Problem #1 Determine the maximum shear stress and rate of twist of the given shaft if a 10 kNm torque is applied to it. If the length of the shaft is 15 m, how much would it rotate by? Let $G = 81$ GPa, $D = 75$ mm

\[ J = \frac{\pi D^4}{32} = \frac{\pi (0.075)^4}{32} = 3.106 \times 10^{-6} \text{ m}^4 \]

\[ \tau = \frac{T \gamma}{J} = \frac{10 \times 10^3 \times 0.0375}{3.1063 \times 10^{-6}} = 120.7 \text{ MPa} \]

\[ \frac{d\theta}{dx} = \frac{T}{GJ} = \frac{10 \times 10^3}{81 \times 10^9 \times 3.1063 \times 10^{-6}} = 0.03974 \text{ rad/m} \]

Which equates to:

\[ \frac{d\theta}{dx} = 2.277 \text{ \theta/m} \]

If the shaft is 15 m long, the angle of rotation at the free end is $34.157^\circ$ degrees.
Problem #2  GERE Problem 3.3-3
Aluminum Bar in Torsion

a) \( k_T = G l_p / L = G \pi d^4 / 32L = (3.8 \times 10^6 \text{psi})(\pi)(1.0 \text{in})^4/(32 \times 48 \text{in}) = 7770 \text{lb-in} \)

b) \( \Phi = 5^\circ = 5^\circ \pi / 180 \text{ rad} = 0.087 \text{ rad} \)
\( \Phi = TL / G l_p \rightarrow T = G l_p \Phi / L \)
\( \tau_{\text{max}} = T r / l_p = T d / 2 l_p \rightarrow \text{plug in } T, \text{ so} \)
\( \tau_{\text{max}} = G d \Phi / 2L = 3450 \text{ psi} \)

Max Shear Strain
Hooke’s law: so \( \gamma_{\text{max}} = \tau_{\text{max}} / G = 909 \times 10^{-6} \text{ rad} \)

Problem #3  GERE Problem 3.4-2
Polar Moments of Inertia for AB, BC and CD
Calc I = \( \pi / 32 * d^4 \)
AB: I = 4.021e6 mm^4
BC: I = 1.272e6
CD: I = 0.2513e6

AB
a) Shear Stresses
AB: \( \tau = T r_{AB} / I p_{AB} = 57.7 \text{ MPa} \)
BC: \( \tau = T r_{BC} / I p_{BC} = 66.0 \text{ MPa} \)
CD: \( \tau = T r_{CD} / I p_{CD} = 63.7 \text{ MPa} \)
\( \tau_{\text{max}} = 66.0 \text{ MPa} \)

b) angle of twist
AB: \( \Phi_{AB} = T r_{AB} L_{AB} / (G l_p)_{AB} = 0.00902 \text{ rad} \)
BC: \( \Phi_{BC} = T r_{BC} L_{BC} / (G l_p)_{BC} = 0.01376 \text{ rad} \)
CD: \( \Phi_{CD} = T r_{CD} L_{CD} / (G l_p)_{CD} = 0.01990 \text{ rad} \)
\( \Phi_D = \Phi_{AB} + \Phi_{BC} + \Phi_{CD} = 0.04268 \text{ rad} = 2.45 \text{ degrees} \)

Stresses and Strains within a Beam

Problem #4  GERE Problem 5.5-3
a) Maximum Bending Stress
\( \alpha \rho = L \) so \( \rho = L / \alpha \)
\( \sigma_{\text{max}} = E y / \rho = E (t/2) \alpha / L = 50,600 \text{ psi} \)

b) Change in stress
If the angle \( \alpha \) increases, the stress \( \sigma_{\text{max}} \) increases.

Problem #5  GERE Problem 5.5-6
\( M_{\text{max}} = P b \)
\( S = \pi / 32 * d^3 \)
\( \sigma_{\text{max}} = M_{\text{max}} / S = 32 P b / (\pi d^3) = 185.0 \text{ MPa} \)
Problem #6 Horizontal Shear Stresses

STATICS & STRENGTH OF MATERIALS

A loaded, simply supported W 10 x 45 beam is shown below. For this beam:

A. Determine the maximum bending stress 6 feet from the left end of the beam.

B. Determine the horizontal shear stress at a point 4 inches above the bottom of the beam cross section and 6 feet from the left end of the beam.

Unless otherwise indicated, all joints and support points are assumed to be pinned or hinged joints.

Solution:

Part A:

STEP 1: Determine the external support reactions:

1.) FBD of structure (See Diagram)
2.) Resolve all forces into x/y components
3.) Apply equilibrium conditions:

Sum $F_x = 0$ none

Sum $F_y = B_y + D_y - 2,000 \text{ lbs/ft (4 ft)} - 5,000 \text{ lbs} = 0$

Sum $T_B = 5,000 \text{ lbs (4 ft)} - 2,000 \text{ lbs/ft (4 ft)} (6 \text{ ft}) + D_y (8 \text{ ft}) = 0$

Solving: $B_y = 9,500 \text{ lbs}$; $D_y = 3,500 \text{ lbs}$

STEP 2: Determine the shear force and bending moment at $x=6 \text{ ft}$.

1.) Cut beam at 6 ft. Draw the FBD of left end of beam, showing and labeling all external forces.
2.) Resolve all forces into x/y directions.
3.) Apply equilibrium conditions:

\[
\begin{align*}
\text{Sum } F_x &= 0 \text{ none} \\
\text{Sum } F_y &= -5,000 \text{ lbs} + 9,500 \text{ lbs} - V_6 = 0 \\
\text{Sum } T_A &= 9,500 \text{ lbs (4 ft)} - 4,500 \text{ lbs (6 ft)} + M_6 = 0 \\
\text{Solving: } V_6 &= 4,500 \text{ lbs; } M_6 = -11,000 \text{ ft-lbs}
\end{align*}
\]

STEP 3: Apply the Flexure Formula to determine the Maximum Bending Stress (MBS) at 6'.

\[
\text{MBS} = \frac{M_6}{S}
\]

(Where \( M_6 \) is the bending moment at 6 ft, and \( S \) is the section modulus for the beam. The section modulus is available from the Beam Tables. The W 10 x 45 beam has a section modulus for the beam from the beam tables is 49.1 in\(^3\).)

\[
\text{MBS} = \frac{-11,000 \text{ ft-lbs}(12 \text{ in/ft})}{49.1 \text{ in}^3} = -2,688 \text{ lbs/in}^2
\]

Part B:
STEP 4: To determine the Horizontal Shear Stress (HSS) at 6 ft from the end of the beam and 4 inches above the bottom of the beam, apply the horizontal shear stress formula.
The form we will use is: \( \text{HSS} = \frac{Vay'}{lb} \)
Where:
\( V = \) Shear force 6 ft from the end of the beam
\( a = \) cross sectional area from 4 in above the bottom of the beam to bottom of beam
\( y' = \) distance from neutral axis to the centroid of area \( a \)
\( l = \) moment of inertia of the beam (249 in\(^4\) for W 10 x 45 beam)
\( b = \) width of beam a 4 in above the bottom of the beam

\[
\text{HSS} = \frac{[(4,500 \text{ lbs})(6.153 \text{ in}^2)(4.37 \text{ in})]}/[(249 \text{ in}^4)(.35 \text{ in})] = 1,388 \text{ psi}
\]