3.11 **Solutions** Problem Set # 6

Problem #1 Determine the maximum shear stress and rate of twist of the given shaft if a 10 kNm torque is applied to it. If the length of the shaft is 15 m, how much would it rotate by? Let G = 81 GPa, D = 75 mm

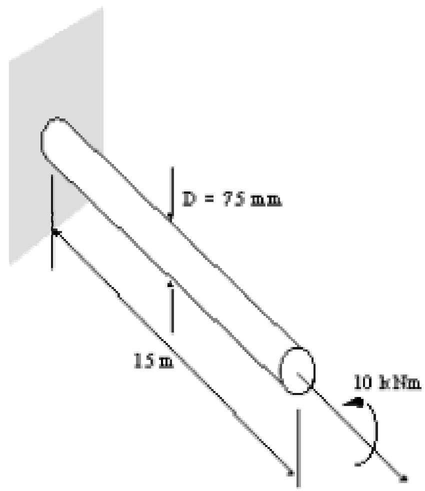


Figure 90: Shaft of example 13, subject to 10 kNm torque

$$J = \frac{\pi D^4}{32} = \frac{\pi\,(0.075)^4}{32} = 3.106 \times 10^{-6}\ m^4$$

$$\tau = \frac{Tr}{J} = \frac{10 \times 10^3\ x\ 0.0375}{3.1063 \times 10^{-6}} = 120.7\ MPa$$

$$\frac{d\theta}{dx} = \frac{T}{GJ} = \frac{10 \times 10^3}{81 \times 10^9\ x\ 3.1063 \times 10^{-6}} = 0.03974\ rad/m$$

Which equates to :

$$\frac{d\theta}{dx} = 2.277\ \theta/m$$

If the shaft is 15 m long, the angle of rotation at the free end is 34.157^o degrees.

Problem #2 GERE Problem 3.3-3
Aluminum Bar in Torsion

a) $k_T = GI_p/L = G\pi d^4/32L$ = (3.8e6psi)(π)(1.0in)^4/(32*48in) = 7770lb-in

b) $\Phi = 5^o = 5*\pi/180$ rad = 0.087rad
$\Phi = TL/GIp \rightarrow T = GIp\Phi/L$
τ max = Tr/Ip = Td/2Ip → plug in T, so
τ max = Gd Φ/2L = 3450psi

Max Shear Strain
Hooke's law: so $\gamma max = \tau max/G$ = 909e-6 rad

Problem #3 GERE Problem 3.4-2
Polar Moments of Inertia for AB, BC and CD
Calc I = π/32 * d^4
AB: I = 4.021e6mm^4
BC: I = 1.272e6
CD: I = 0.2513e6
AB
a) Shear Stresses
AB: $\tau = T_{AB}\, r_{AB}/Ip_{AB}$ = 57.7 MPa
BC: $\tau = T_{BC}\, r_{BC}/Ip_{BC}$ = 66.0 MPa
CD: $\tau = T_{CD}\, r_{CD}/Ip_{CD}$ = 63.7 MPa
τ max = 66.0Mpa

b) angle of twist
AB: $\Phi_{AB} = T_{AB}L_{AB}/G(Ip)_{AB}$ = 0.00902rad
BC: $\Phi_{BC} = T_{BC}L_{BC}/G(Ip)_{BC}$ = 0.01376rad
CD: $\Phi_{CD} = T_{CD}L_{CD}/G(Ip)_{CD}$ = 0.01990rad
$\Phi_D = \Phi_{AB} + \Phi_{BC} + \Phi_{CD}$ = 0.04268 rad = 2.45degrees

Stresses and Strains within a Beam

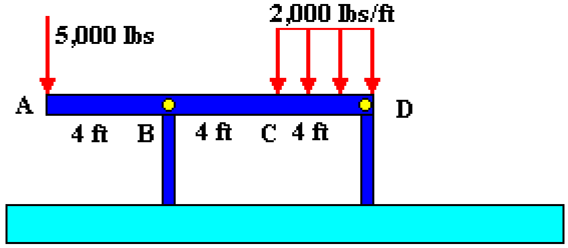
Problem #4 GERE Problem 5.5-3
a) Maximum Bending Stress
$\alpha*\rho = L$ so $\rho = L/\alpha$
$\sigma max = Ey/\rho = E(t/2)\alpha/L$ = 50,600psi
b) Change in stress
If the angle α increases, the stress σmax increases.

Problem #5 GERE Problem 5.5-6
Mmax = Pb
S = π/32 * d^3
σmax = Mmax/S = 32Pb/ (πd^3) = 185.0MPa

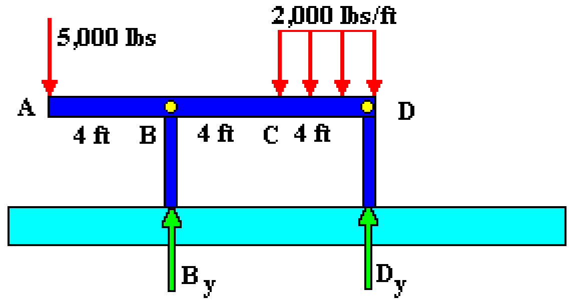
Problem #6 Horizontal Shear Stresses

STATICS & STRENGTH OF MATERIALS

A loaded, simply supported W 10 x 45 beam is shown below. For this beam:
A. Determine the maximum bending stress 6 feet from the left end of the beam.
B. Determine the horizontal shear stress at a point 4 inches above the bottom of the beam cross section and 6 feet from the left end of the beam.
Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.



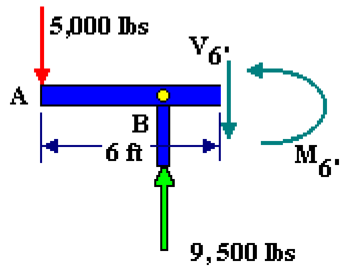
Solution:
Part A:
STEP 1: Determine the external support reactions:



1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:

Sum F_x = 0 none
Sum F_y = B_y + D_y - 2,000 lbs/ft (4 ft) - 5,000 lbs = 0
Sum T_B = 5,000 lbs (4 ft) - 2,000 lbs/ft (4 ft) (6 ft) + D_y(8 ft) = 0
Solving: B_y = 9,500 lbs; D_y = 3,500 lbs

STEP 2: Determine the shear force and bending moment at x=6 ft.
1.) Cut beam at 6 ft. Draw the FBD of left end of beam, showing and labeling all external forces.

2.) 

Resolve all forces into x/y directions.

3.) Apply equilibrium conditions:

Sum F_x = 0 none
Sum F_y = -5,000 lbs + 9,500 lbs - V_6 = 0
Sum T_A = 9,500 lbs (4 ft) - 4,500 lbs (6 ft) + M_6 = 0
Solving: V_6 = 4,500 lbs; M_6 = -11,000 ft-lbs

STEP 3: Apply the Flexure Formula to determine the Maximum Bending Stress (MBS) at 6'.

MBS = $M_{6'}$/S (Where $M_{6'}$ is the bending moment at 6 ft, and S is the section modulus for the beam. The section modulus is available from the Beam Tables. The W 10 x 45 beam has a section modulus for the beam from the beam tables is 49.1 in^3.)

MBS = **-11,000 ft-lbs(12 in/ft)/49.1 in^3** = **-2,688 lbs/in^2**

Part B:
STEP 4: To determine the Horizontal Shear Stress (HSS) at 6 ft from the end of the beam and 4 inches above the bottom of the beam, apply the horizontal shear stress formula.
The form we will use is: **HSS = Vay'/Ib**
Where:
V = Shear force 6 ft from the end of the beam
a = cross sectional area from 4 in above the bottom of the beam to bottom of beam
y' = distance from neutral axis to the centroid of area **a**
I = moment of inertia of the beam (249 in^4 for W 10 x 45 beam)
b = width of beam a 4 in above the bottom of the beam

HSS = [(4,500 lbs)(6.153 in^2)(4.37 in)]/[(249 in^4)(.35 in)] = 1,388 psi