

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics of Solids II —6.732

PROBLEM SET # 3

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Due: October 1, 2001

1. (a) Using Fermi statistics, find the temperature dependence of the Fermi level in a metal. Ref. Reif: *Fundamentals of Statistical and Thermal Physics*, p. 394-397. You do not need to write anything for (a) since this problem is solved in many texts. Just review the proof given in your favorite text book.
- (b) Suppose that we have an ideal 2D metal film one atomic layer thick, taken as one layer from a metal crystal. In practice, we can imagine this film as floating on an insulating fluid (both an electrical and a thermal insulator). The mass density of the bulk metal is ρ_m for which the mass is m^* and the Fermi surface is a sphere. Assume further that the total carrier density n results from one conduction electron per atom. Using this model, find an expression for the temperature dependence of the 2D metal for
 - i. the electronic density of states,
 - ii. the Fermi energy,
 - iii. the electrical conductivity, and
 - iv. the Seebeck coefficient.

Compare your results with the corresponding expressions for the 3D metal.

2. Suppose that you have a semimetal with three equivalent electron pockets along the $(1, \bar{1}, \bar{1})$, $(\bar{1}, 1, \bar{1})$ and $(\bar{1}, \bar{1}, 1)$ directions with

$$E(k) = \frac{\hbar^2 k_{el}^2}{2m_{el}} + \frac{\hbar^2 k_{et}^2}{2m_{et}}$$

(where $m_{el} = m_0$ and $m_{et} = 0.01m_0$), and a single hole pocket along the $(1,1,1)$ direction

$$E(k) = \frac{\hbar^2 k_{hl}^2}{2m_{hl}} + \frac{\hbar^2 k_{ht}^2}{2m_{ht}}$$

(where $m_{hl} = m_0$ and $m_{ht} = 0.1m_0$). We note here that this semimetal (which is a simplification of the Bi electronic structure) does not strictly have cubic symmetry, although we use a cubic coordinate system for simplicity. Use the above values for the longitudinal and transverse effective mass components for electrons and holes, and a band overlap of 100 meV:

- (a) Find the carrier density for electrons and holes for this semimetal at $T = 0$ and at low but finite temperature T . State your definition of low T .
- (b) Find the Fermi energy for electrons and holes.
- (c) Find the electrical conductivity in the (111) direction at temperature T , using your definition is (a).

3. (a) Suppose that we apply stress to a highly degenerate Si sample where the Fermi level lies at an energy E_F above the conduction band minimum. Assume that the stress Σ is applied along the (1,0,0) direction, such that the conduction band along the stress direction is lowered, while maintaining the same indirect band gap E_g for the (0, Δ , 0) and (0, 0, Δ) conduction band minima as before the stress was applied. Assume that the change in band gap with stress for the (Δ , 0, 0) minima is linear with stress ($\partial E_g / \partial \Sigma = \alpha \Sigma$) and assume that the total carrier density $n = 6 \times 10^{18} / \text{cm}^3$ is independent of stress. Write an expression for the stress level at which all the carriers will be in the two (Δ , 0, 0) conduction band minima at $T = 0$ K? Explain qualitatively the effect of increasing temperature on this system.
- (b) What is the electrical conductivity for the Si sample in part (a) when all the electrons are in the ($\Delta 00$) and ($\bar{\Delta}00$) carrier pockets and when the electric field is oriented along the $\vec{E} \parallel (011)$ direction? Assume that the longitudinal and transverse effective mass components for Si are $m_l = 0.92m_0$ and $m_t = 0.19m_0$, respectively.
- (c) Find the Seebeck coefficient corresponding to part (b).
4. The valence band edge of lead telluride (PbTe) consists of 4 degenerate ellipsoidal hole pockets at the L -points (effective mass components $m_t = 0.03m_0$ and $m_\ell = 0.24m_0$). For this simplified version of PbTe, assume that the conduction band edge is a mirror image of the valence band edge (i.e., the same effective masses and degeneracies and that the carrier pockets are at the same points in the Brillouin zone). The direct band gap E_g and the static dielectric constant ϵ_0 are $E_g = 0.310$ eV and $\epsilon_0 = 412$, respectively, at 300 K, and assume for simplicity that E_g and ϵ_0 are independent of temperature. In this problem, you are asked to evaluate the effect of T and dopant concentration on E_F :
- (a) First find E_d , the energy of the donor level with respect to the bottom of the conduction band. In which temperature range are most of the donor states ionized?
- (b) Find $E_F(0 \text{ K})$, $E_F(300 \text{ K})$, and $n_i(300 \text{ K})$ for intrinsic PbTe.
- (c) Find $E_F(0 \text{ K})$, $E_F(300 \text{ K})$, and $n_i(300 \text{ K})$ for lightly doped ($N_d = 10^{16}$ carriers/cm³) PbTe. Justify your choice of the distribution function.
- (d) Repeat (c) for heavily doped ($N_d = 10^{19}$ carriers/cm³) PbTe.