6.263/16.37 Problem Set 4

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Due: 10/13/05

Problem 2.2 Consider the M/G/∞ queue in which each customer always finds a free server. Let $P_k(t) = P[N(t) = k]$ and assume $P_0(0) = 1$. Show that

$$P_k(t) = \sum_{n=k}^{\infty} e^{-\lambda t} \frac{(\lambda t)^n}{n!} \binom{n}{k} \left[\frac{1}{t} \int_0^t [1 - F_X(x)] dx \right]^k \left[\frac{1}{t} \int_0^t F_X(x) dx \right]^{n-k},$$

where F_X is the cumulative distribution function of the service time X.

Problem 2.3 Consider an M/G/1 queue in which bulk arrivals occur at rate λ and with a probability g_r that r customers arrive together at an arrival instant.

(a) Show that the z-transform of the number of customers arriving in an interval of length t is $e^{-\lambda t[1-G(z)]}$ where $G(z) = \sum g_r z^r$.

(b) Show that the z-transform of the random variables v_n, the number of arrivals during the service of a customer, is $X^*[\lambda - \lambda G(z)]$.

(c) Show that the generating function for queue size is

$$Q(z) = \frac{(1-\rho)(1-z)X^*[\lambda - \lambda G(z)]}{X^*[\lambda - \lambda G(z)] - z}.$$

Using Little's result, find the ratio $W/\overline{x}$ of the expected wait on queue to the average service time.