6.263/16.37 Problem Set 7

MIT, Fall 2005

Issued: Thursday, Nov. 3
Due: Thursday, Nov. 10

Problem 7.1

Prove the following: ***If an M×N nonblocking network is composed of n_{12} 1×2 nodes, n_{21} 2×1 nodes, n_{22} cells, plus possibly crosspoints, then***

$$n_{12} + n_{21} + 4n_{22} = 2MN - M - N$$

Problem 7.2

1. Consider the Bell System number 5 crossbar network with a network structure shown in figure 15. It is constructed using $n \times n$ switches. A switch frame (in dotted boxes) is an $n^2 \times n^2$ network with 2 stages with n switches in each stage. The entire network has 2 stages of n frames, with n^3 inputs and n^3 outputs.

   a. Compute the number of crosspoints for the number 5 crossbar. For $n=10$, compare that with the number of crosspoints needed by a 3 stage rearrangeably non-blocking network and the Benes network. (Use the approximation $2^{10} \approx 1000$. For the 3 stage network, assume there are 32 32×32 switches in each of the 3 stages.)

   b. How many paths are there between an input and an output?

   c. Show that the number of permutations realized by the network is upper bounded by $(n!)^{4n^2}$. Why is this only an upper bound?

   d. Upper bound the fraction of permutations which can be realized by the network. Get a rough estimate for $n=10$ by using Sterling's approximation

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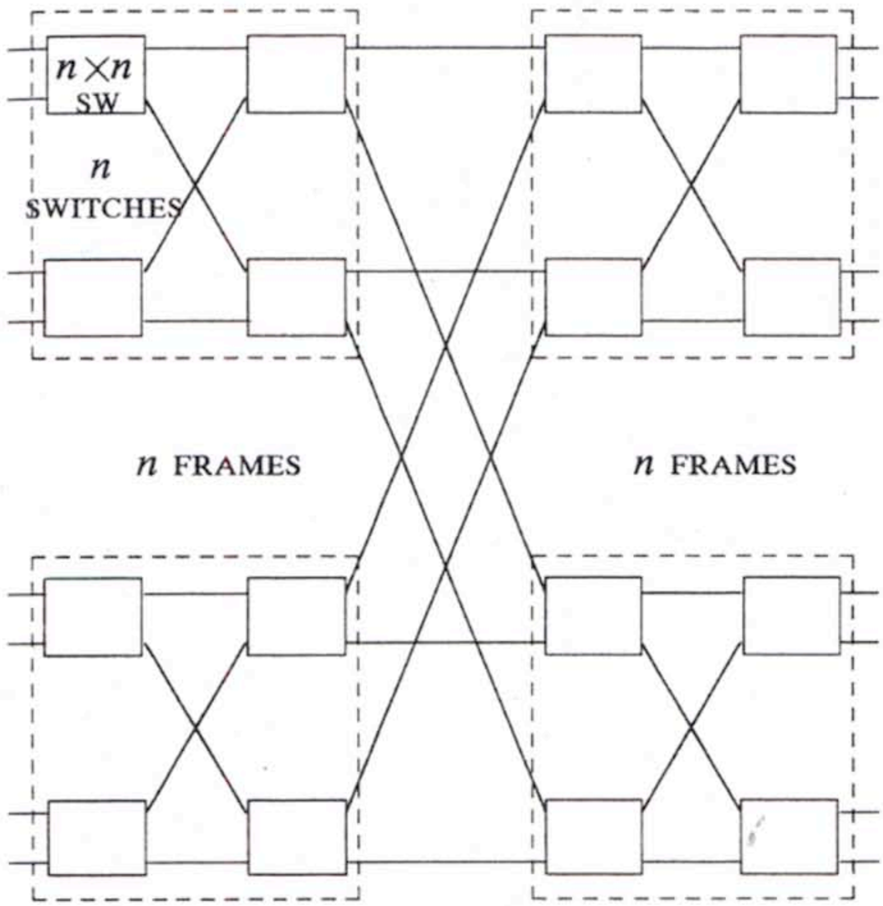
Problem 7.2 - cont.



The No. 5 Crossbar Network

$$\log_{10} x! \approx (x+\frac{1}{2})\log_{10} x + x\log_{10} e$$

$$= (x+0.5)\log_{10} x + 0.4343x$$