MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.334 Power Electronics — Practice Exam A

Problem 1

Figure 1 shows the internal structure and dimensions of a power diode mounted in an axial lead package. The diode is cooled by conduction through its leads, which are soldered to terminals that are assumed to be at temperature T_A. Heat is generated at the junction of the diode, which is planar and centered between the two surfaces. The thermal resistivity of Silicon is 1.2 °C-cm/W, and the thermal resistivity of copper is 0.25 °C-cm/W.

a. Draw the analog circuit model for the thermal system of Fig. 1
b. If the maximum permissible junction temperature of the diode is T_j is 225 °C, determine the maximum permissible dissipation for T_A = 75 °C.

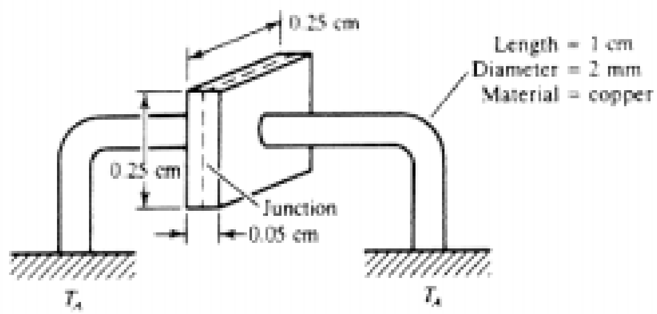

Figure 1

Problem 2

The switch S of the circuit of Fig. 2 is operated at a *constant* duty ratio D and a switching frequency f_s. At $t = 0$ the capacitor is charged to an initial voltage V_o and the inductor current is zero. Calculate and sketch the ***local averages*** of $i_L(t)$ and $v_C(t)$ for $t > 0$ under the assumption that $2\pi f_{sw} >> (1-D)(LC)^{-1/2}$.

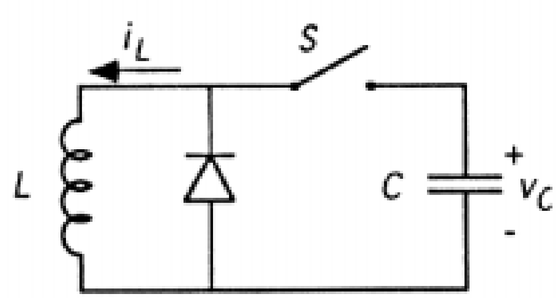

Figure 2

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Problem 1

The switch in the circuit of Fig. 1 is operated at a constant duty ratio D and a switching frequency f_s. At $t = 0$, the inductor is carrying a current I_o and the capacitor is uncharged. The switching frequency is much higher than $(1-D)(LC)^{-1/2}$. Calculate and sketch $i_L(t)$ and $v_C(t)$ for $t > 0$.

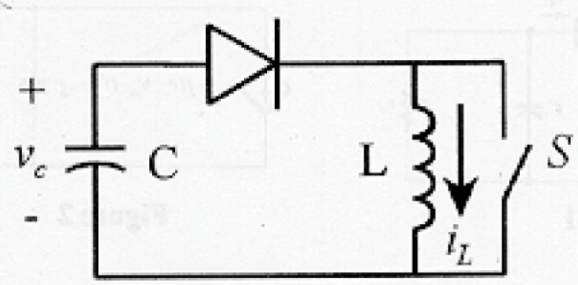

Figure 1

Problem 2

Derive an averaged model for the up/down converter of Fig. 2 under duty ratio control. You may derive such a model by either direct circuit averaging or by state space averaging. Is the model linear in terms of the control variable d? Why or why not?

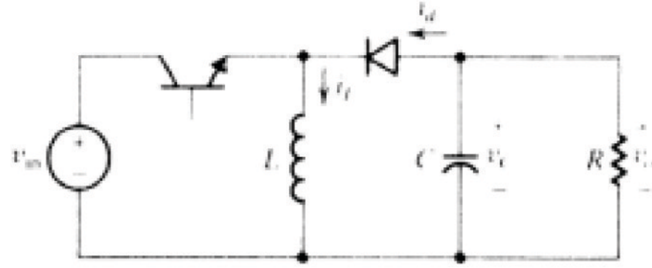

Figure 2

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6.334 Power Electronics — Practice Exam C

Problem 1

Fig. 3 shows an input filter for a switching power converter. The current drawn by the converter is represented as i_X, and the voltage supplying the converter is v_Y. The converter switching frequency 1/T is 100 kHz. You may assume the filter components are ideal.

a. Select component values L, C, and R for the filter such that

   i. The maximum output impedance of the filter, Z_O, is 1 Ω or less at all frequencies

   ii. The filter achieves an attenuation of approximately 40 dB (a factor of 100) in current at the switching frequency. That is, $|i_Y / i_X| \approx 0.01$ at the switching frequency.

   iii. The filter is well damped, such that it has less than 10 dB of peaking in $|i_Y / i_X|$ near the undamped natural frequency of the filter.

b. Suppose that the load on the filter (slowly) adjusts the local average current $\langle i_X \rangle$ drawn based on the local average voltage $\langle v_X \rangle$ to maintain a constant average power draw P_O. Please find a (low-frequency) equivalent small-signal resistance r_E for this load for the operating point P_O = 10 W and V_X = 100 V. Will this load greatly affect the filter damping?

c. Please propose a simple filter modification (e.g., an addition of no more than one component) that would provide higher-order attenuation performance (and increased attenuation) at and above the switching frequency, without harming filter damping. Please specify values for any added or modified components.

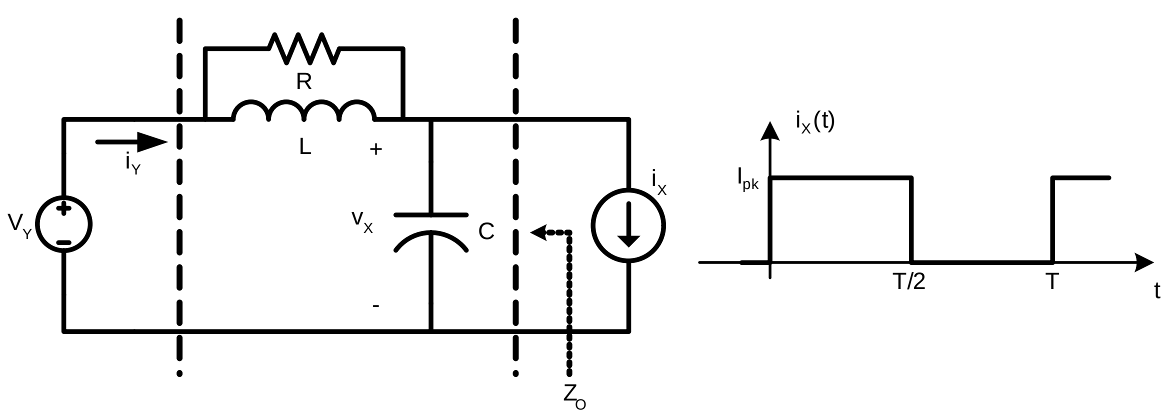

Figure 3

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6.334 Power Electronics | Practice Exam D

Problem 1

Consider the buck/boost converter of Fig. 1 operating in the discontinuous conduction mode. (That is, operating such that the inductor current returns to zero before the end of each switching cycle.)

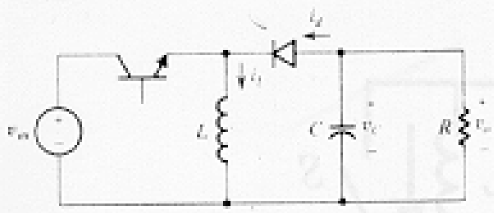

Figure 1

Figure 2

a. Find an averaged model for the system of the form shown in Fig. 2 assuming that the inductor current and output voltage do not vary much over a switching cycle.

b. Assume R = 2 Ohms, C = 220 uF, L = 0.25 mH, V_{in} = 12 V, V_o = -9 V. Find a linearized model for the system for perturbations in the duty ratio about this operating point, and identify the open loop pole location.

Problem 2

A double-insulated window is made of panes of glass 4 mm thick spaced 1 cm apart. Window glass has a thermal resistivity of 100 °C-cm/W, and still air has a thermal resistivity of 3050 °C-cm/W. If the interior of the building is at 25 °C, and the exterior is at 0 °C, what is the rate of heat loss by conduction through the window in kW/m^2?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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6.334 Power Electronics — Practice Exam E

Problem 1

The dc/dc converter of Figure 1 takes in an input voltage V_1, and generates an output current I_2. Derive an averaged model for this converter in continuous conduction under duty ratio control. You may derive such a model by either direct circuit averaging or by state space averaging, but you should express your results as a pair of state-space equations in terms of the local averages of state variables i_L and v_C.

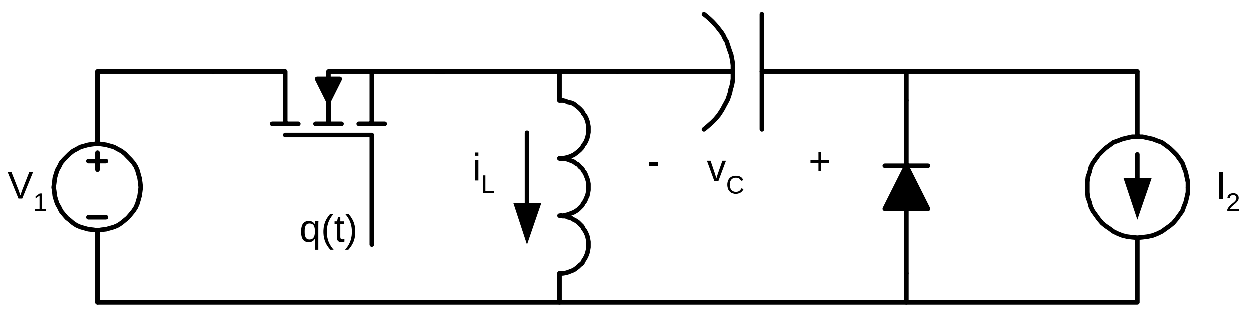


Figure 1

Problem 2

Figure 2 shows an RF power amplifier, an L-section matching network, and a resistive load.

a. Derive an expression for the input impedance Z_{in} of the matching network and load in terms of L, C, R, and ω.

b. Find the frequency ω for which the specified matching network components provide an impedance match between the RF amplifier and the load resistance *R*.

c. Design a different L-section (2 component) matching network that will match the RF power amplifier to the load *at an operating frequency of 100 MHz*. Specify the component values to be used. (*Hint: use a "high-pass" L-section, rather than the "low-pass" L-section that is shown in Fig. 2.)*

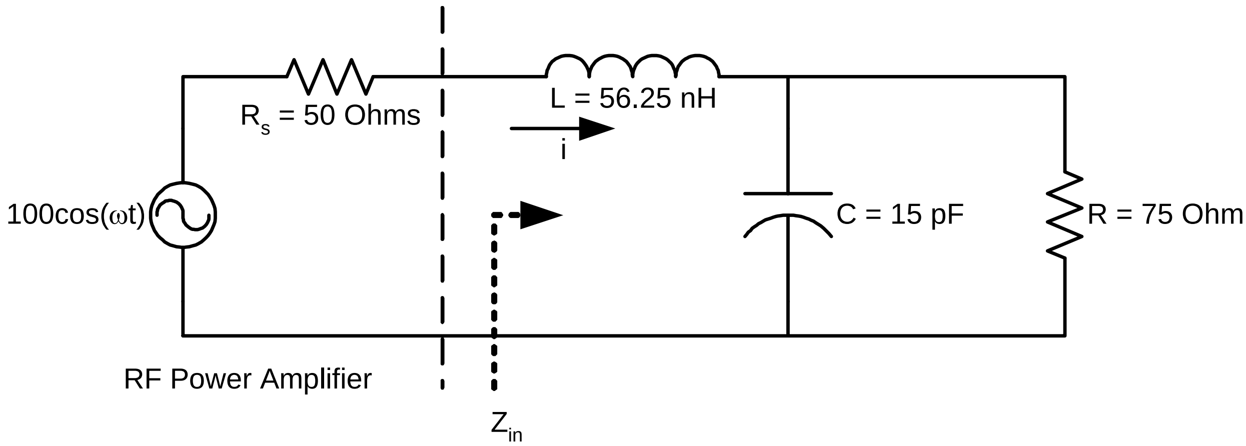


Figure 2

Problem 3

Figure 3 shows a semiconductor device mounted to a heatsink via an insulating pad. The Semiconductor device has a junction-to-case thermal resistance of $R_{\theta jc}$ = 1.2˚C/W, and the thermal insulating pad results in a case-to-sink thermal resistance of $R_{\theta cs}$ = 0.8 ˚C/W. The heat sink has a sink-to-ambient thermal resistance of $R_{\theta sa}$ = 4 ˚C/W, and a thermal capacitance $C_{\theta s}$ = 70 J/˚C. The thermal capacitance of the device and thermal pad are negligible. The system operates at an ambient temperature T_A = 100 ºC. We denote the semiconductor device dissipation in Watts as P_D, and the junction temperature of the device as T_j.

a. Draw the thermal model for the system of Fig. 3.

b. What is the largest average device dissipation P_D that is permissible if the junction temperature T_j of the device is to be kept below 175 ºC?

c. At startup (after being at zero dissipation for a long time), the device is subjected to a dissipation of 30 W for 0.1 s before dropping to a much lower steady-state level (below that of part B). Please calculate the peak device junction temperature reached during the startup transient for an ambient temperature T_A = 100 ºC. (*Note: An accuracy of 1*ºC *is sufficient. You may make reasonable approximations as long as you justify them.*)

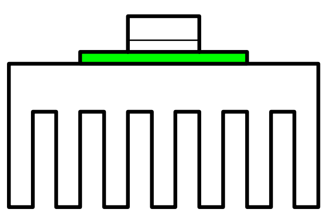
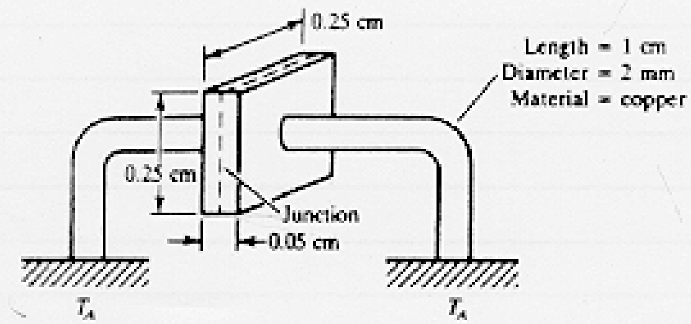


Figure 3

Jamie C. Byrum
5/1/00

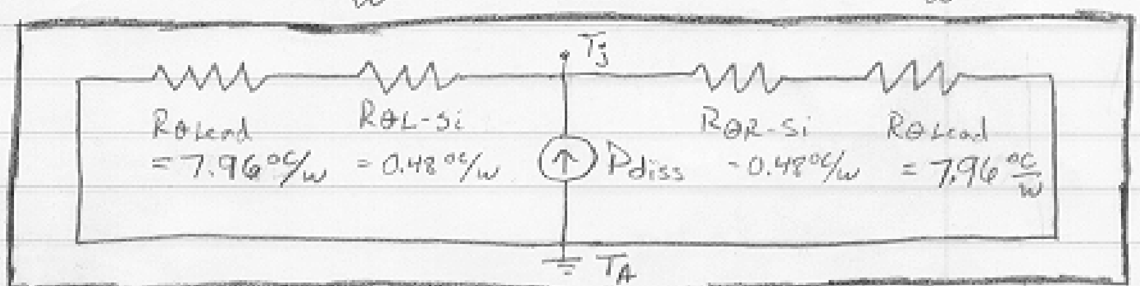
6.334 Practice Exam 2 - Solutions

PA.1



$\rho_{\theta Si} = 1.2°\frac{C\text{-cm}}{W}$ $\quad \rho_{\theta cu} = 0.25°\frac{C\text{-cm}}{W}$

a).



$R = \frac{\rho l}{A} \Rightarrow R_{\theta L\text{-}Si} = R_{\theta R\text{-}Si} = \frac{(1.2)(0.025\text{cm})}{(0.25\text{cm})^2}$

$= 0.48\ °C/W$

$R_{Lead} = \frac{(0.25)(1\text{cm})}{\pi\left(\frac{.2}{2}\text{cm}\right)^2} = 7.96\ °C/W$

---

b). $T_{J_{max}} = 225°C$

$T_A = 75°C$

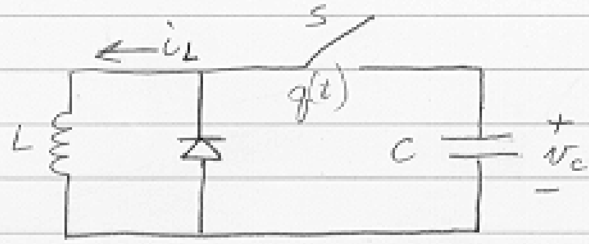
KCL of analog circuit:

$\frac{2(T_J - T_A)}{R_{\theta Lead} + R_{\theta Si}} = P_{diss}$

$\frac{2(225° - 75°)}{7.96°C/W + 0.48°C/W} = P_{diss}$

$\boxed{35.6\ W = P_{diss}}$

PA.2.



$v_c(\emptyset) = V_0$ constant duty ratio D
$i_L(\emptyset) = \emptyset$ switching frequency f_s

Assume: $2\pi f_{sw} \gg (1-D)(LC)^{-\frac{1}{2}}$

Recall local average $\overline{x}(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)\,d\tau$

☆ $v_L(t) = L\frac{di_L(t)}{dt} = q(t)\,v_c(t)$

$L\frac{d\overline{i_L}(t)}{dt} = \overline{q(t)\,v_c(t)} = D\,\overline{v_c}(t)$

☆ $i_c(t) = q(t)\left(-i_L(t)\right) = C\frac{dv_c(t)}{dt}$

$\overline{-q(t)\,i_L(t)} = C\frac{d\overline{v_c}(t)}{dt} = -D\,\overline{i_L}(t)$

True as long as $v_c > \emptyset$.
When $v_c \le \emptyset$, diode stays on regardless of the position of the switch.

$$C\frac{d\overline{v_c}(t)}{dt} = \frac{LC}{D}\frac{d^2\overline{i_L}(t)}{dt^2} = -D\,\overline{i_L}(t)$$

$$\frac{d^2\overline{i_L}(t)}{dt^2} + \frac{D^2}{LC}\overline{i_L}(t) = \emptyset$$

$$r^2 + \frac{D^2}{LC} = \emptyset \rightarrow r = \pm j\frac{D}{\sqrt{LC}}$$

$$\longrightarrow \overline{i_L}(t) = K_1\sin\left(\frac{D}{\sqrt{LC}}t\right) + K_2\cos\left(\frac{D}{\sqrt{LC}}(t)\right)$$

$$\longrightarrow \overline{v_c}(t) = \frac{L}{D}\frac{d\overline{i_L}(t)}{dt} = K_1\sqrt{\frac{L}{C}}\cos\left(\frac{D}{\sqrt{LC}}t\right) - K_2\sqrt{\frac{L}{C}}\sin\left(\frac{D}{\sqrt{LC}}t\right)$$

(continued)

PA.2

$$\bar{i}_L(\emptyset) = \emptyset = \cancelto{\emptyset}{k_1 \sin\left(\frac{D}{\sqrt{LC}}(\emptyset)\right)} + k_2 \cancelto{1}{\cos\left(\frac{D}{\sqrt{LC}}(\emptyset)\right)} = k_2 = \emptyset$$

$$\bar{v}_C(\emptyset) = V_0 = k_1 \sqrt{\frac{L}{C}}\, \cancelto{1}{\cos\left(\frac{D}{\sqrt{LC}}(\emptyset)\right)} \Rightarrow k_1 = V_0\sqrt{\frac{C}{L}}$$

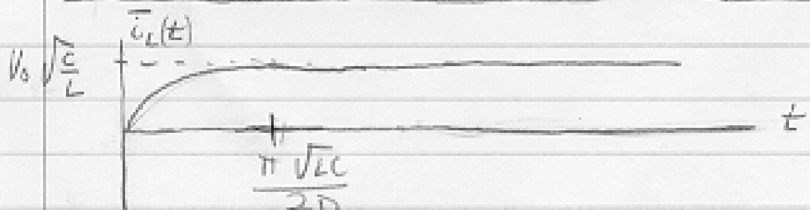
$$\boxed{\begin{aligned} &\Rightarrow \bar{i}_L(t) = V_0\sqrt{\frac{C}{L}} \sin\left(\frac{D}{\sqrt{LC}}t\right) \\ &\Rightarrow \bar{v}_C(t) = V_0 \cos\left(\frac{D}{\sqrt{LC}}t\right) \end{aligned}}$$

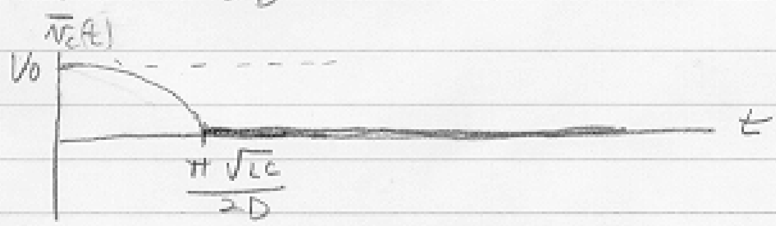
while $\bar{v}_C(t) > \emptyset$

$\bar{v}_C(t) = \emptyset$ at $\frac{D}{\sqrt{LC}}t = \frac{\pi}{2} \Rightarrow t = \frac{\pi\sqrt{LC}}{2D}$

At $t = \frac{\pi\sqrt{LC}}{2D}$, the diode clamps $D\bar{v}_C(t)$ to $\emptyset$, meaning that $\bar{i}_L(t)$ remains at its peak value and $\bar{v}_C(t)$ remains at $\emptyset$ forever.

$$\boxed{\begin{aligned} \bar{i}_L(t) &= \begin{cases} V_0\sqrt{\frac{C}{L}}\sin\left(\frac{D}{\sqrt{LC}}t\right) & \text{for } \emptyset \le t \le \frac{\pi\sqrt{LC}}{2D} \\ V_0\sqrt{\frac{C}{L}} & \text{for } t > \frac{\pi\sqrt{LC}}{2D} \end{cases} \\ \bar{v}_C(t) &= \begin{cases} V_0\cos\left(\frac{D}{\sqrt{LC}}t\right) & \text{for } 0 \le t \le \frac{\pi\sqrt{LC}}{2D} \\ \emptyset & \text{for } t > \frac{\pi\sqrt{LC}}{2D} \end{cases} \end{aligned}}$$





Jamie C. Byrum
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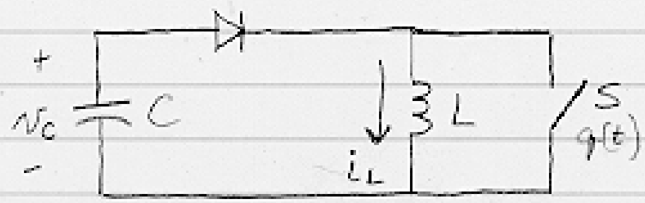
PB.1



$i_L(\emptyset) = I_o$ constant duty ratio D

$v_c(\emptyset) = \emptyset$ switching frequency f_s

Assume $2\pi f_{sw} \gg (1-D)(LC)^{-\frac{1}{2}}$

$$\star \quad v_L(t) = L\frac{di_L(t)}{dt} = v_c(t)(1-q(t))$$

$$L\frac{d\bar{i}_L(t)}{dt} = \overline{(v_c(t)(1-q(t)))} = D'\bar{v}_c(t)$$

$$\star \quad i_c(t) = C\frac{dv_c(t)}{dt} = -i_L(t)(1-q(t))$$

$$C\frac{d\bar{v}_c(t)}{dt} = \overline{-i_L(t)(1-q(t))} = -D'\bar{i}_L(t)$$

True as long as $i_L > \emptyset$. If i_L tries to go in the opposite direction, it cannot flow through the diode.

$$C\left(\frac{L}{D'}\frac{d^2\bar{i}_L(t)}{dt^2}\right) = -D'\bar{i}_L(t)$$

$$\frac{d^2\bar{i}_L(t)}{dt^2} + \frac{D'^2}{LC}i_L(t) = \emptyset$$

$$r^2 + \frac{D'^2}{LC} = \emptyset \rightarrow r = \pm j\frac{D'}{\sqrt{LC}} = \pm j\frac{1-D}{\sqrt{LC}}$$

$$\rightarrow \quad \bar{i}_L(t) = K_1\sin\left(\frac{1-D}{\sqrt{LC}}t\right) + K_2\cos\left(\frac{1-D}{\sqrt{LC}}(t)\right)$$

$$\rightarrow \quad v_c(t) = \frac{L}{1-D}\frac{d\bar{i}_L(t)}{dt} = K_1\sqrt{\frac{L}{C}}\cos\left(\frac{1-D}{\sqrt{LC}}t\right) - K_2\sqrt{\frac{L}{C}}\sin\left(\frac{1-D}{\sqrt{LC}}t\right)$$

(continued)

$$\bar{i}_L(0) = I_0 = k_1 \cancelto{0}{\sin\left(\frac{1-D}{\sqrt{LC}}(0)\right)} + k_2 \cancelto{1}{\cos\left(\frac{1-D}{\sqrt{LC}}(0)\right)} \Rightarrow k_2 = I_0$$

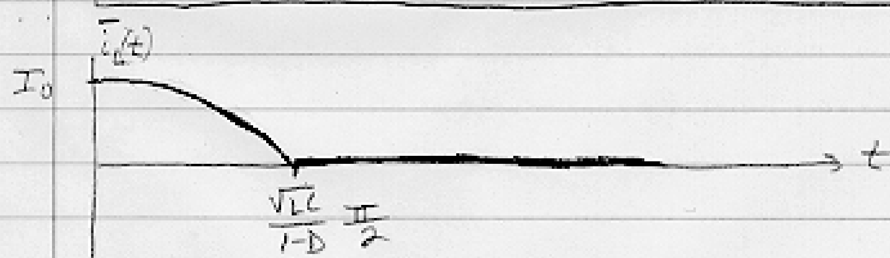
$$\bar{v}_C(0) = 0 = k_1 \sqrt{\tfrac{L}{C}} \cancelto{1}{\cos\left(\frac{1-D}{\sqrt{LC}}(0)\right)} \Rightarrow k_1 = 0$$

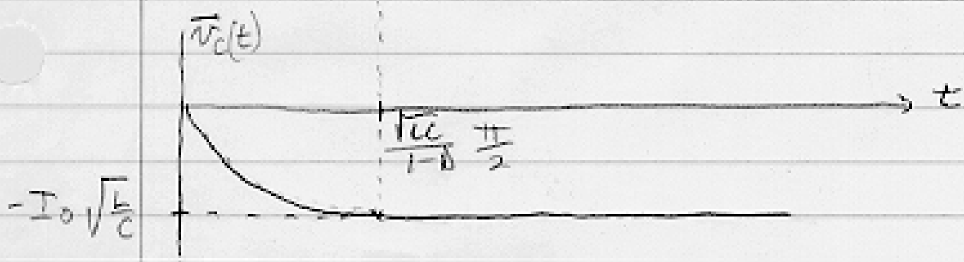
$$\Rightarrow \bar{i}_L(t) = I_0 \cos\left(\frac{1-D}{\sqrt{LC}}\, t\right)$$

$$\Rightarrow v_C(t) = -I_0 \sqrt{\tfrac{L}{C}} \sin\left(\frac{1-D}{\sqrt{LC}}\, t\right)$$

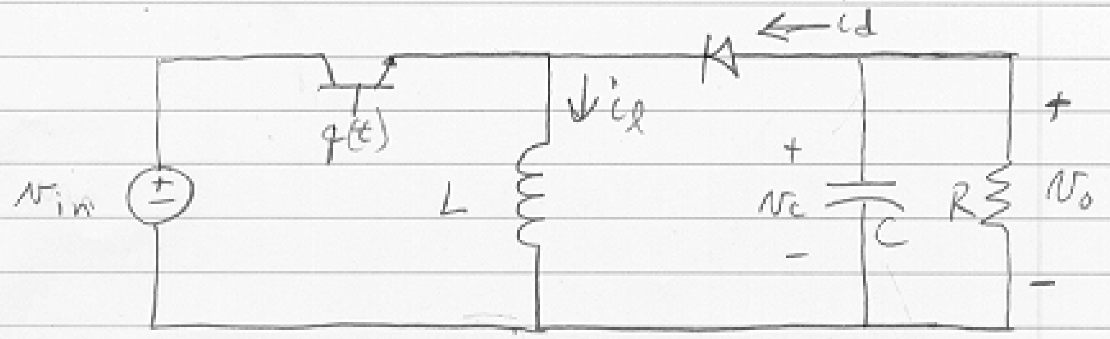
$\bar{i}_L(t) = 0$ at $t = \frac{\sqrt{LC}}{1-D}\frac{\pi}{2}$. At this time, current attempts to circulate the opposite way through the capacitor, which is impossible because of the diode. Thus, $i_L = 0$ for $t > \frac{\pi}{2}\frac{\sqrt{LC}}{1-D}$, and v_C remains at its (negative) peak value.

$$\boxed{\begin{aligned}
\bar{i}_L(t) &= \begin{cases} I_0 \cos\left(\frac{1-D}{\sqrt{LC}}\, t\right) & \text{for } 0 \le t \le \frac{\sqrt{LC}}{1-D}\frac{\pi}{2} \\ 0 & \text{for } t > \frac{\sqrt{LC}}{1-D}\frac{\pi}{2} \end{cases} \\
\bar{v}_C(t) &= \begin{cases} -I_0 \sqrt{\frac{L}{C}} \sin\left(\frac{1-D}{\sqrt{LC}}\, t\right) & \text{for } 0 \le t \le \frac{\sqrt{LC}}{1-D}\frac{\pi}{2} \\ -I_0 \sqrt{\frac{L}{C}} & \text{for } t > \frac{\sqrt{LC}}{1-D}\frac{\pi}{2} \end{cases}
\end{aligned}}$$





PB.2.



$$\star \quad \frac{di_\ell}{dt} = \frac{v_{in}}{L} q(t) + \frac{v_o}{L}(1-q(t))$$

$$\star \quad \frac{dv_c}{dt} = \frac{-v_o}{RC} - \frac{i_\ell}{C}(1-q(t))$$

$$\star \quad \frac{d\bar{i}_\ell}{dt} = \overline{\frac{v_{in}}{L} q(t)} + \overline{\frac{v_o}{L}(1-q(t))}$$

$$(1) \quad \boxed{\frac{d\bar{i}_L}{dt} = \frac{\bar{v}_{in}}{L} d(t) + \frac{\bar{v}_o}{L} d'(t)}, \quad \text{where } \overline{q(t)} = d(t)$$

$$\star \quad \frac{d\bar{v}_c}{dt} = \frac{-\bar{v}_o}{RC} - \overline{\frac{i_\ell}{C}(1-q(t))}$$

$$(2) \quad \boxed{\frac{d\bar{v}_c}{dt} = \frac{-\bar{v}_o}{RC} - \frac{\bar{i}_\ell}{C} d'(t)}$$

Consider equation (1):

$$\left(d(t) + d'(t)\right)\frac{d\bar{i}_L}{dt} = \frac{\bar{v}_{in}}{L} d(t) + \frac{\bar{v}_o}{L} d'(t)$$

$$\left(\frac{d\bar{i}_L}{dt} - \frac{\bar{v}_{in}}{L}\right) d(t) = \left(\frac{\bar{v}_o}{L} - \frac{d\bar{i}_L}{dt}\right) d'(t)$$

(continued)

PB.2

$$\left(\frac{\overline{v_0}}{L} - \frac{d\overline{i_L}}{dt}\right) = \frac{d(t)}{d'(t)}\left(\frac{d\overline{i_L}}{dt} - \frac{\overline{v_{in}}}{L}\right)$$

Looks like a transformer with turns ratio

$$\frac{N_2}{N_1} = \frac{d(t)}{d'(t)}$$ ✓

Consider equation (2): By average model, we need average circuit to the left of the capacitor to provide current $-\overline{i_L}\, d'(t)$ to the capacitor.

If we model switch and diode together as a transformer with $N_2 = d(t)$ and $N_1 = d'(t)$, then the current through the secondary winding will be:

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} \Rightarrow I_2 = \frac{N_1}{N_2} I_1$$

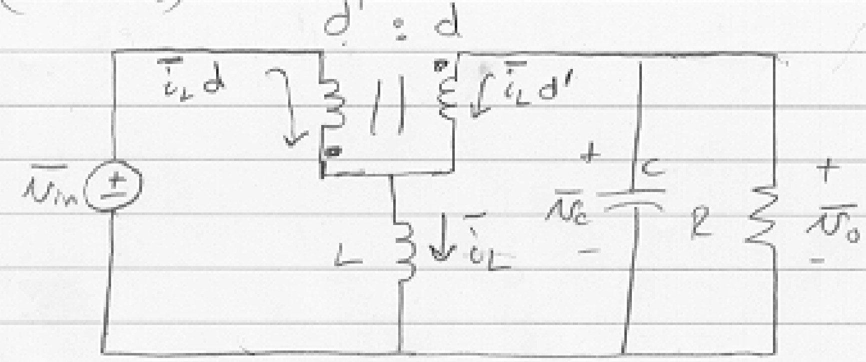
$$= \frac{d'(t)}{d(t)} I_1 = -\overline{i_L} \cdot d'(t)$$

so, $I_1 = d(t)\,\overline{i_L}$

and $I_2 - I_1 = -\overline{i_L}\, d'(t) - d(t)\,\overline{i_L} = -\overline{i_L}$ ✓

Thus, all of the above is consistent with an average model containing a transformer with $N_1 = d'(t)$, $N_2 = d(t)$, and whose connection between the primary and secondary sides connects to L.

PB. 2 (continued)



This model is <u>nonlinear</u> in terms of the control variable d, because the control d (or $d' = 1-d$) is multiplied by the state variables $\bar{i}_L$ and $\bar{v}_o$ $(= \bar{v}_c)$ in the two average-model state equations.

Practice Exam C.
Problem 1

a) Select L, C, R such:

$+ |Z_o|_{max} = 1\,\Omega \qquad + \left|\frac{i_Y}{i_X}\right|\Big|_{\omega=\omega_s} = 0.01 \qquad + \left|\frac{i_Y}{i_X}\right|_{max} = 10C$

The impedance Z_o is: $\quad Z_o(\omega) = R // \omega L // \omega C$

$|Z_o|(\omega)$ will be maximum at resonance $\omega_o = \dfrac{1}{\sqrt{LC}}$

and will be equal to $|Z_o|_{max} = R$

Hence we want $R \leq 1\,\Omega$

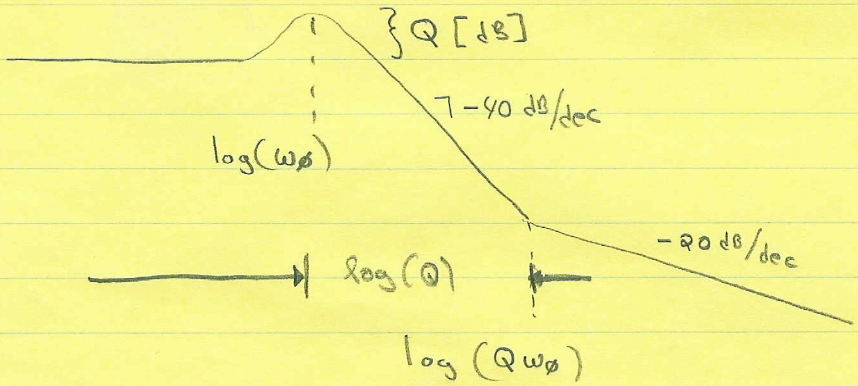
Now let's take a look at $\left|\frac{i_Y}{i_X}\right|(\omega)$:

$$\frac{i_y}{i_x} = \frac{\frac{1}{sC}}{\frac{1}{sC} + \frac{sL}{1+s\frac{L}{R}}} = \frac{1}{1 + \frac{s^2LC}{1+s\frac{L}{R}}} = \frac{1+s\,L/R}{1+s\frac{L}{R}+s^2LC}$$

$$\frac{i_y}{i_x}(s) = \frac{1+s\,L/R}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}} = \frac{1+\frac{s}{Q\omega_o}}{1+\frac{s}{Q\omega_o}+\frac{s^2}{\omega_o^2}}$$

where $\quad \omega_\varnothing = \dfrac{1}{\sqrt{LC}} \qquad Q = R\sqrt{\dfrac{C}{L}}$

The Bode Plot of $\left|\frac{i_Y}{i_X}\right|$ is :



Hence to achieve a maximum of 10 dB peaking near wo I need

$$Q = R\sqrt{\frac{C}{L}} < 3.1622 \text{ [10 dB]}$$

The attenuation at $\omega = Q\omega_\emptyset$ is

$$\left|\frac{i_Y}{i_X}\right| \cong -40 \log(Q) \qquad \text{for } Q = 3.1622$$

$$\left|\frac{i_Y}{i_X}\right| = -20 \text{ dB}$$

Hence to achive -40 dB attenuation at $\omega = \omega_S$ I need

$$Q\omega_\emptyset = 0.1\,\omega_S = R/L$$

$$\omega_\emptyset = \frac{0.1\,(2\pi \cdot 100\text{k})}{3.1622} = 19.869 \times 10^3 \text{ rad/s} = \frac{1}{\sqrt{LC}}$$

with $R = 1$:

$$L = \frac{1}{0.1(2\pi \times 100\text{k})} = \underline{15.9155\,\mu\text{H}}$$

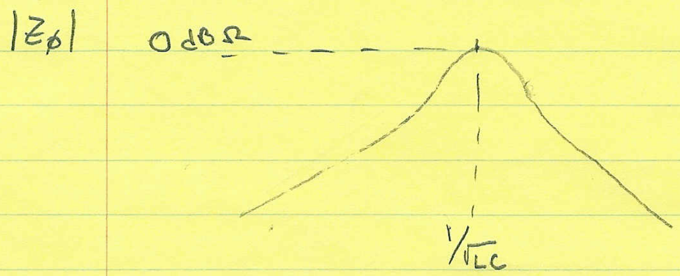
$$\omega_0^2 = \frac{1}{LC} = \left(\frac{0.1\,\omega_s}{Q}\right)^2$$

hence $$C = \frac{1}{L\left[\frac{0.1\,\omega_s}{Q}\right]^2} = \frac{1}{15.9155\mu\left[\frac{0.1\times 2\times\pi\times 100k}{3.1622}\right]^2}$$

$$\boxed{C = 159.1549\ \mu F}$$

The component values for the filter are:

$R = 1\Omega \quad L = 15.9155\mu H \quad C = 159.1549\mu F$



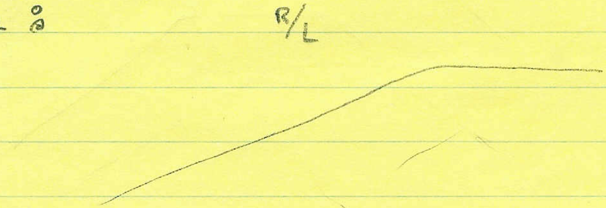
b) $i = \frac{P_o}{v}$ $\quad R_{eq} = \left(\left. \frac{di}{dv} \right|_{V_x} \right)^{-1} = \left. -\frac{P_o}{v_x^2} \right|_{V_x} = \left(-\frac{P_o}{V_x^2} \right)^{-1}$

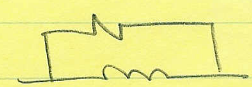
$$R_{eq} = \left(-\frac{10}{(100)^2} \right)^{-1} = -\frac{(100)^2}{10} = -1k\,\Omega$$

$|R_{eq}| >> |Z_{Filt}|_{max}$ hence it won't affect the filter damping

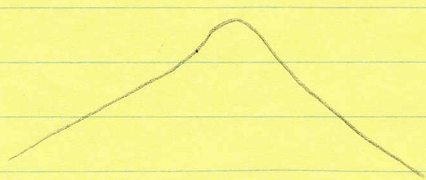
c) $\frac{i_y}{i_x} = \frac{Z_C}{Z_C + Z_{R\|L}} \cdot \frac{Z_{R\|L}}{Z_{RL}} = \frac{Z_C \| Z_{R\|L}}{Z_{RL}}$

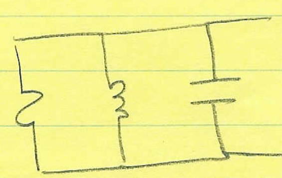
$Z_{R\|L}$:



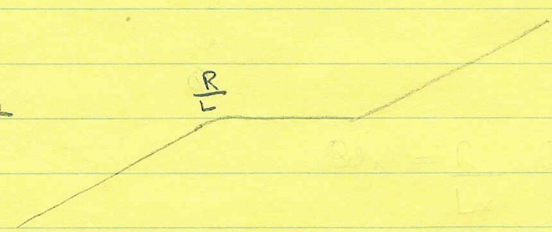


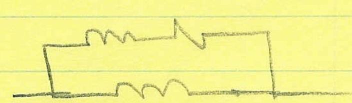
$Z_C \| Z_{R\|L}$





Desired $Z_{R\|L}$





$L_x < L_1$

PD.1

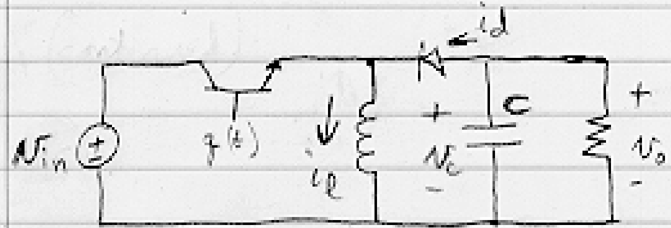


Fig 1

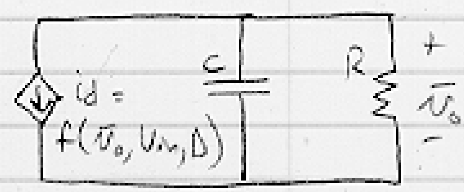
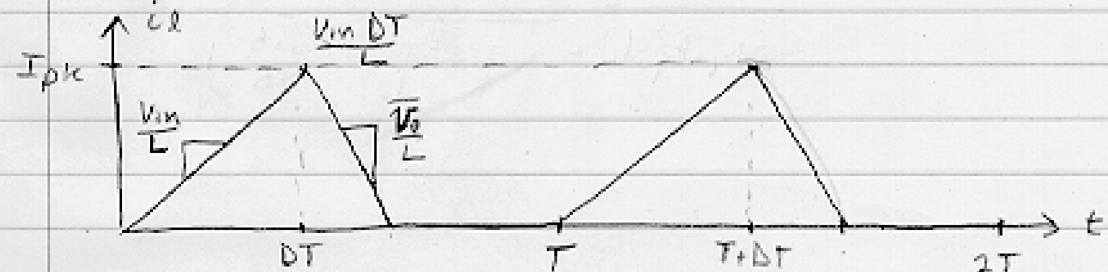
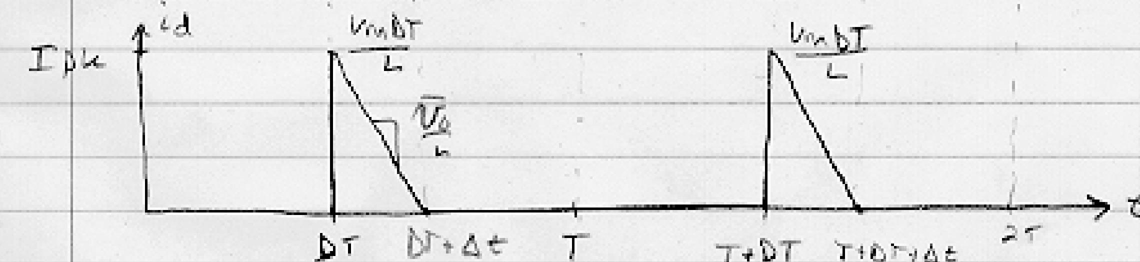


Fig 2

Discontinuous Conduction — Assume constant $v_{in} = V_{in}$





$$\frac{\bar{v}_o}{L}\Delta t = -\frac{V_{in}DT}{L} \Rightarrow \Delta t = -\frac{V_{in}}{\bar{v}_o}DT \quad (\text{since } \bar{v}_o \text{ is negative})$$

a). Find $\bar{i}_d = f(\bar{v}_o, V_{in}, D)$ geometrically

$$\bar{i}_d = \frac{1}{T}(\text{Area of triangle}) = \frac{1}{T}\left(\frac{1}{2}\right)(\text{base})(\text{height})$$

$$= \frac{1}{2T}\left(\frac{-V_{in}}{\bar{v}_o}DT\right)\left(\frac{V_{in}DT}{L}\right) = \boxed{\frac{-(DV_{in})^2 T}{2\bar{v}_o L} = \bar{i}_d = f(\bar{v}_o, V_{in}, D)}$$

The average model is in Figure 2

b). $R = 2\Omega$, $C = 220\mu F$, $L = 0.25\,mH$, $V_{in} = 12V$, $V_o = -9V$

Linearize $i_d = I_d + \tilde{i}_d$

$$v_o = V_o + \tilde{v}_o \rightarrow \tilde{v}_o = v_o - V_o$$

$$d = D + \tilde{d} \rightarrow \tilde{d} = d - D$$

(continued)

PD.1

Operating Point: $I_d = f(V_o, V_{in}, \Delta) = \dfrac{-(DV_{in})^2 T}{2V_o L}$

First order Taylor expansion

$i_d = I_d + \tilde{i}_d$

$= f(V_o, V_{in}, \Delta) + \left.\dfrac{\partial f}{\partial d}\right|_{V_o, V_{in}, \Delta} \overbrace{(d - \Delta)}^{\tilde{d}} + \left.\dfrac{\partial f}{\partial v_o}\right|_{V_o, V_{in}, \Delta} \overbrace{(v_o - V_o)}^{\tilde{v}_o}$

$\cancel{I_d} + \tilde{i}_d = \cancel{I_d} - \dfrac{DV_{in}^2 T}{V_o L}\tilde{d} + \dfrac{(DV_{in})^2 T}{2V_o^2 L}\tilde{v}_o$

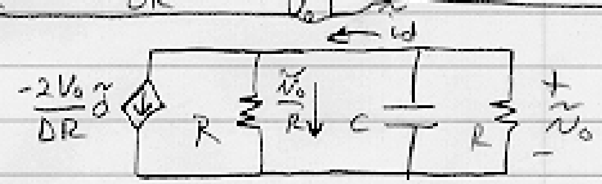
$$\boxed{\tilde{i}_d = \frac{(DV_{in})^2 T}{2V_o^2 L}\tilde{v}_o - \frac{DV_{in}^2 T}{V_o L}\tilde{d}}$$

At operating point, capacitor is an open circuit.

$\therefore\ I_d = \dfrac{-V_o}{R} = \dfrac{-(DV_{in})^2 T}{2V_o L} \Rightarrow R = \dfrac{2V_o^2 L}{(DV_{in})^2 T}$

$\dfrac{1}{R} = \dfrac{(DV_{in})^2 T}{2V_o^2 L} \Rightarrow \dfrac{2V_o}{DR} = \dfrac{DV_{in}^2 T}{V_o L}$

$$\therefore\ \boxed{\tilde{i}_d = \frac{\tilde{v}_o}{R} - \frac{2V_o}{DR}\tilde{d}}$$

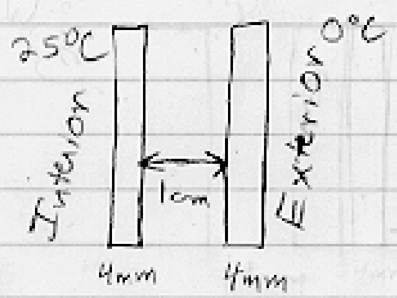


$\tilde{v}_o = \dfrac{-2V_o}{DR}\tilde{d}\,(Z(s))$ $\qquad Z(s) = \dfrac{\frac{R}{2sC}}{\frac{1}{2}R + \frac{1}{sC}} = \dfrac{R}{sRC + 2}$

$\dfrac{\tilde{v}_o}{\tilde{d}} = -\dfrac{2V_o}{D\cancel{R}}\dfrac{\cancel{R}}{sRC+2} = \dfrac{-2V_o}{sRC+2} = \dfrac{\tilde{v}_o}{\tilde{d}}$

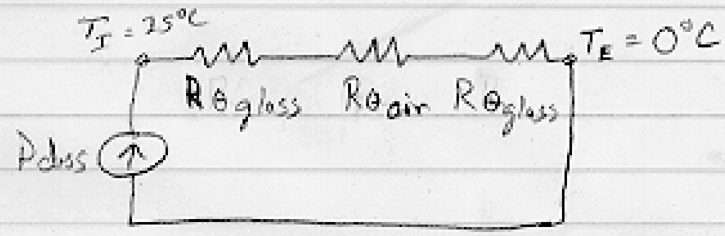
Open loop pole is at $s = \dfrac{-2}{RC} = -4545.45$
Left-hand plane, so open-loop stable.

P D.2



$\rho_{\theta glass} = 100 \frac{°C\text{-}cm}{W}$

$\rho_{\theta air} = 3050 \frac{°C\text{-}cm}{W}$



$$R\theta_{glass} = \frac{\rho_{\theta glass}\, l}{A} = \frac{\left(100 \frac{°C\text{-}cm}{W}\right)(0.4\ cm)}{A\ cm^2} = \frac{40}{A}\ °C/W$$

$$R\theta_{air} = \frac{\rho_{\theta air}\, l}{A} = \frac{\left(3050 \frac{°C\text{-}cm}{W}\right)(1\ cm)}{A\ cm^2} = \frac{3050}{A}\ °C/W$$

$$T_I - T_E = P_{diss}\left(2R\theta_{glass} + R\theta_{air}\right)$$

$$25°C = P_{diss}\left(\frac{80}{A}\ °C/W + \frac{3050}{A}\ °C/W\right)$$

$$25°C = P_{diss}\left(\frac{3130}{A}\ °C/W\right)$$

$$P_{diss} = 0.00799\ A\ (W) \qquad A \text{ is in } cm^2$$

$$= 0.00000799 A\ (kW)$$

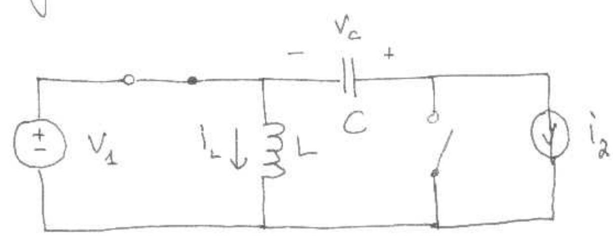
$$\frac{P_{diss}}{cm^2} = 0.00000799\ \frac{kW}{cm^2} \cdot \frac{(100 cm)^2}{m^2} = \boxed{0.0799\ \frac{kW}{m^2}}$$

Practice Exam E SOLUTIONS

Problem 1:

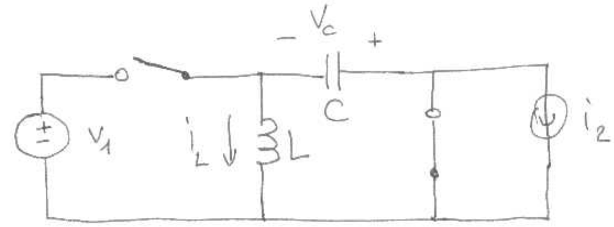
Using method of assumed states:

$q(t) = 1$



$$\begin{cases} L\dfrac{di_L}{dt} = V_1 \\ C\dfrac{dv_c}{dt} = -i_2 \end{cases}$$

$q(t) = 0$



$$\begin{cases} L\dfrac{di_L}{dt} = -V_c \\ C\dfrac{dv_c}{dt} = i_L \end{cases}$$

Averaging:

$$\begin{cases} L\dfrac{d\bar{i}_L}{dt} = d\,\bar{V}_1 - (1-d)\,\bar{V}_c \\ C\dfrac{d\bar{v}_c}{dt} = -d\,\bar{i}_2 + (1-d)\,\bar{i}_L \end{cases} \qquad \text{where } d = \overline{q(t)}$$

Problem 2

(a) $\boxed{Z_{in} =} \; sL + \left(\dfrac{1}{sC} + R\right)$

$$= R\,\frac{s^2LC + \frac{L}{R}s + 1}{sRC + 1} = \boxed{R\,\frac{(1-LC\omega^2) + j\omega\frac{L}{R}}{1 + j\omega RC}}$$

(b) Separating the real and imaginary part of Z_{in}:

$$Z_{in} = R\,\frac{(1-LC\omega^2) + j\omega\frac{L}{R}}{1+j\omega RC} \cdot \frac{1-j\omega RC}{1-j\omega RC}$$

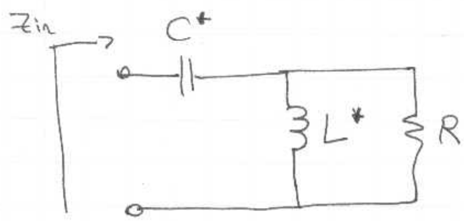
$$= \frac{R}{1+\omega^2R^2C^2} + j\,\frac{\omega\left[R^2C + R^2C^2L\omega^2 + L\right]}{1+\omega^2R^2C^2}$$

Trusting the correct design of the maching network:

$$\frac{R}{1+\omega_0^2R^2C^2} = R_s$$

$$\Rightarrow \boxed{\omega_0 =} \frac{1}{RC}\cdot\sqrt{\frac{R}{R_s}-1} = \boxed{2\cdot\pi\cdot 100\cdot 10^3 \ \text{rad/sec}}$$

(c) We will design a L-section network of the form:



The values of C^*, L^* can be easily obtained from the low-pass design by choosing these values:

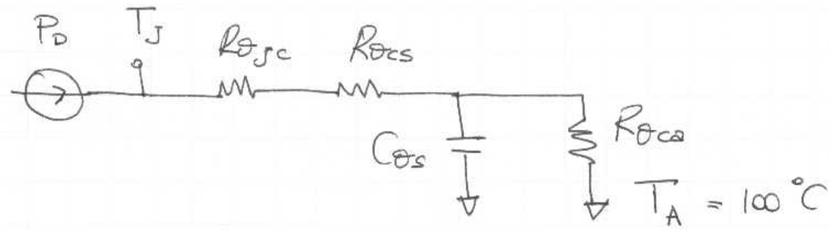
$$\begin{cases} \dfrac{1}{sC^*} = -sL \\ sL^* = -\dfrac{1}{sC} \end{cases} \Rightarrow \begin{cases} C^* = \dfrac{1}{\omega^2 L} \\ L^* = \dfrac{1}{\omega^2 C} \end{cases}$$

Problem

$\therefore$ $\boxed{\begin{aligned} C^* &= 45.0\ pF \\ L^* &= 169\ nH \end{aligned}}$

Problem 3

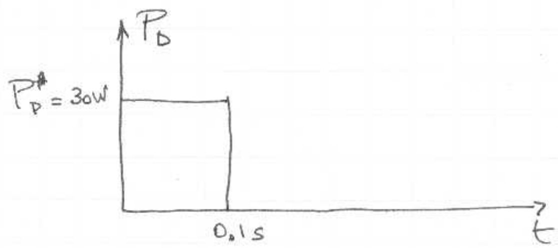
(a)



(b) $T_{J,max} = 175\,°C$

$$P_{D,max} = \frac{T_{J,max} - T_A}{R_{\theta JC} + R_{\theta CS} + R_{\theta CA}} = 12.5\,W$$

(c) At startup we can assume $T_J = 0$ as well as T_s (sink temperature).



The amount of energy dissipated in this pulse is:

$E_D = 3\,J$

Therefore : T_s must always remain below $\frac{E_D^*}{C_{\theta S}} = 43\,m°C$

Approximating: $T_J \approx P_D^{*} \cdot \left(R_{\theta JC} + R_{\theta CS}\right) + \frac{E_D^{*}}{C_{\theta S}} + T_A$

$$\approx P_D^{*}\left(R_{\theta JC} + R_{\theta CS}\right) + T_A$$

$$= 160\,^{\circ}C$$