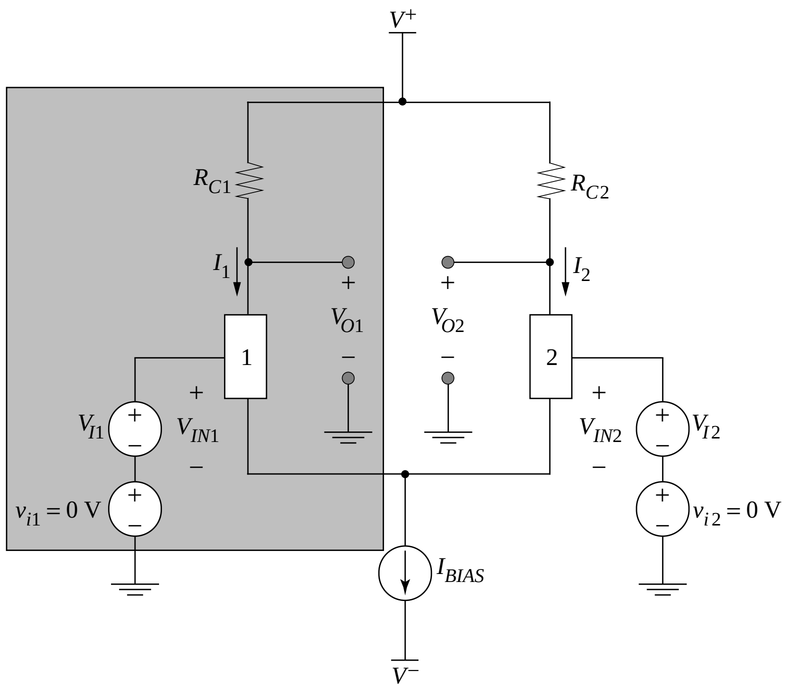
I. General Concepts for Differential Amplifiers

A. General Structure



B. Biasing

- Assume input current to device is 0; $V_{I1} = V_{I2}$; $R_{C1} = R_{C2} = R_C$

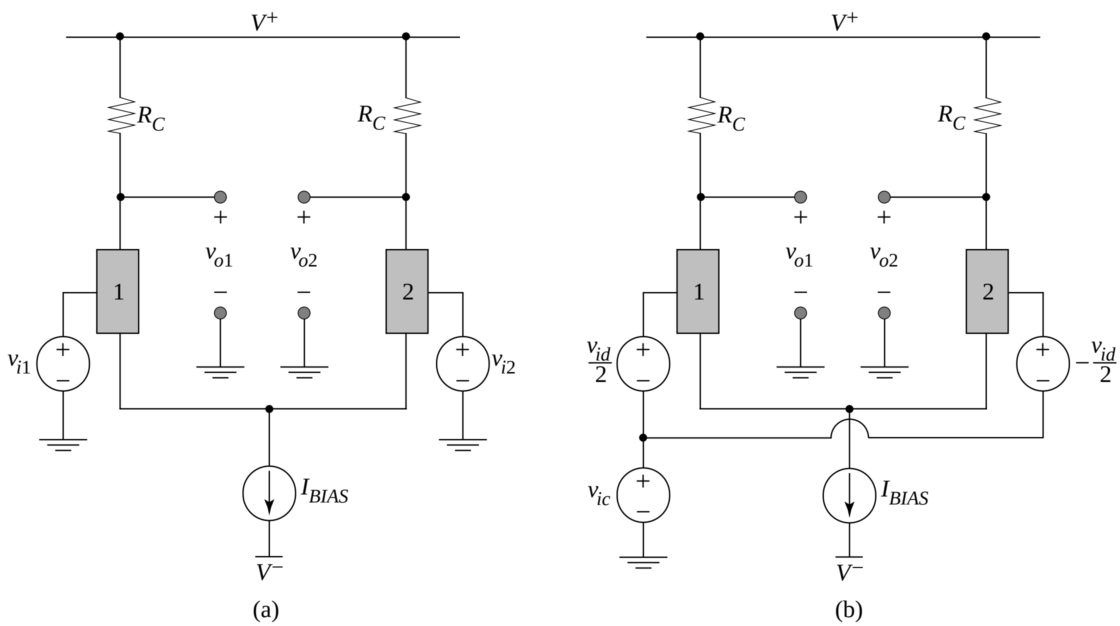
$$I_1 = \frac{V^+ - V_{O1}}{R_C} \approx \frac{V^+}{R_C}$$

$$I_2 = \frac{V^+ - V_{O2}}{R_C} \approx \frac{V^+}{R_C}$$

$$I_{BIAS} = I_1 + I_2 = 2V^+/R_C$$

- This condition yields $V_{O1} = V_{O2} = 0$ and $I_1 = I_2 = I_{BIAS}/2$

C. Differential and Common Mode Signals



- Write the input voltages in terms of the differential and common mode signals

$$v_{i1} = v_{ic} + v_{id}/2$$

$$v_{i2} = v_{ic} - v_{id}/2$$

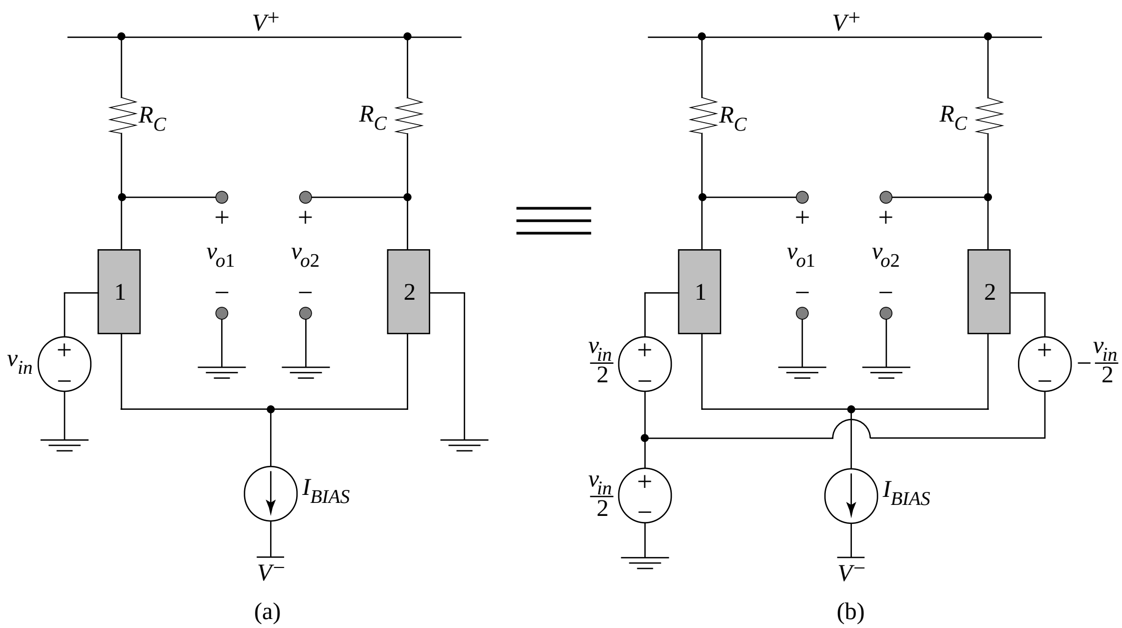
- Write the differential and common mode signals in terms of the input voltages

$$v_{id} = v_{i1} - v_{i2}$$

$$v_{ic} = (v_{i1} + v_{i2}) / 2$$

D. Example

- Decompose a single ended input voltage into its differential and common mode parts

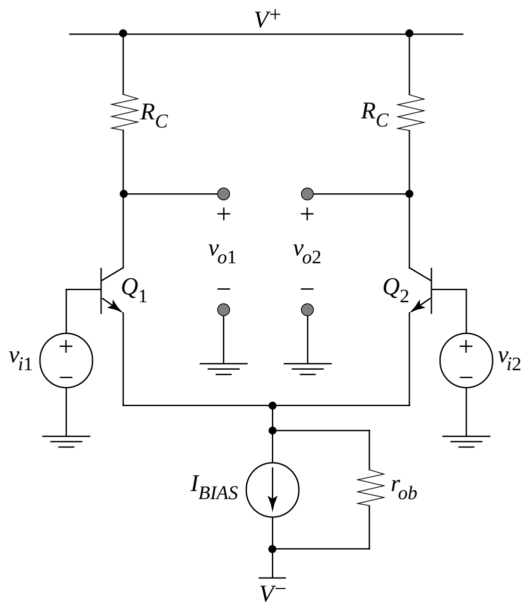


$$v_{id} = v_{in}$$

$$v_{ic} = v_{in}/2$$

II. Small Signal Analysis of the Differential Amplifier

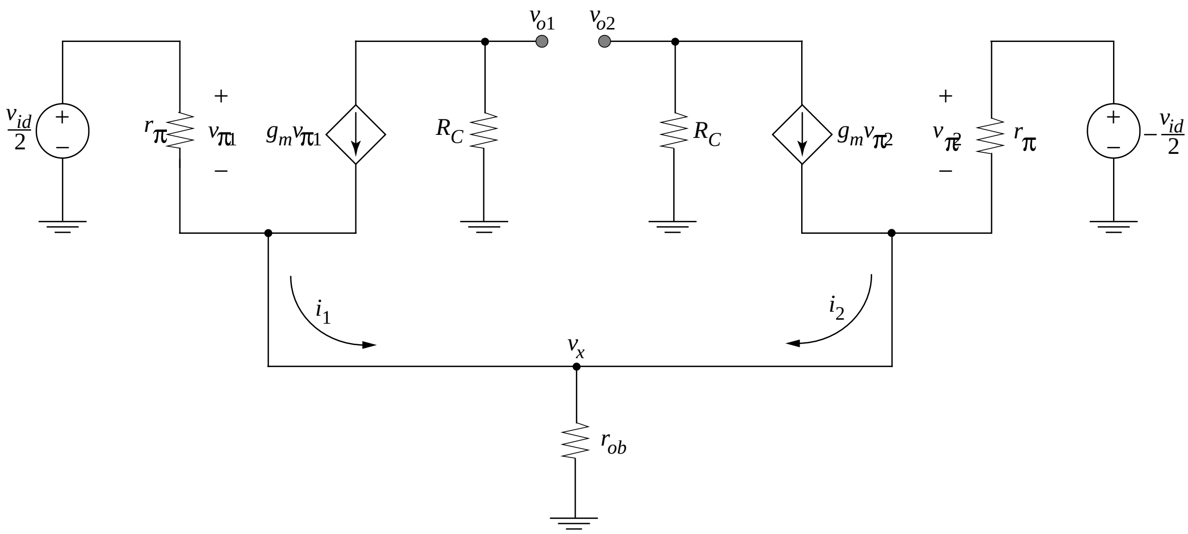
A. Bipolar Differential Amplifier with Resistors



- Set large signal bias voltage sources $V_{I1} = V_{I2} = 0\text{V}$
- Set $I_{BIAS} = 2V^{+}/R_C$ so that $V_{O1} = V_{O2} = 0$
- Finite output resistance of I_{BIAS} is modeled with r_{ob}

B. Small Signal Model for Purely Differential Mode Input

- $v_{i1} = v_{id}/2$, $v_{i2} = -v_{id}/2$, $v_{ic} = 0$

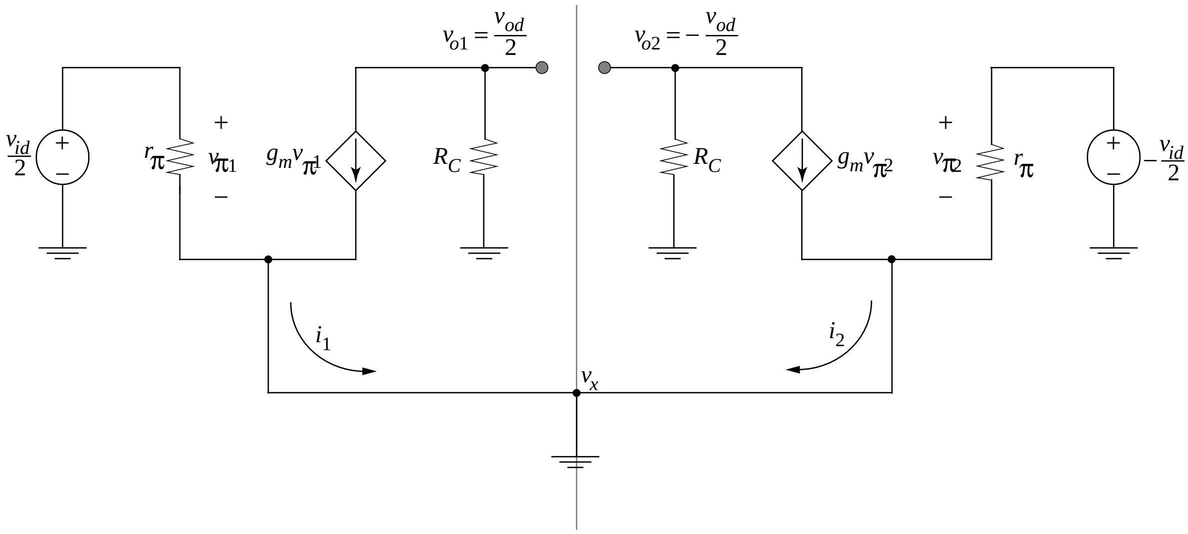


- Assume $r_o >> R_C$
- By symmetry $i_1 = -i_2$; $g_{m1} = g_{m2} = g_m$; $r_{\pi 1} = r_{\pi 2} = r_\pi$
- $v_x = 0$ for a purely differential mode signal

$$v_{o1} = -g_m R_C \frac{v_{id}}{2}$$

$$v_{o2} = g_m R_C \frac{v_{id}}{2}$$

C. Half Circuit Technique - Purely Differential Mode Input

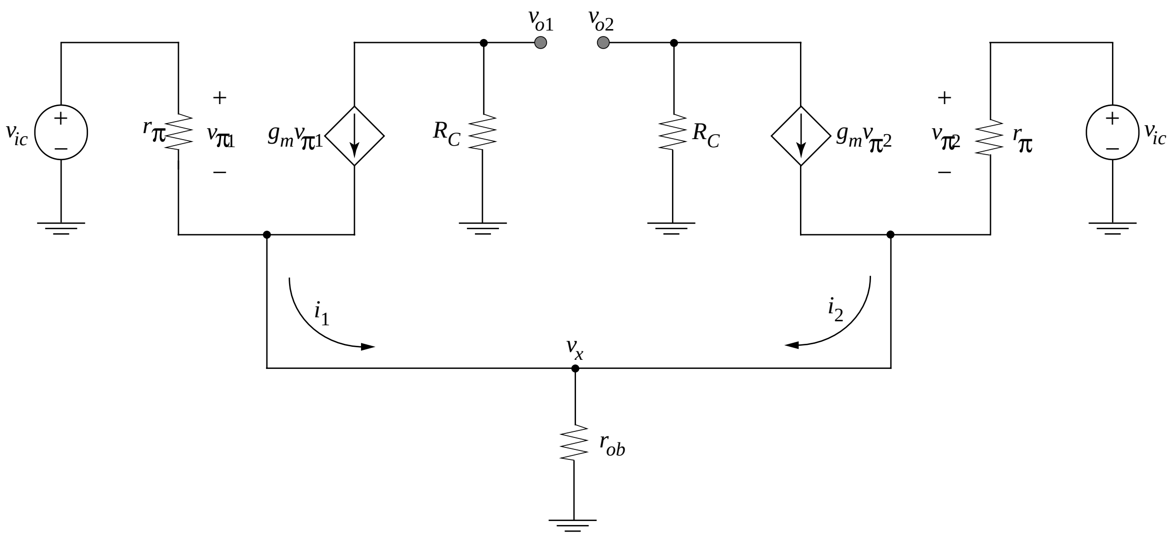


$$\frac{v_{od}}{2} = -g_m R_C \frac{v_{id}}{2}$$

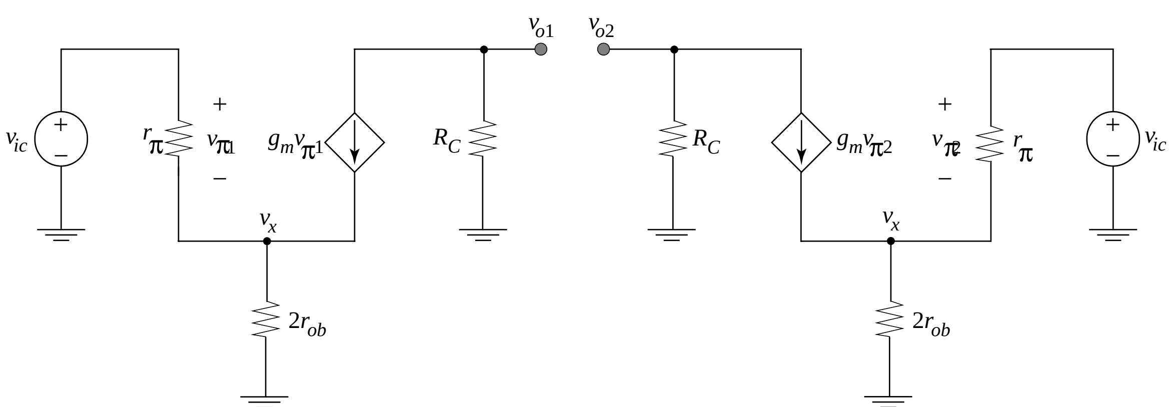
$$a_{dm} = \frac{v_{od}}{v_{id}} = -g_m R_C$$

D. Small Signal Model for Purely Common Mode Input

- $v_{i1} = v_{i2} = v_{ic}, v_{id} = 0$



- $i_1 = i_2$-----> $v_x = (i_1 + i_2)r_{ob} = 2i_1 r_{ob} = 2i_2 r_{ob}$



$$a_{cm} \equiv \frac{(v_{o1} + v_{o2})/2}{v_{ic}} = \frac{-g_m R_C}{(1 + 2g_m r_{ob})}$$

E. Common Mode Rejection

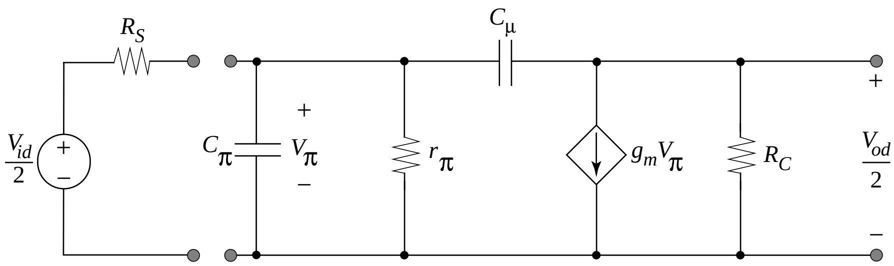
- Figure of Merit for differential amplifiers

$$CMRR \equiv \frac{a_{dm}}{a_{cm}} = \left(1 + 2g_m r_{ob}\right)$$

- Large r_{ob} ----> Large CMRR

III. Frequency Response of Differential Amplifiers

A. Differential Mode Frequency Response-Half Circuit



- DC Response

$$\frac{v_{od}}{v_{id}} = -\left(\frac{r_\pi}{r_\pi + R_S}\right)\left(g_m R_C\right)$$

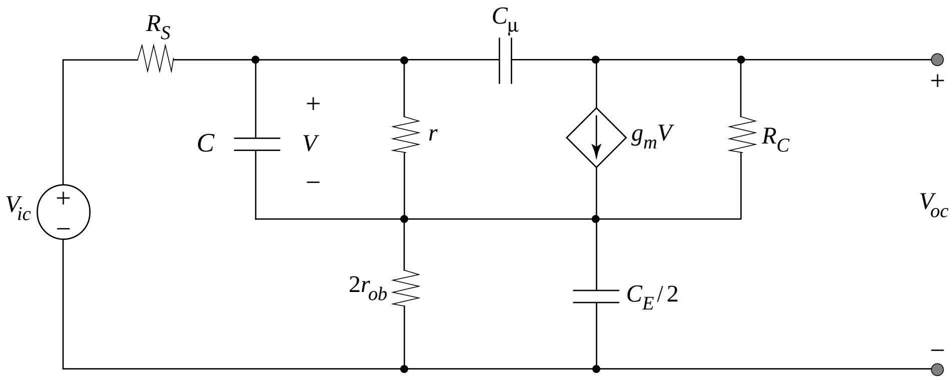
- Use Miller Approximation to find frequency response

$$C_T = C_\pi + \left(1 + g_m R_C\right) C_\mu$$

$$R_T C_T = (R_S \| r_\pi) C_T$$

$$\frac{V_{od}}{V_{id}}(j\omega) \cong -\left(\frac{r_\pi}{r_\pi + R_S}\right)\left(g_m R_C\right)\left[\frac{1}{1 + j\omega\left(R_S \| r_\pi\right)\left[C_\pi + \left(1 + g_m R_C\right)C_\mu\right]}\right]$$

B. Common Mode Frequency Response - Half Circuit



- DC Response

$$\frac{v_{oc}}{v_{ic}} \cong \frac{-g_m R_C}{1 + g_m 2r_{ob}}$$

- In common mode half circuit C_E ---> $C_E/2$ since it must be 2X the impedance

$$Z_E = \frac{2r_{ob}}{1 + j\omega r_{ob} C_E}$$

$$\frac{V_{oc}}{V_{ic}}(j\omega) = \frac{-g_m R_C}{1 + g_m Z_E}$$

- For $g_m Z_E >> 1$

$$\frac{V_{oc}}{V_{ic}}(j\omega) \cong \frac{-R_C}{2r_{ob}}\left(1 + j\omega r_{ob} C_E\right)$$

C. CMRR Frequency Response

- Recall CMRR is a_{dm}/a_{cm}
- At DC

$$CMRR \equiv \frac{a_{dm}}{a_{cm}} = \left(1 + 2g_m r_{ob}\right) \approx 2g_m r_{ob}$$