MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics of Solids II —6.732

PROBLEM SET #5

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1. Consider the Burstein shift phenomena for heavily doped semiconductors, determine the energy at which appreciable interband (band-to-band) absorption begins at 300K for a p-type GaAs sample $E_g = 1.42\,\text{eV}$ with a hole concentration of 1×10^{20} cm^{-3}. Take into account both the light- and heavy-hole bands, which are degenerate at the Γ point $(m_{lh} = 0.074m_0,\ m_{hh} = 0.62m_0)$.

2. Suppose that you prepare a quantum well structure by molecular beam epitaxy (MBE) from GaAs $(E_g = 1.42\text{eV})$ and Al$_x$Ga$_{1-x}$As $(E_g = 1.80\text{eV})$ where $x = 0.3$ so that Al$_x$Ga$_{1-x}$As is a direct gap semiconductor. Assume that the band off–set of the conduction band is three times greater than in the valence band $(m_e = 0.067m_0)$.

    (a) Assuming a width of the quantum well of L$_z$ = 150Å, at what photon energies can optical absorption take place due to optical transitions between the highest heavy hole and light hole bound states to the lowest conduction band bound state? Use the approximation of an infinite rectangular well in obtaining the energy levels in the bound state.

    (b) Are there selection rules that suppress the transitions between selected valence and conduction band bound states?

    (c) What is the dependence of the threshold photon energy for optical transitions on L_z? For a lightly n-doped system $(n = n_0,\ \tau = \tau_0)$.

    (d) What is the free carrier contribution to the dielectric function at room temperature?

    (e) What is the free carrier contribution to the optical absorption coefficient?

3. For a two level system with energy levels E_1 and E_2 (where $E_1 < E_2$), assume that before $t = 0$ when a light wave of frequency ω and intensity I_0 is applied, the system is in the ground state E_1.

    (a) Find the transition probability for transitions to the state E_2 as a function of time. Consider the response of the system as ω is tuned in energy through resonance, $\hbar\omega_R = E_2 - E_1$.

    (b) Suppose that the system is in state E_2 at time t_0 when the light wave is switched off, find an expression for the probability that state E_2 is still occupied after a time $(t_f - t_0)$.

    (c) Sketch the occupation of states E_1 and E_2 over the time interval $0 \le t \le t_f$ and indicate the change in behavior occurring at time t_0.

4. In many physical cases the momentum matrix element coupling the highest lying valence band and the lowest lying conduction band vanishes by symmetry at the extremal point $\vec{k}_0$. Thus, optical transitions at $\vec{k}_0$ are "forbidden". However, in these cases the momentum matrix element is non–zero as we move away from $\vec{k}_0$ by an arbitrary amount. This gives rise to "forbidden direct interband transitions" which have a different frequency dependence for the optical absorption coefficient than their "allowed counterparts".

(a) By making a Taylor expansion of the wave function $\psi_{n\vec{k}}(\vec{r})$ about the band extremum $\vec{k}_0$ where $\vec{k} = \vec{k}_0 + \vec{\kappa}$, find the dependence of the matrix element $\langle v|\vec{p}|c\rangle$ on the magnitude of $\vec{\kappa}$, assuming that the matrix element vanishes by symmetry at $\vec{K}_0$.

(b) Using the result for part (a), find the frequency dependence of the optical absorption coefficient for the case of a forbidden direct interband transition around an M_0 type critical point.

(c) Compare the frequency dependence of the optical absorption coefficient in (b) to that for direct allowed interband transitions and for indirect optical transitions.

(d) What is the frequency dependence of the optical absorption coefficient for a two-dimensional electron gas for allowed and forbidden interband transitions?