MASSACHUSETTS INSTITUTE OF TECHNOLOGY

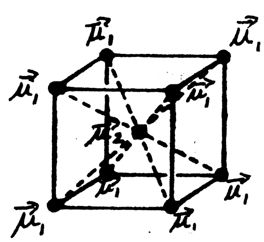
Physics of Solids II —6.732

PROBLEM SET #8

Issued: November 21, 2001 Due: November 28, 2001

1. (a) Find the diamagnetic contribution to the susceptibility of a mole of hydrogen atoms in the ground state at 300K and 1 atmosphere pressure (assuming that such a system could be prepared).
   (b) What is the contribution to the diamagnetic susceptibility of a hydrogen atom in a $2s$ state relative to that in a hydrogen atom in a $1s$ state?
   (c) Now find the paramagnetic contribution to the susceptibility of a mole of hydrogen atoms in the ground state at 300K and 1 Atmosphere pressure. This is called "Van Vleck paramagentism".
   (d) Give a physical reason why one cannot readily prepare a mole of hydrogen atoms in the ground state at 300K and 1 atmosphere pressure.
   (e) Would you expect molecular hydrogen at room temperature to be diamagnetic or paramagnetic? Why?
   (f) What is the paramagnetic contribution to the susceptibility for a mole of gaseous helium (^{4}He)? If the helium gas is a mixture of 50 atomic % ^{3}He and ^{4}He isotopes, what is the paramagnetic contribution to the susceptibility for this helium gas mixture?

2. Consider a magnetic moment $\vec{\mu}_2$ at the center of a cube of other magnetic moments $\vec{\mu}_1$ at the cube corners as shown in the diagram below.



Using the dipole–dipole interaction for each pair of magnetic dipoles

$$E = -\vec{\mu}_2 \cdot \frac{3(\vec{\mu}_1 \cdot \vec{r})\vec{r} - r^2\vec{\mu}_1}{r^5},$$

compare the dipole–dipole energy due to the interaction of the central moment with its 8 nearest neighbors for $\vec{\mu}_2 \parallel \hat{x}$ and for

(a) $\vec{\mu}_1 \parallel \hat{x}$
(b) $\vec{\mu}_1 \parallel \hat{y}$
(c) $\vec{\mu}_1 \parallel \hat{z}$
(d) all $\vec{\mu}_1$ vectors are radially inward.
(e) Which of these configurations has the lowest energy?

3. Consider that a rare earth neodymium ion (which is the key component in a Nd:YAG laser) enters a solid substitutionally as an Nd^{3+} ion with electronic configuration $4f^3$.

   (a) Using Hund's rule, find s, ℓ, j.

   (b) Suppose that the Nd^{3+} ion is put into a magnetic field. What is the expected low field Zeeman pattern and energy separation between adjacent magnetic levels associated with the ground state and with the first excited state?

   (c) How does the low magnetic field pattern change for the case of the Nd^{2+} ion in the ground state?

   (d) Find the paramagnetic susceptibility of a mole of $NdCl_3$.

4. (a) For angular momentum $j = 3/2$, there are four possible values for m_j. To what values of θ in the classical limit do these values of m_j correspond?

   (b) The angular momentum states for right and left circular polarization correspond to $m_j = \pm 1$ and to the spherical harmonics $Y_{1,1}(\theta, \phi)$ and $Y_{1,-1}(\theta, \phi)$. What are the spherical harmonics corresponding to linear polarization along x and y?

   (c) Compare the total j, s and ℓ values for the ground state of the multiplet for a $2s^2 2p$ electronic configuration, a $2s2p^2$ configuration and a $2s2p3d$ configuration.