I. Review Law of the Junction

A. Definitions for Excess Carriers *n'*, *p'*

- $n = n_o + n'$
- $n \equiv$ Total electron concentration (cm^{-3})
- $n_o \equiv$ Equilibrium electron concentration (cm^{-3})
- $n' \equiv$ Excess electron concentration (cm^{-3})
- $p = p_o + p'$
- $p \equiv$ Total hole concentration (cm^{-3})
- $p_o \equiv$ Equilibrium hole concentration (cm^{-3})
- $p' \equiv$ Excess hole concentration (cm^{-3})

B. Law of the Junction: Expressed as Excess Carriers

$$n_p(-x_p) = N_d e^{-\phi_B/V_{th}} e^{V_D/V_{th}} = n_{po} e^{V_D/V_{th}}$$

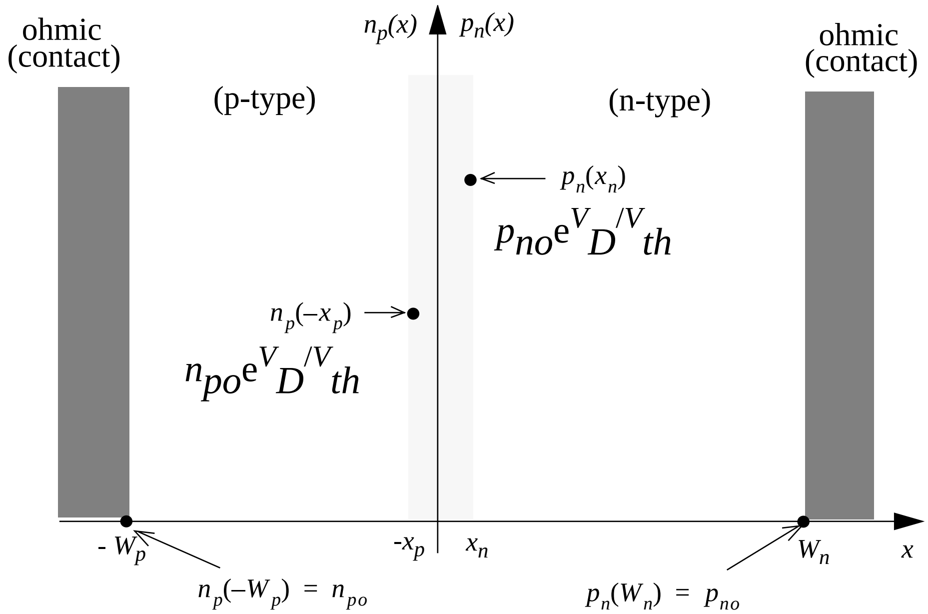
$$n_p'(-x_p) = n_{po}\,(e^{V_D/V_{th}} - 1)$$

$$p_n(x_n) = N_a e^{-\phi_B/V_{th}} e^{V_D/V_{th}} = p_{no} e^{V_D/V_{th}}$$

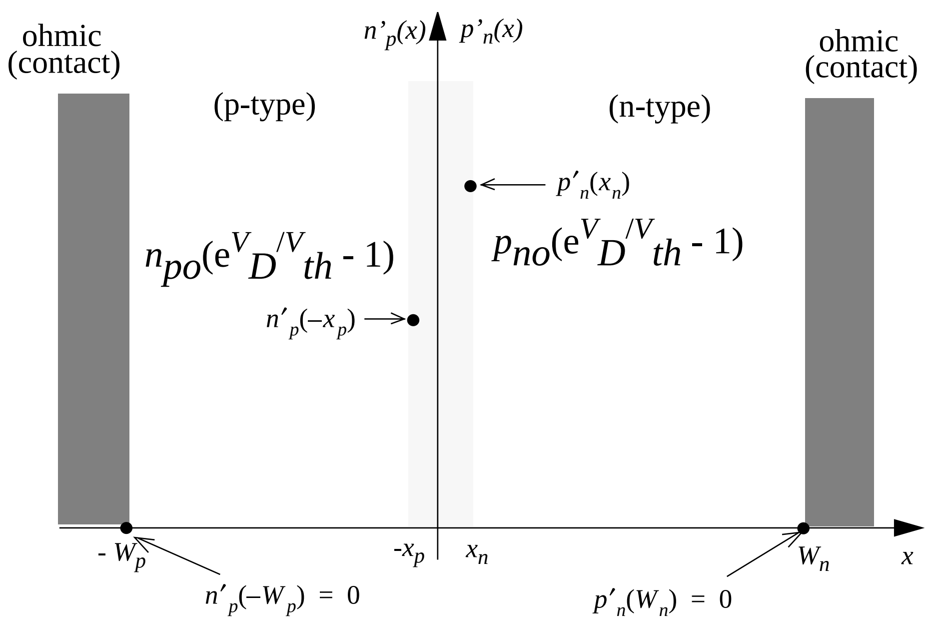
$$p_n'(x_n) = p_{no}\,(e^{V_D/V_{th}} - 1)$$

C. Qualitative View of Total and Excess Concentrations

- Total Carrier Concentration



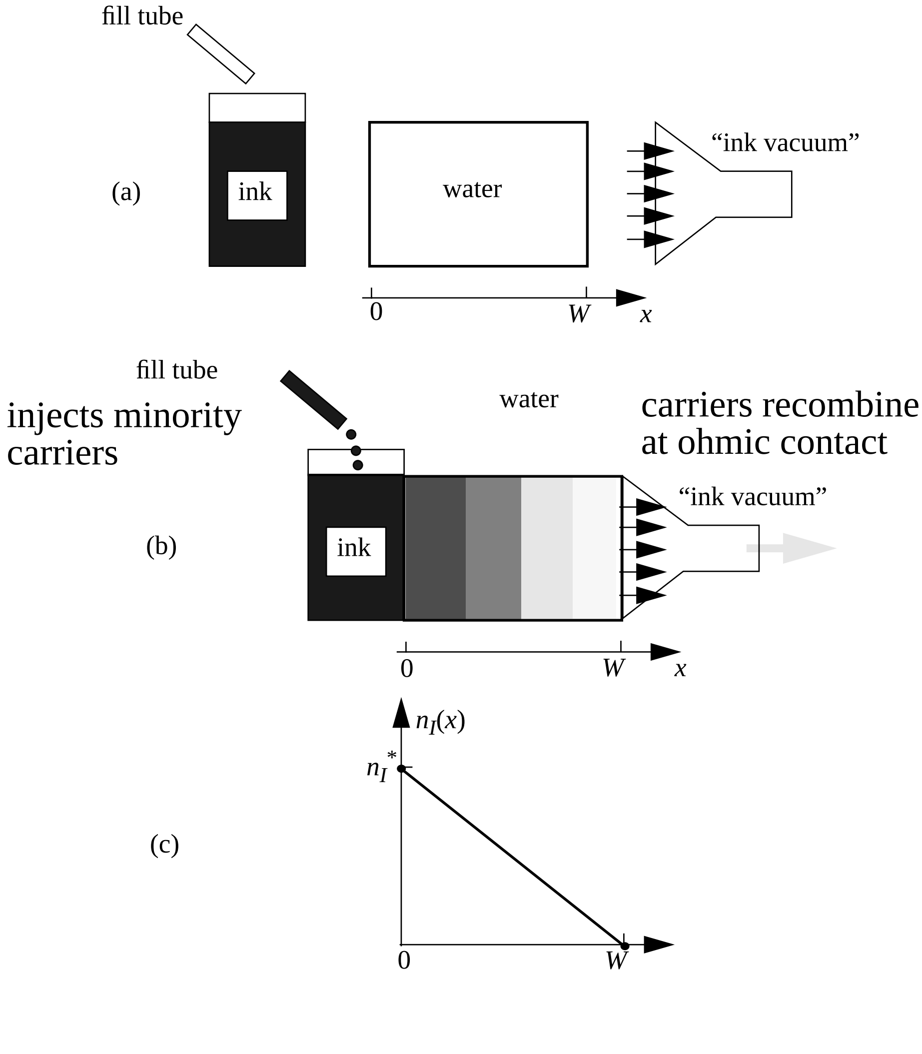
- Excess Carrier Concentration - Subtract n_{po} and p_{no}



II. Motivation for Steady-State Diffusion Equation

A. Ink Diffusion Example

- Flux is number of ink molecules passing a plane/cm^2-sec
- No molecules vanish in the water (NO RECOMBINATION)
- Ink concentration is a constant n_I^* at x=0
- Ink concentration is zero at x=W
- RESULT - Ink flux is constant - $F \propto d(ink)/dx$

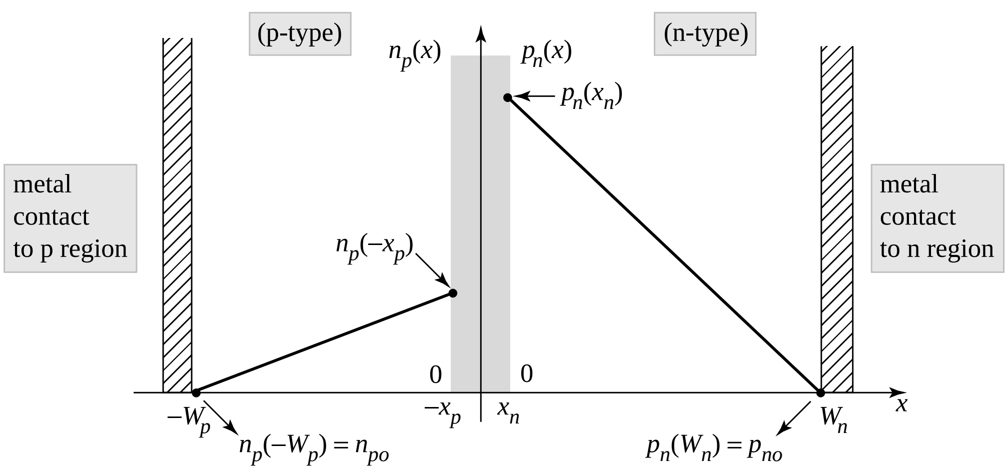


B. Equation for Minority Carrier Spatial Distribution

- Concentration is linearly decreasing from the depletion region edge to the ohmic contact. This expression assumes **no recombination** of minority carriers.

$$n'_p(x) = n'_p(-x_p) + \left(\frac{n'_p(-x_p)}{W_p - x_p}\right)(x + x_p)$$

$$p'_n(x) = p'_n(x_n) - \left(\frac{p'_n(x_n)}{W_n - x_n}\right)(x - x_n)$$



- Steady-state --> minority carriers must be continuously injected across the junction to keep $p_n(x_n) >> p_{no}$ and $n_p(-x_p) >> n_{po}$ while the same number is continously extracted at the ohmic contacts

C. Review Derivation Steps

- **Step 1**: Find the minority carrier concentrations at the edges of depletion region as a function of forward bias V_D
- **Step 2**: Find the minority carrier concentration at the ohmic contacts. All excess carriers **recombine** at ohmic contacts. The carrier concentrations return to their equilibrium value.
- **Step 3**: Find the spatial distribution of the minority carrier concentrations, $n_p(x)$, (electrons in the p region), and $p_n(x)$, (holes in the n region.)

- We need to do steps 4 and 5
- Step 4: Find the minority carrier diffusion currents at the edges of the depletion region.
- Step 5: Find the total diode current $J = J_n^{diff}(-x_p) + J_p^{diff}(x_n)$

III. Minority Carrier Diffusion Currents

A. Transport of minority carriers by diffusion

- Gradient in minority carrier concentrations across the n & p quasi-neutral regions

$$n = n_o + n' \rightarrow \frac{dn}{dx} = \frac{dn'}{dx}$$

- Transport occurs by *diffusion*. since number is small

B. Evaluate Minority Carrier Diffusion Currents at Edges of Space Charge Region

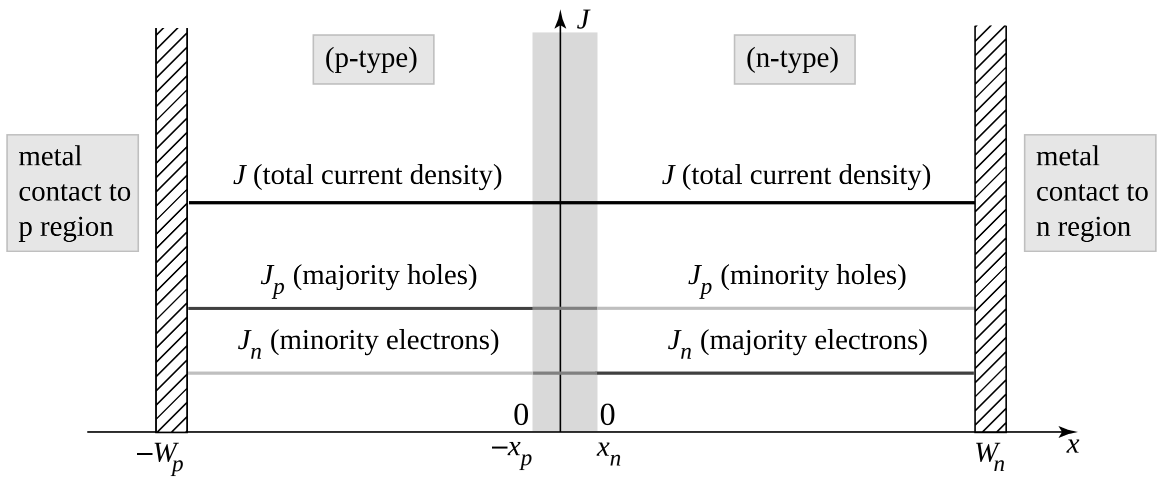
- At x_n: $J_p^{diff} = - qD_p dp_n / dx$ = constant --> $p_n(x)$ is *linear*
- At $-x_p$: $J_n^{diff} = qD_n dn_p / dx$ = constant --> $n_p(x)$ is *linear*
- $J = J_p^{diff} + J_n^{diff}$

$$J_n = qD_n \frac{dn'_p}{dx} = qD_n \left(\frac{n'_p(-x_p) - 0}{W_p - x_p} \right) = \left(\frac{qD_n n_{po}}{W_p - x_p} \right) \left(e^{V_D / V_{th}} - 1 \right)$$

$$J_p = -qD_p \frac{dp'_n}{dx} = -qD_p \left(\frac{0 - p'_n(x_n)}{W_n - x_n} \right) = \left(\frac{qD_p p_{no}}{W_n - x_n} \right) \left(e^{V_D / V_{th}} - 1 \right)$$

C. Picture of Total Diode Current

- Minority carriers are injected from the other side of the junction
- Minority carriers diffuse from depletion region to the ohmic contact with no recombination
- Excess majority carriers have the same spatial distribution as minority carriers ***n' = p'***
- Majority carriers are transported to the junction from the ohmic contact by **drift and diffusion**
- To find total current sum the diffusion currents at the depletion region edge and assume the current across the depletion region is constant



$$J = J_n + J_p = \left(\frac{qD_n n_{po}}{W_p - x_p} + \frac{qD_p p_{no}}{W_n - x_n}\right)\left(e^{V_D/V_{th}} - 1\right) = J_s\left(e^{V_D/V_{th}} - 1\right)$$

IV. Large Signal Model for PN Juntion Diode

$$I = I_s\left(e^{V_D/V_{th}} - 1\right)$$

where

$$I_s = qAn_i^2\left(\frac{D_n}{N_a(W_p - x_p)} + \frac{D_p}{N_d(W_n - x_n)}\right)$$

V. Small Signal Model - Forward Biased

A. DC

$$i_D = I_s e^{v_D/V_{th}} \quad \text{-->} \quad I_D + i_d = I_s e^{(V_D + v_d)/V_{th}}$$

$$I_D + i_d = I_s e^{V_D/V_{th}} e^{v_d/V_{th}} = I_D e^{v_d/V_{th}}$$

$$I_D + i_d = I_D\left(1 + \frac{v_d}{V_{th}} + \frac{1}{2}\left(\frac{v_d}{V_{th}}\right)^2 + \frac{1}{6}\left(\frac{v_d}{V_{th}}\right)^3 + \ldots\right)$$

$$i_d \cong \left(\frac{I_D}{V_{th}}\right)v_d = g_d v_d$$

B. Capacitance - Depletion

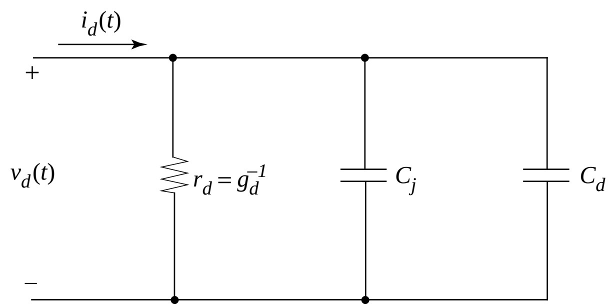
- Zero Biased Depletion Region Capacitance

$$C_{jo} = A\sqrt{\frac{q\varepsilon_s N_a N_d}{2(N_a + N_d)\phi_B}}$$

- NOTE: The depletion approximation is not valid with forward bias since there is a large number of carriers flowing in the depletion region. Assume $V_D = \phi_B/2$ for forward bias.

$$C_j \cong \frac{C_{jo}}{\sqrt{1 - 1/2}} = \sqrt{2}C_{jo}$$

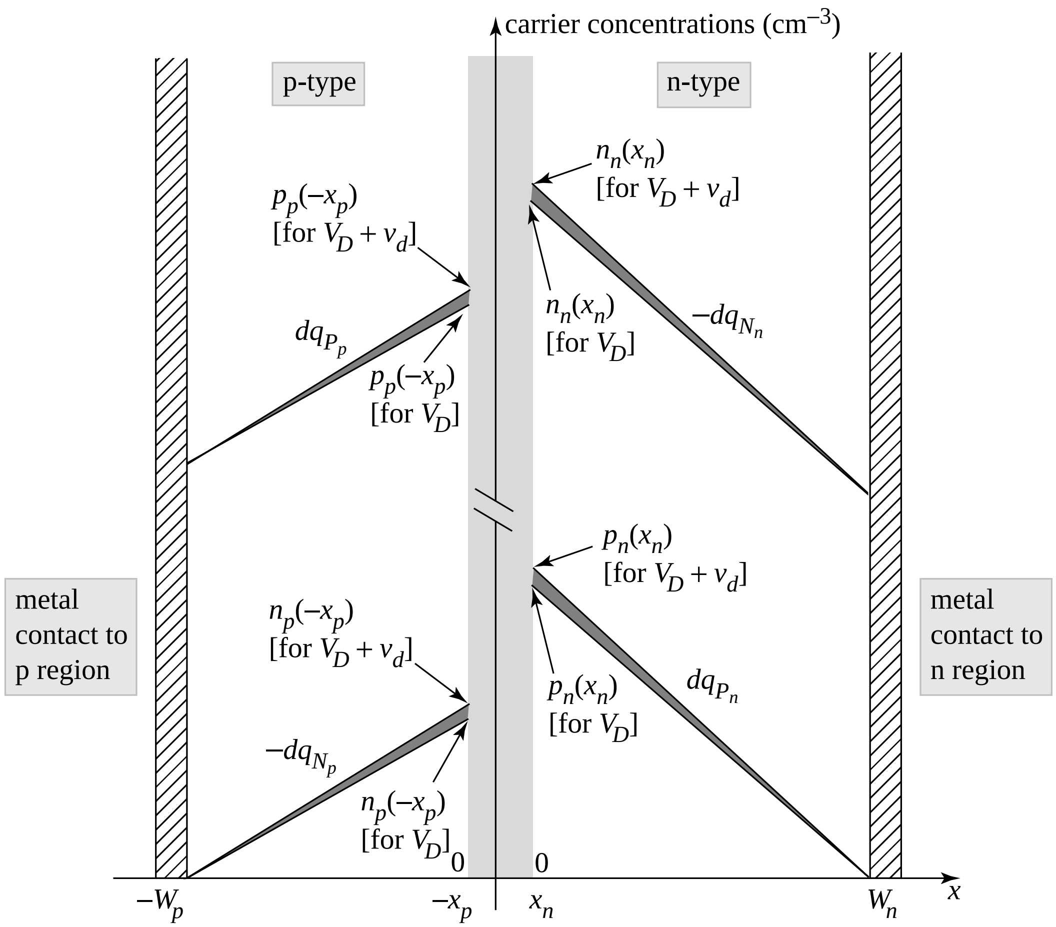
C. Small Signal Circuit Model



- Find Diffusion Capacitance: C_d

D. Capacitance - Diffusion

- Concept - dV_D changes minority **AND** majority concentration on **BOTH** sides of the junction. Majority carrier concentration follows minority carrier concentration. $\boldsymbol{p_n = n_n}$ **AND** $\boldsymbol{n_p = p_p}$



- **Quantitative Analysis**

$$q_N = A \int_{-W_p}^{-x_p} -qn'_p(x)dx = \frac{-qA}{2}\left(W_p - x_p\right)n_{po}\left(e^{v_D/V_{th}} - 1\right)$$

$$q_P = A \int_{x_n}^{W_n} qp'_n(x)dx = \frac{qA}{2}\left(W_n - x_n\right)p_{no}\left(e^{v_D/V_{th}} - 1\right)$$

$$C_d = \left.\frac{-dq_N}{dv_D}\right|_{V_D} + \left.\frac{dq_P}{dv_D}\right|_{V_D}$$

$$\boxed{C_d = \frac{qA}{2V_{th}}\left(\left(W_p - x_p\right)n_{po} + \left(W_n - x_n\right)p_{no}\right)e^{V_D/V_{th}}}$$