## I. NMOS Inverter with Resistor Pull-Up



## II. NMOS Inverter with Current-Source Pull-Up

## A. Motivation

- With the resistor pull-up we could increase R to sharpen transfer characteristic BUT it slows down inverter operation.


## B. Idealized Current-Source "Pull-Up"



- Incremental resistance can be large --> high small-signal gain
- Current is large ---> Fast transitions


## C. NMOS Inverter with Current-Source Pull-Up

- Replace resistor with current source

- Find the voltage transfer curve graphically by superimposing $i_{S U P}$ Vs. $v_{O U T}$ (load line) on top of the drain characteristics



## D. Load Line Analysis

- Voltage transfer curve is much closer to that of the ideal inverter



## E. PMOS as a Current-Source Pull-Up

- Substitute p-channel MOSFET (with bulk connected to the source to make $V_{S B}=0$ and source connected to the supply voltage)
- The bias voltage $V_{B}$ is selected so that the appropriate source-gate voltage drop is obtained: $V_{S G 2}=V_{D D}-V_{B}$. The supply current $i_{S U P}$ as a function of the supply voltage $v_{S U P}=V_{S D 2}=V_{D D}-V_{O U T}$ is:

(a)

(b)


- In order to find the slope at $V_{I N}=V_{M}$, we note that both transistors are saturated there (near point 3) and that our small-signal models from Chapter 4 are valid


## F. Calculate midpoint - $\mathbf{V}_{\mathbf{M}}$

- $V_{M}$ is the input voltage $V_{I N}$, where the output voltage $V_{O U T}=V_{I N}$
- Both transistors are saturated
- Equate drain currents, omitting the channel length modulation terms

$$
\begin{gathered}
I_{D n}=\frac{1}{2} \mu_{n} C_{o x}\left(\frac{W}{L}\right)_{n}\left(V_{M}-V_{T_{n}}\right)^{2} \\
-I_{D p}=\frac{1}{2} \mu_{p} C_{o x}\left(\frac{W}{L}\right)_{p}\left(V_{D D}-V_{B}+V_{T p}\right)^{2}
\end{gathered}
$$

## G. Find slope of transfer characteristic at $\mathbf{V}_{\mathbf{M}}$



## III. Complementary MOS (CMOS) Inverter

## A. Motivation

- When input is high, the n -channel MOSFET is in the triode region conducting $I_{D}=I_{L}$ from $V_{D D}$ to ground.
- Power dissipation of $P=I_{L} \times V_{D D}$
- Need a current source load which itself is switchable -- turning itself off when the output is low.


## B. Concept


(a)

(b)

(c)

## C. Practical realization:

- Connect input to gate of p-channel device.
- $V_{I N}=V_{D D}{ }^{-->} V_{S G 2}=V_{D D}-V_{I N}=0\langle | V_{T p} \mid$--> cutoff
- $V_{I N}=0-->V_{S G 2}=V_{D D}-V_{I N}=V_{D D} \gg\left|V_{T p}\right|-->$ on (triode region)
- NMOS transistor is off when PMOS is in triode region
- PMOS transistor is off when NMOS is in triode region


## D. CMOS Transfer Characteristic

- Plotting the p-channel load on the n-channel "driver's" drain characteristics allows us to find the input-output voltage pairs which satisfy the constraint
- $I_{D n}=-I_{D p}$


(a)

(b)
- We can arrange to have the upper and lower noise margins be identical, if the transistor geometries are selected appropriately


## III. Quantitative Noise Margins

## A. Calculate $V_{M}$

- $V_{M}$ is the input voltage $V_{I N}$, where the output voltage $V_{O U T}=V_{I N}$
- Both transistors are saturated
- Equate drain currents, omitting the channel length modulation terms

$$
\begin{gathered}
I_{D n}=\frac{1}{2} \mu_{n} C_{o x}\left(\frac{W}{L}\right)_{n}\left(V_{M}-V_{T n}\right)^{2} \\
-I_{D p}=\frac{1}{2} \mu_{p} C_{o x}\left(\frac{W}{L}\right)_{p}\left(V_{D D}-V_{M}+V_{T p}\right)^{2}
\end{gathered}
$$

- Let $k_{n}=\mu_{n} C_{o x}(W / L)_{n}$ and $k_{p}=\mu_{p} C_{o x}(W / L)_{p}$

$$
\frac{1}{2} k_{n}\left(V_{M}-V_{T n}\right)^{2}=\frac{1}{2} k_{p}\left(V_{D D}-V_{M}+V_{T p}\right)^{2}
$$

- Result:

$$
V_{M}=\frac{V_{T n}+\sqrt{\frac{k_{p}}{k_{n}}}\left(V_{D D}+V_{T p}\right)}{1+\sqrt{\frac{k_{p}}{k_{n}}}}
$$

## B. Small-Signal Model: CMOS Inverter at $V_{I N}=V_{M}$



- p-channel MOSFET small-signal source-gate voltage is $v_{s g 2}=-v_{i n}$



## C. Approximate Transfer Curve

- The small-signal gain (which is the slope of the transfer curve when the input is equal to the mid-point voltage) is:

$$
v_{o u t} / v_{i n}=-\left(g_{m n}+g_{m p}\right)\left(r_{o n} \| r_{o p}\right)=A_{v}
$$

- An inverter optimized for gain can have values in the 100 's.
- Real inverters have a channel length which is as small as possible (to minimize the area ... and maximize speed)
- Output resistance is lowered and a typical value is $v_{\text {out }} / v_{\text {in }}=-5$.

$$
\begin{gathered}
V_{I L}=V_{M}+\frac{V_{D D}-V_{M}}{A_{v}} \\
V_{I H}=V_{M}-\frac{V_{M}}{A_{v}}
\end{gathered}
$$



