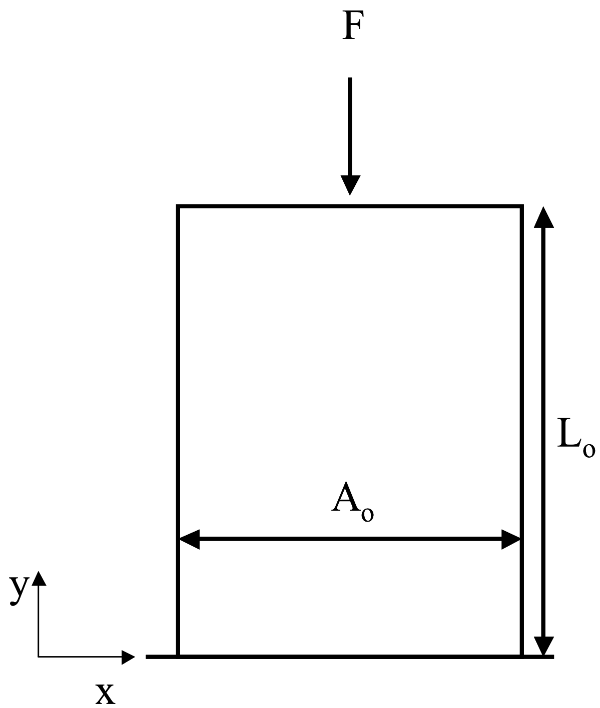
Problem Set #4
3.11 Fall 2003

1. Thermal Expansion Problem: Calculate the force needed to compress the steel bar shown below to Lo/2 if the bar is simultaneously heated up by 45°C ($E = 200GPa$, $\alpha_L = 12*10^{-6}/°C$, $L_o = 0.5m$, $A_o = 0.05m^2$)



$\sigma = F/A_o = E\varepsilon_\sigma \rightarrow \varepsilon_\sigma = F/EA_o$

$\varepsilon_{total} = \varepsilon_{thermal} + \varepsilon_\sigma = +\alpha_L\Delta T - F/EA_o$

$\varepsilon_{total} = \Delta L/L_o = -0.5L_o/L_o = -0.5$

$-0.5 = +\alpha_L\Delta T - F/EA_o$

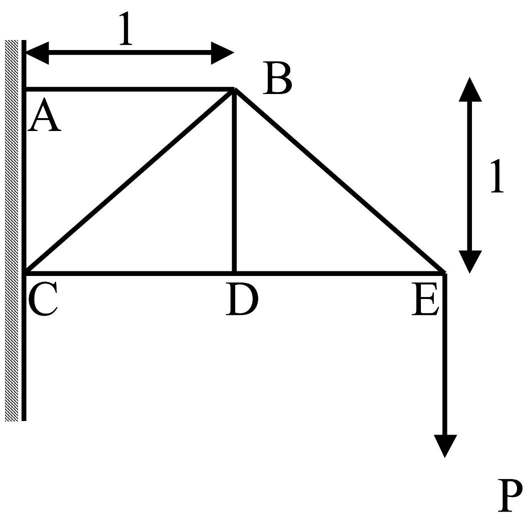
solve for: $F = [0.5 + \alpha_L\Delta T]EA_o$

substitute in numerical values:

$F = [0.5+(12*10^{-6}/°C*45°C)]200GPa*0.05m^2$

$F = 5.0054GN$

2. For the diagram below:



(a) Determine forces of all members in terms of P. Label whether or not members are in tension or compression. Show all work.

The first thing to do is to find the reaction forces at the support points A and C. They are both fixed supports, so we assume that both vertical and horizontal forces are present. Using the conditions of static equilibrium is required for this kind of problem. They are:

$$\Sigma M = 0$$
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

These conditions are always applicable for a system in static equilibrium.

Taking the sum of the moments around joint C, and assuming first that all reaction forces go either to the right or upwards, we obtain the relationship:

$- (2) P - (1) [F_A]_x = 0$

The rest of the reaction forces have no effect, since their distance to joint C is zero (there is no moment arm).

This gives us:

$$\mathbf{[F_A]_x = - 2P}$$

which means that the horizontal reaction force at joint A goes to the left, not to the right, as first assumed.

Next, by inspecting joint A, we find that there are two possible horizontal forces, $[F_A]_x$ and F_{AB}, and only one force that has a vertical component, $[F_A]_y$. Since static equilibrium conditions, which hold at all joints, require that the sum of all forces in the vertical direction be zero, the one force that has a vertical component must be zero. Thus,

$$[F_A]_y = 0$$

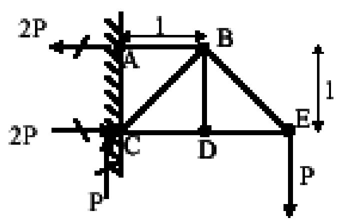
<u>This will be true at every joint where only one possible force has a vertical or horizontal component.</u>

Knowing this, going back to determining the reaction forces, we find with the sum of forces in the x and y-directions:

$\Sigma F_x = [F_A]_x + [F_C]_x = 0$
➔ $\mathbf{[F_C]_x} = - [F_A]_x = - (- 2P) = \mathbf{2P}$

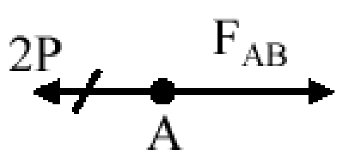
$\Sigma F_y = [F_C]_y + [F_A]_y - P = [F_C]_y + 0 - P = 0$
➔ $\mathbf{[F_C]_y = P}$

The FBD below is updated with the reaction forces in their correct directions.



Now we start to examine individual joints in order to solve the truss. Note that when writing static equilibrium equations, it will be assumed that forces pointing to the right or upwards are positive, and forces pointing to the left or downwards are negative.

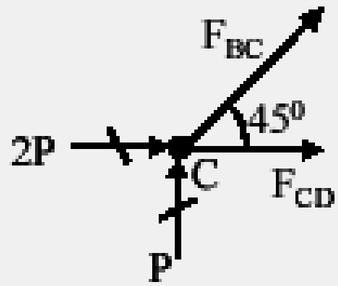
<u>Joint A:</u>



The only two forces acting here are horizontal, so static equilibrium dictates that the two forces are equal and opposite.

$\Sigma F_x = - [F_A]_x + F_{AB} = 0$ (force going right is positive, left is negative)
➔ $\mathbf{F_{AB} = [F_A]_x = 2P}$ **in tension** (F_{AB} is pulling away from the joint)

Joint C:

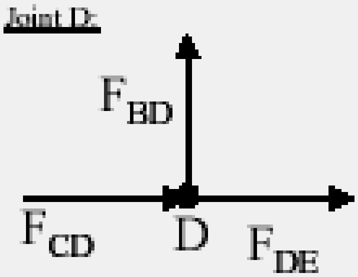


At point C, F_{BC} can be split into its horizontal and vertical components, with $[F_{BC}]_x = F_{BC} \cos 45°$ and $[F_{BC}]_y = F_{BC} \sin 45°$. That gives us only two vertical forces, $[F_C]_y$ (= P) and $[F_{BC}]_y$. Thus,

$\Sigma F_y = [F_C]_y + [F_{BC}]_y = [F_C]_y + F_{BC} \sin 45° = 0$
➔ $F_{BC} = - [F_C]_y / \sin 45° = - \sqrt{(2)}\, P$ (direction is opposite that previously assumed above)
➔ **$F_{BC} = \sqrt{(2)}\, P$ in compression** (actual direction of force is towards the joint)

$\Sigma F_x = [F_C]_x - [F_{BC}]_x + F_{CD} = 2P - \sqrt{(2)}\, P \cos 45° + F_{CD} = 0$
➔ $F_{CD} = - 2P + P = -P$
➔ **F_{CD} = P in compression**

Joint D:



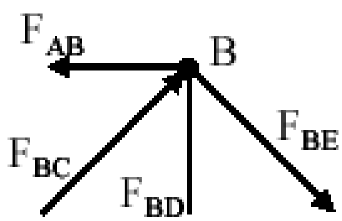
Note: the direction of F_{CD} is already known, since we know it is a compressive force.

Only one force with y-component, F_{BD} ➔ **$F_{BD} = 0$**.

Note: When the force in a member is zero, it means that member might as well not be there, and that wouldn't change anything in the truss. Of course, this is with the assumption that members are weightless. In the real world, practically any member in a truss bears some kind of load, be it only its weight.

$\Sigma F_x = F_{CD} + F_{DE} = P + F_{DE} = 0$
➔ $F_{DE} = -P$
➔ **F_{DE} = P in compression**

Joint B:



F_{BD} is zero, as previously determined.

$\Sigma F_y = [F_{BC}]_y - [F_{BE}]_y = F_{BC} \sin 45° - F_{BE} \sin 45° = 0$
➔ $F_{BE} = F_{BC} = \sqrt{(2)}\ P$
➔ **$F_{BE} = \sqrt{(2)}\ P$ in tension**

As a check, the summation of all forces in the x-direction should be zero.

$\Sigma F_x = - F_{AB} + [F_{BC}]_x + [F_{BE}]_x$
$= - 2P + F_{BC} \cos 45° + F_{BE} \cos 45°$
$= - 2P + \sqrt{(2)}\ P * \sqrt{(2)}/2 + \sqrt{(2)}\ P * \sqrt{(2)}/2$
$= - 2P + P + P =$ **0**

The forces in all members are summarized in the following table:

Member	Force	C or T
AB	2P	T
BC	sqrt(2) P	C
BD	0	---
BE	sqrt(2) P	T
CD	P	C
DE	P	C

Note: sqrt(A) = square root of A

(b) Calculate the total strain energy, $U_{T,}$ contained in the truss as a function of P, where the lengths are in meters, and the cross-sectional area of each member is 9^{-4} m^2. All members are made with a material with elastic modulus of 175GPa.

The strain energy U_T for a truss containing n members is given by the following expression:

$$U_T = \sum_i^n \frac{F_i^2 L_i}{2 A_i E_i}$$

where F_i is the magnitude of the load in the member, L_i is the length of the member, A_i is the cross-sectional area of the member and E_i is the elastic modulus of the member.
The cross-sectional area of every member and their modulus are the same throughout the truss, so we can factor them out of the summation. We then obtain:

U_T = 1/(2* 9^-4 m^2 * 175x10^9Pa) * [1m(2P)^2+√(2)m(√(2)P)^2+√(2)m(√(2)P)^2 + 1m(P)^2+1m(P)^2]

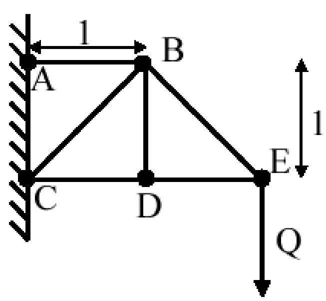
= 1/(2* 9^-4 m^2* 175x10^9Pa) * [6m + 4√(2)m]P^2

U_T = 2.185 x 10^{-7} [m/N]*P^2

(c) What is the displacement, δ, in joint E if load P = 45kN? (Use load Q in exchange for P at joint of interest, E.)

The displacement of a joint is given by:

$\delta = \frac{dU_T}{dQ} = \sum_i^n \frac{F_i L_i}{A_i E_i}\frac{dF_i}{dQ}$, where Q is a load that is applied at the joint of interest in the direction in which we are looking for the displacement. Therefore, in this case, the load Q applied vertically at joint E can replace the load P, as shown in the following diagram.

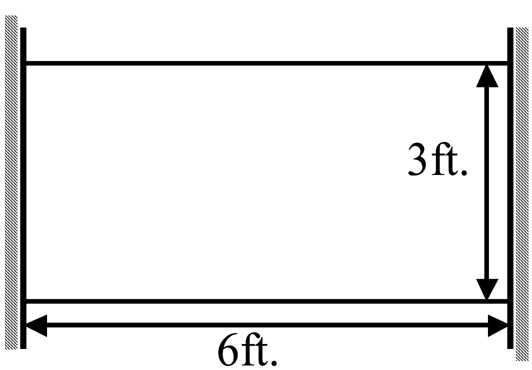


Finding the loads within each member with a load Q applied instead of P will yield the same expressions for all loads, except that Q replaces P. We can then use the results from the previous problem set in order to make up the following table:

Q in N	Member	Pi	dPi/dQ	Li/(AiEi)	PiLi(AiEi)(dPi/dQ)
45000	AB	2Q	2	1m/(AE)	
	BC	sqrt(Q)	sqrt2	sqrt2/(AE)	
	BD	2Q	0	1m/(AE)	
	BE	sqrt(Q)	sqrt2	sqrt2/(AE)	
	CD	Q	1	1m/(AE)	
	DE	Q	1	1m/(AE)	

Memberlength	Member	Pi	dPi/dQ	Li/(AiEi)	PiLi(AiEi)(dPi/dQ)	
1	AB	90000	2	3.74953E-08	0.006749156	
1.414213562	BC	63639.61	1.414214	5.30264E-08	0.004772374	
1	BD	90000	0	3.74953E-08	0	
1.414213562	BE	63639.61	1.414214	5.30264E-08	0.004772374	
1	CD	45000	1	3.74953E-08	0.001687289	
1	DE	45000	1	3.74953E-08	0.001687289	
				deflection =	0.019668483	meters

3. An aluminum alloy cylinder 3ft. in diameter and 6 ft. long is placed between two walls at room temperature (25°C) as shown below. Assuming no friction between the cylinder and walls, calculate the load (in lbs.) applied on the cylinder by the walls when it is heated up to 500°C.



Since the cylinder is constrained by the walls in the x-direction(horizontal direction), we can say that the sum of all the strains in that direction must be zero, i.e. $\varepsilon x = 0$. We know that there will be a thermal strain, εT due to the heating of the cylinder, and the cylinder will tend to expand. We also consider the fact that because of the expansion, the walls would exert a pressure back on the cylinder, so that there is a mechanical strain, εM associated with that pressure (stress). We can use the principle of linear superposition (i.e. the final state can be taken as the sum of the different contributions when those contributions are considered separately) to find that: $\varepsilon x = 0 = \varepsilon T + \varepsilon M$.

Using the definition of thermal strain, the stress-strain relationship and definition of stress, we find that

$\varepsilon T = \alpha L \Delta T$

$E \varepsilon M = \sigma$

$\sigma = F/A$

$F = A*E* (-\alpha L \Delta T)$

The area, $A = \pi (1.5ft)^2 * (144in^2/ft^2)$ E is 10,000ksi = $10x10^6$ psi, $\alpha L = 23x10^{-6}/C$, and $\Delta T = (500-25)C = 475C$. (Material Properties from Appendix H in Gere)

$F = -1.112x10^8$ lbs.

Or $1.112x10^8$lbs. in Compression