MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics of Solids II —6.732

PROBLEM SET #8

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1. Starting from the matrix elements for the position and momentum of the harmonic oscillator states in §5.3,

$$\langle \ell | x | \ell' \rangle = \sqrt{\frac{\hbar}{2m_c^* \omega_c^*}} \left[\sqrt{\ell + 1}\, \delta_{\ell', \ell+1} + \sqrt{\ell}\, \delta_{\ell', \ell-1} \right]$$

and

$$\langle \ell | p_x | \ell' \rangle = \sqrt{\frac{\hbar m_c^* \omega_c^*}{2}} \left[\sqrt{\ell + 1}\, \delta_{\ell', \ell+1} + \sqrt{\ell}\, \delta_{\ell', \ell-1} \right]$$

   (a) Find the corresponding matrix elements for $\langle \ell \mid x^2 \mid \ell' \rangle$ and $\langle \ell \mid p^2 \mid \ell' \rangle$.

   (b) Show that the equipartition theorem applies to harmonic oscillator states: half the total energy goes into kinetic energy, and half into potential energy.

   (c) Using the matrix elements in part (a), explain the degeneracy of the Landau levels in the limit $B \to 0$, as (ℓ, k_x) goes into (k_x, k_y).

2. Suppose that a magnetic field of 10T is applied along a (100) direction to a silicon crystal doped n-type with $10^{18}/\text{cm}^3$ arsenic impurity atoms.

   (a) What is the electron concentration for each of the 6 conduction band carrier pockets in zero magnetic field and in a field of 10 T? Note that carriers empty out of the carrier pockets that have large Landau level separations.

   (b) At what magnetic field will the carrier pockets with the light cyclotron masses be completely emptied out?

   (c) Is there a magnetic field direction for which there is no transfer of carriers between carrier pockets?

3. Suppose that you have a modulation doped (n–type) quantum well structure composed of layers of GaAs/$Ga_{1-x}Al_xAs$ such that the bulk carrier density in the GaAs is $10^{16}/\text{cm}^3$ and the width of the quantum well is 80Å. (Use $m_e^* = 0.07 m_0$, $E_{g1} = 1.42$eV for GaAs, and $E_{g2} = 1.70eV$ for $Ga_{1-x}Al_xAs$ and $\Delta E_c = 3\Delta E_v$ for the band offsets). For simplicity in calculating the energy levels, use the energy eigenvalues of the infinite well.

   (a) What is the quantum well widths range so that two bound states are contained in the quantum well at zero magnetic field. How many Landau levels are occupied at a field of 10 Tesla applied normal to the two dimensional electron gas? What is the fractional occupation of the last Landau level? The fractional occupation refers to the number of occupied states in the Landau level compared to the total number of states in the Landau level obtained from the degeneracy factor.

   (b) Give design parameters for a quantum well that has only 1 bound state level, and this level has a filling factor or fractional occupation of 1/3.

4. Bismuth nanowires grow along a (012) direction in alumina templates. For large diameters these wires are semimetals and as the diameter is decreased to about 50 nm, the wires undergo a semimetal to semiconductor transition. The crystal structure of bismuth for this problem is simplified to be two equivalent inter-penetrating face centered cubic lattices in the NaCl structure. Assume that the electron carriers are located at the L points in the fcc Brillouin zone. Approximate the effective mass components for the electrons as $m_1 = m_3 = 0.001m_0$ and $m_2 = 0.26m_0$.

   (a) Neglecting quantum confinement effects, find E_F for an electron concentration of 10^{18} electrons/cm^3.

   (b) What is the effective mass (or masses) along the wire direction for transport in a magnetic field applied along the wire axis direction?

   (c) What are the corresponding cyclotron effective masses for the various ellipsoidal carrier pockets?