MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics of Solids II —6.732

PROBLEM SET #4

Issued: October 10, 2001 Due: October 15, 2001

1. Contrast the values for holes and electrons regarding the Hall coefficient and the transverse magnetoresistance for the holes in silicon for degenerate p-type material with 10^{17} hole carriers/cm^3 in the valence bands and for electrons in degenerate n-type material with 10^{17} electron carriers/cm^3 in the conduction bands. Model the valence bands as a doubly degenerate heavy hole band and a non-degenerate light hole band with spherical Fermi surfaces and effective mass components $m_h = 0.50m_0$ and $m_\ell = 0.16m_0$. Model the electrons to occupy conduction states associated with the 6 electron carrier pockets of Si using the mass components $m_t = 0.19m_0$ and $m_\ell = 0.98m_0$ and take $\vec{H} \parallel (001)$ and $\vec{j} \parallel (100)$.

2. (a) Derive a formula for the reflectivity for a thin film of thickness t and optical constants n and k. Assume that t is within a factor of 2 of the wavelength of light λ.

   (b) Find the complex dielectric constant due to free carriers in a magnetic field assuming the zero field dispersion relation $E(\vec{k}) = \hbar^2 k^2/2m^*$. Consider the Poynting vector for the light along $\vec{B}$ (neglect interband effects), and consider right and left circular polarization.

   (c) Find the dependence of the plasma frequency on magnetic field.

   (d) Sketch the optical reflectivity in a magnetic field and in zero field for right and left circularly polarized radiation.

   (e) Cyclotron resonance is observed when an electromagnetic wave is incident on a sample at $\omega = \omega_c$ where $\omega_c = eB/m^*c$. What happens to an electromagnetic wave incident on a sample at $\omega = \omega_c$? (Is it reflected, absorbed or transmitted?) The cyclotron resonance experiment is normally carried out by varying the magnitude of B. Why does this cyclotron resonance in germanium occur at different values of B for electrons and holes?

3. (a) Assuming one free electron per Cu atom, calculate the plasma frequency for metallic Cu. Why then does Cu have a characteristic reddish color?

   (b) ReO_3 is a reddish semi-transparent material to visible radiation, yet a good electrical conductor. How can this be explained?

   (c) Sapphire is a good thermal conductor, yet an optically transparent material to visible light. How can this be explained?

   (d) Doped SiO_2 is used as a transparent electrode. Explain how it is possible that this material can be used to make good electrical contacts and yet is transparent to visible light.

4. Find the plasma frequency ω_p for a semimetal for light incident in an in-plane direction (such as the x-direction) for the two light polarization directions $\vec{E} \perp z$ and $\vec{E} \parallel z$. The electron pockets are located about the K points at the center of the Brillouin zone edges HKH and the hole pockets are also along the HKH edges as shown. Assume the effective mass components for the electron and hole carriers to be $m_t = 0.06m_0$ and $m_\ell = m_0$, where m_0 is the free electron mass, and $E_F^e = 50\,\text{meV}$ above the conduction band minima for electrons. As an intermediate step you will need to find E_F^h for holes.