6.433 Recursive Estimation
6.435 System Identification

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Problem Set 7

Problem 1.

Pseudo-random Binary Sequences (PRBS)

A pseudo-random binary sequence taking the values 0 and 1 can be generated by

$$x(t+1) = \begin{pmatrix} a_1 & \cdots & a_n \\ 1_1 & 0 & \vdots \\ & \ddots & \\ 0 & & 10 \end{pmatrix} x(t) \tag{1}$$

$$u(t) = (0, \ldots 01)x(t)$$

Here $a_i \in \{0, 1\}$ and addition $\oplus$ is mod 2. The initial state can be arbitrary excepting $x(0) \equiv 0$ is not allowed.

(i) Draw a shift-register realization corresponding to the state-space representation Eq. (1).

(ii) Show that the maximum period of shift register with n-states is $M = 2^n - 1$. Such a PRBS is called maximum length.

(iii) Show that the PRBS can be generated by

$$H(q)u(t) = 0 \tag{2}$$

for an appropriate polynomial $H(q)$. (Notational convention as in book.)

(iv) Eq. (2) has a solution of period $M = 2^n - 1$ iff (a) $H(q)$ is irreducible and (b) $H(q)$ is a factor of $q \oplus q^{-M}$ but not a factor of $q \oplus q^{-p}$ for any $p < M$.

(v) If $u(t)$ is a maximum length PRBS of period $M = 2^n - 1$, then within one period it contains $(M + 1)/2$ ones and $(M - 1)/2$ zeroes.

(vi) For $k = 1, \ldots, M-1$

$$u(t) \oplus u(t-k) = u(t-\ell) \quad \text{for} \quad \ell \in [1, M-1]$$

that depends on k.

(vii) Let $u(t)$ be PRBS that shifts between a and $-a$ and has period M. Compute its covariance function and spectral density.

(viii) In what sense does a PRBS behave similar to white noise? Discuss this by comparing

$$\int_{-\pi}^{\pi} f(\omega)\Omega(\omega)\mathrm{d}\omega$$

and f being a continuous function and when Ω corresponds to the spectral density of a white noise process (variance 2) and a PRBS which shifts between a and $-a$ with period M.

(ix) Carry out this comparison for a filtered PRBS and a filtered white noise corresponding to the filter

$$y_1(t) - 0.9y_1(t-1) = u_1(t) \quad (\text{or } u_2(t))$$

($u_1(\cdot)$ white noise with variance λ^2 and $u_2(t)$ is a PRBS of amplitude λ and period M) by comparing the covariance functions in the two cases for $M = 500, 200, 100, 50$ and 20.

Problem 2.

Consider the class of scalar models

$$A(q()Y_k = B(q)U_k + \xi_k \ , \quad k = 0, \ldots, N$$

where the disturbances ξ_k are generated by

$$F\xi_k = C(q)E_k \qquad (q = z^{-1})$$

and the E_k's are a white Gaussian sequence and $Y_k = U_k = \xi_k = 0$, $k < 0$. The polynomials $A(q)$, $B(q)$, $C(q)$ are given by

$$\begin{aligned} A(q) &= 1 + a_1 q + \cdots + a_n q^n \\ B(q) &= b_1 q + \cdots + b_n q^n \\ C(q) &= 1 + c_1 q + \cdots + c_n q^n \end{aligned}$$

Let $\theta = (a_1, \ldots, a_n; b_1, \ldots, b_n; c_1, \ldots, c_n)$. A parameter value θ which minimizes

$$\frac{1}{N}\sum_{k=1}^{N} \epsilon_k^2(\theta)$$

is sought where ϵ_k is the prediction error. If the c_i or the pair (a_i, b_i) are fixed then the minimization problem corresponds to a standard least squares problem.

(a) Derive a generalized recursive least squares algorithm for the general case of correlated disturbances.

(b) Note that if the past disturbances are known then one could derive a recursive least squares algorithm for the parameters: Now derive an algorithm for the determination of the parameters where the past disturbances are estimated.

(c) Finally, note that is $C(q)$ is a known stable polynomial then the determination of the parameters $(a_i, b_i)_{i=1,\dots,n}$ is a least-squares problem. Derive a three-stage least squares algorithm for parameter estimation.

Discuss a modification to this algorithm when the system order is unknown.