosea value near the lower bound. Dispersed particle composites are generally wear composites.

$$E \approx 100\,GPa$$

3. Do Problem 2.2-8 from Gere. (Simple Truss Problem)

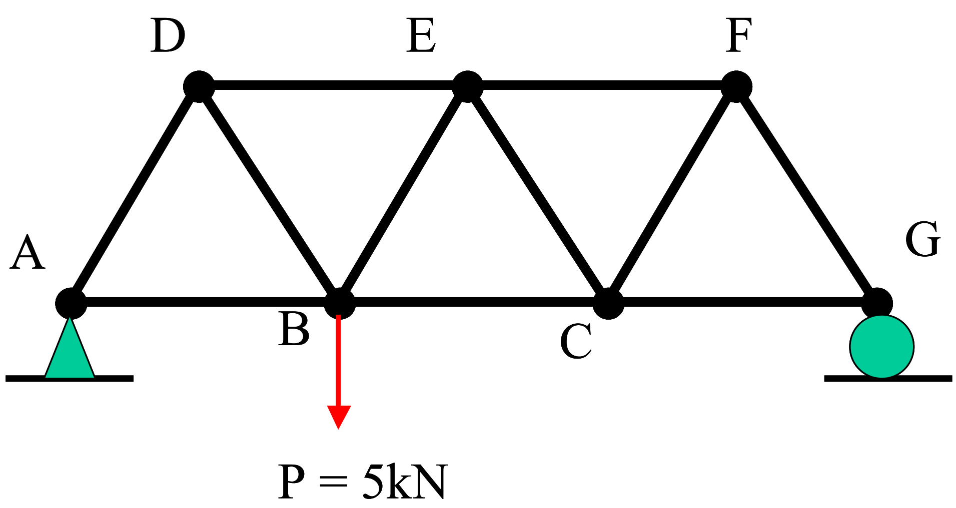
a.

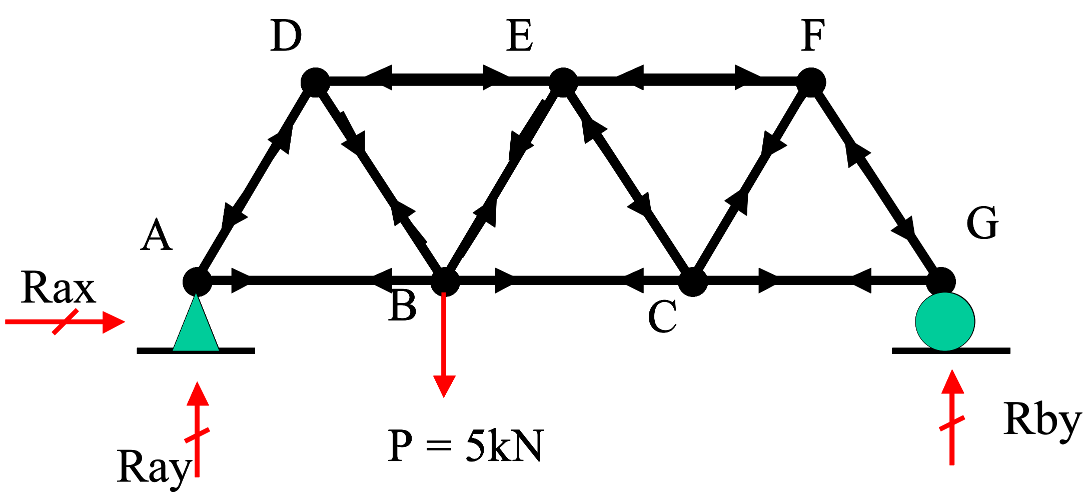
δ = PL/EA = [(120000N)(6m)]/[(200x10^9N/m^2)(0.003m^2)] = .0012m = 1.20mm

b.

P = δEA/L = (.002m)(200x10^9N/m^2)(.003m^2)/6m = 200,000 Pmax = 200kN

4. Calculate the forces in all members of the following truss (each member length is 1m), indicating whether each member is in tension or compression.





METHOD OF JOINTS:
Look at Force and Moment about A

ΣFxa = 0 = Rax

ΣFya = 0 = Ray + Rgy – 5kN

ΣMa = 0 = -1(5kN) + 3(Rgy)

Rax = 0

Rgy = 1.67 kN

Ray = 3.33kN

Joint A

ΣFx = Rax + Tadcos60 + Tab = 0

ΣFy = 0 = Ray + Tadsin60

$T_{ad} = -3.85kN$
$T_{ab} = 1.925kN$

Perform similar operations for Joint D, B, E, C and F.

Joint D

$\Sigma F_x = 0 = T_{de} + T_{db}\cos 60 - T_{ad}\cos 60$
$\Sigma F_y = 0 = -T_{bd}\sin 60 - T_{ad}\sin 60$

$T_{bd} = 3.85kN$
$T_{de} = -3.85kN$

Joint B

$\Sigma F_x = 0 = T_{bc} + T_{be}\cos 60 - T_{ab} - T_{bd}\cos 60$
$\Sigma F_y = 0 = T_{be}\sin 60 + T_{bd}\sin 60 - 5kN$

$T_{be} = 1.92kN$
$T_{bc} = 2.89kN$

Joint E

$\Sigma F_x = 0 = T_{ef} + T_{ec}\cos 60 - T_{de} - T_{be}\cos 60$
$\Sigma F_y = 0 = -T_{ec}\sin 60 - T_{be}\sin 60$

$T_{ec} = -1.92kN$
$T_{ef} = -1.92kN$

Joint C

$\Sigma F_x = 0 = T_{cg} + T_{cf}\cos 60 - T_{bc} - T_{ec}\cos 60$
$\Sigma F_y = 0 = T_{cf}\sin 60 + T_{ec}\sin 60$

$T_{cf} = 1.92kN$
$T_{cg} = 0.96kN$

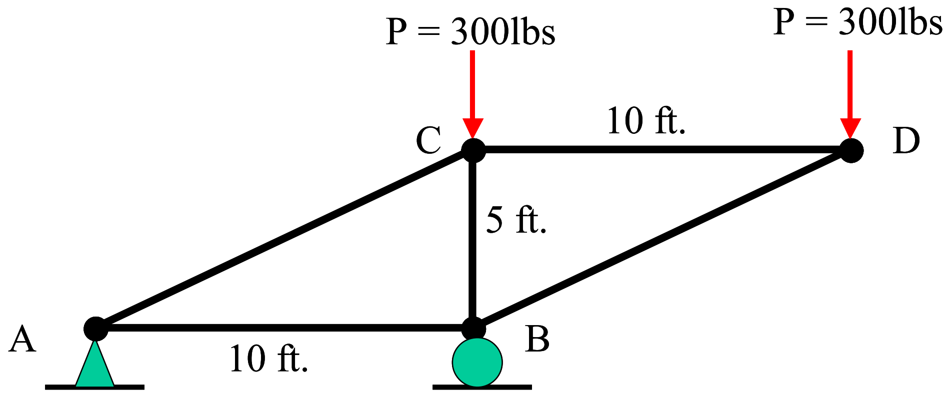
Joint F

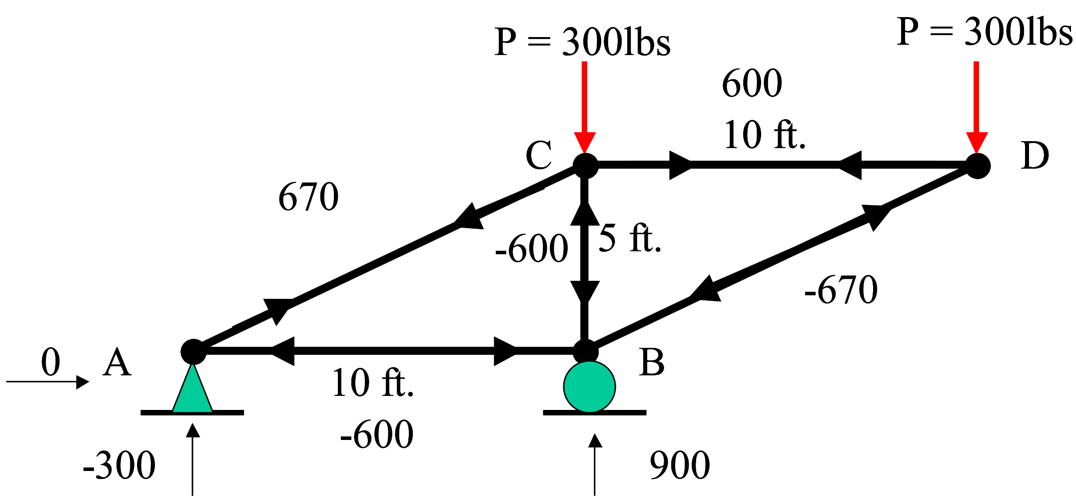
$\Sigma F_x = 0 = -T_{ef} - T_{cf}\cos 60 + T_{fg}\cos 60$
$\Sigma F_y = 0 = -T_{cf}\sin 60 - T_{fg}\sin 60$

$T_{fg} = -1.92kN$

Remember that Compression and Tension relate to the direction the force acts upon the joint, not the member. Therefore, if your $T_{fg} = -1.92kN$, your forces will be compressing the pins of f and g.

4. Calculate the forces in all members of the following truss, indicating whether each member is in tension or compression.





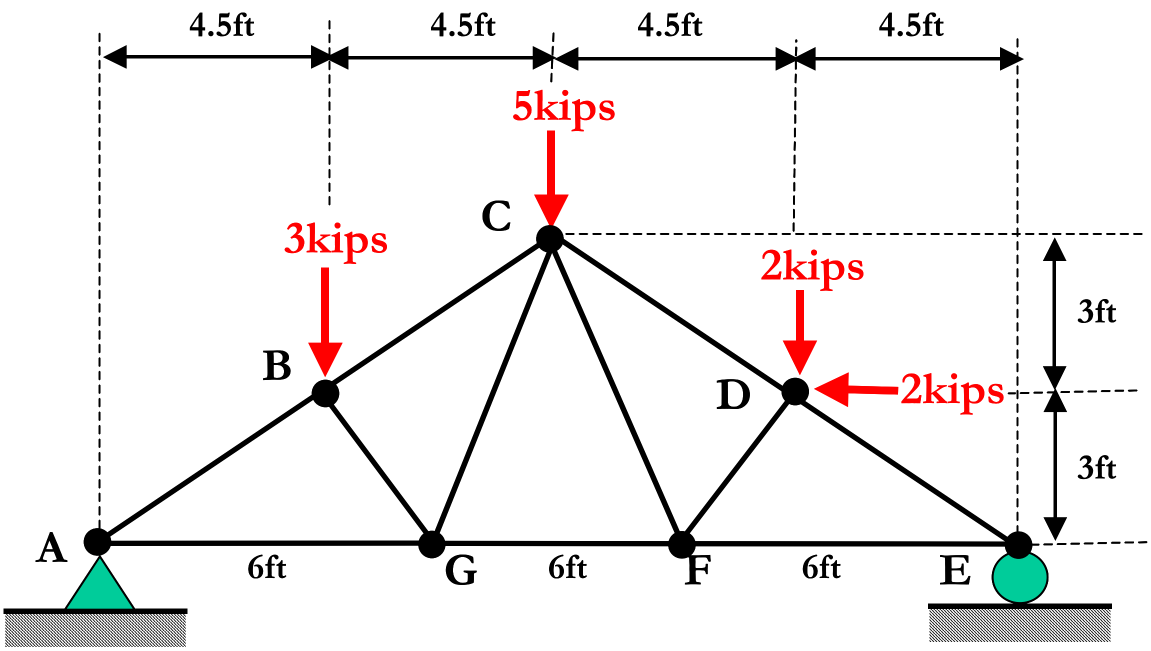
Angle at A = tan-1 (5/10) = 26.6°
Label Rax, Ray, Rby

Forces and Moment about A
ΣFx = 0 = Rax
ΣFy = 0 = Ray + Rby – 300 –300
ΣMa = 0 = 10(Rby) – 10(300) – 20(300)
Rby = 900lbs
Ray = -300lbs

Joint A.
ΣFx = 0 = Rax + Tab + Tac(cos(26.6))
ΣFy = 0 = Ray + Tac(sin(26.6))
Tac = 670lbs

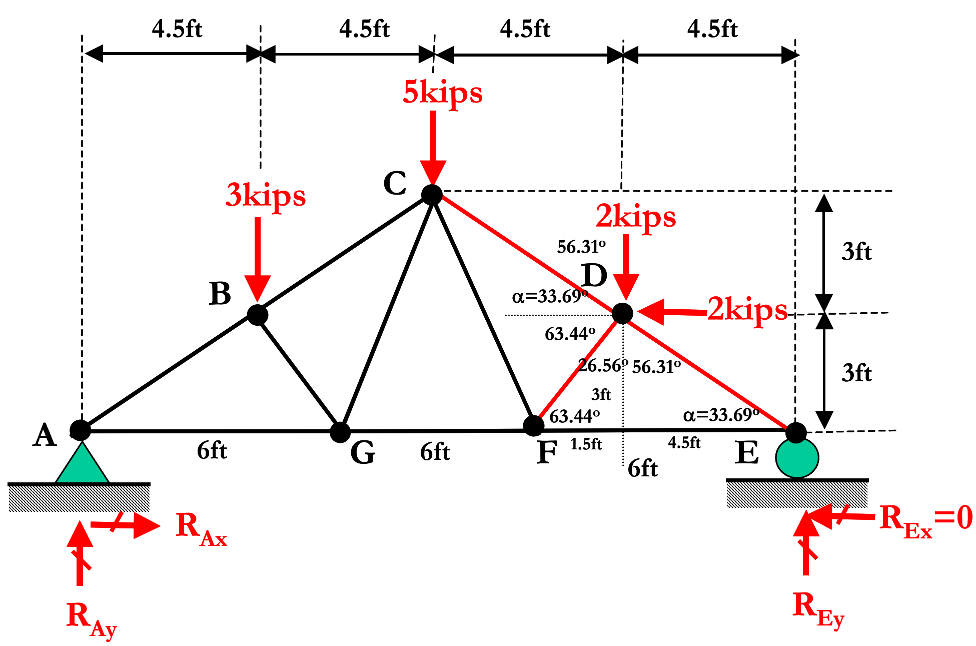
Perform similar operations for Joint B, C and D
$\Sigma Fx = 0$
$\Sigma Fy = 0$

5. Calculate the magnitude of the forces in member DE of the truss shown below. Indicating whether each member of the diagram is in tension or compression.



5 Solution <u>**METHOD OF JOINTS :**</u>

1. Draw a free-body diagram of the entire truss *(*shown below)*



2. Determine support reactions using the equations of static equilibrium
$\Sigma F_x=0$, $\Sigma F_y=0$, $\Sigma M_{xy}=0$ (CCW+, CW-) :

By definition : **R_{Ex}=0 kips**

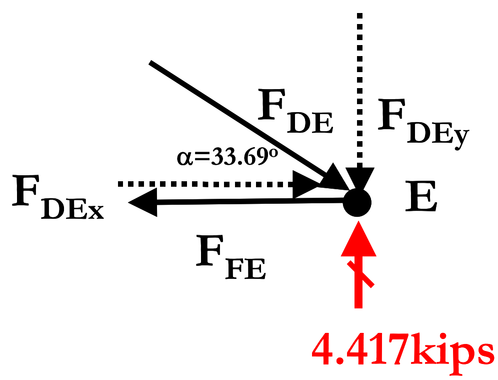
$\Sigma F_x=0= R_{Ax}+R_{Ex}-2$
R_{Ax}=2 kips

$\Sigma M_A=0= -3(4.5)-5(4.5)(2)-2(4.5)(3)+2(3)+R_{Ey}(4.5)(4)$
$\Sigma M_A=0= -13.5-45-27+6+ R_{Ey}18$
$\Sigma M_A=0= -79.5+ R_{Ey}18$
$R_{Ey}=79.5/18$
R_{Ey}=4.417 kips

$\Sigma F_y=0=-3-5-2+R_{Ey}+R_{Ay}= -10+4.417+R_{Ay}$
R_{Ay}=5.583 kips
3. Identify a joint where you know the maximum amount of forces (e.g. a support with two members).

Start at Joint E.

4. Draw a free-body diagram of the joint and determine whether forces are compressive or tensile. We know that there has to be a force in the downwards y-direction counteracting the upwards reaction force at the roller. Hence, F_{DE} must be in compression, which also indictaes that F_{FE} must be in tension to counteract F_{DEx}.



5. Write and solve equations of static equilibrium* for diagram drawn in step 4.
From geometry : $F_{DEx}=F_{DE}\cos\alpha=0.832F_{DE}$
$F_{DEy}=F_{DE}\sin\alpha=0.5546F_{DE}$
where : $\alpha=33.69$
$\Sigma F_y=0=4.417-F_{DEy}=4.417-0.5546F_{DE}$
F_{DE}=7.963 kips (COMPRESSION) *Ans.*