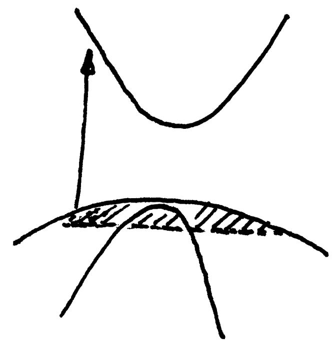
P.S. #5 Solutions

(1) $n_{lh} = \frac{1}{4\pi^3} \cdot \frac{4\pi}{3} k_F^3 = \frac{1}{3\pi^2} \cdot \left(\frac{2 m_{lh} E}{\hbar^2} \right)^{3/2}$



$$n_h = \frac{1}{3\pi^2} \left(\frac{2 m_0}{\hbar^2} \right)^{3/2} \left[\left(\frac{m_{lh}}{m_0} \right)^{3/2} + \left(\frac{m_{hh}}{m_0} \right)^{3/2} \right] E_F^{h\,3/2}$$

$\Downarrow$

$E_F^h = 122 \text{ meV}$

Assume that significant electron population starts at

$E_h = E_F^h - 2kT = 122 - 52 = 70 \text{ meV}$

Find k_0^h - from light holes

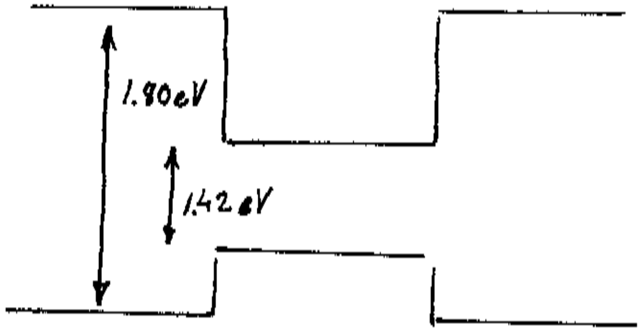
$$k_0^h = \sqrt{\frac{2 m_{lh} E_h}{\hbar^2}}$$

$$k^e = k^h = \sqrt{\frac{2 m_e E_e}{\hbar^2}} \quad \Rightarrow \quad E_e = E_h \frac{m_{lh}}{m_e} = 77 \text{ meV}$$

$\Downarrow$

$$\hbar\omega_{BURSTEIN} = E_g + E_h + E_e = 1.567 \text{ eV}$$

② ⓐ



$$E_e^{(1)} = \frac{\hbar^2 \pi^2}{2 m_e L_z^2} = \frac{(1.05 \cdot 10^{-34})^2 \cdot (3.14)^2}{2 \cdot 0.067 \cdot 9.1 \cdot 10^{-31} \cdot (150 \cdot 10^{-10})^2} = 25\ meV$$

$$E_{hh}^{(1)} = 25 \cdot \frac{m_e}{m_{hh}} = 2.7\ meV$$

$$E_{lh}^{(1)} = 22.5\ meV$$

$$\hbar\omega_{lh-e} = E_g + E_e^{(1)} + E_{lh}^{(1)} = 1.467\ eV$$

$$\hbar\omega_{hh-e} = 1.448\ eV$$

ⓑ Selection Rule For Q.W. Interband Transitions

$$\Delta n_z = 0$$

$$P_{if} = |\langle \Psi_i | \bar{p} | \Psi_f \rangle|^2 \quad \text{transition probability}$$

$$\Psi_i = \varphi(n_{band}, k_x, k_y)\, \phi(n_z)$$

$$\langle \Psi_i | \bar{p} | \Psi_f \rangle = \underbrace{\langle \varphi_i | \varphi_f \rangle}_{= 0 \ \leftarrow \text{ interband transition}} \langle \phi_i | \bar{p} | \phi_f \rangle + \langle \varphi_i | \bar{p} | \varphi_f \rangle \underbrace{\langle \phi_i | \phi_f \rangle}_{= \delta(n_z^i, n_z^f)}$$

(c) From (a) $\hbar\omega = E_g + \frac{\hbar^2 \pi^2}{2} \left[\frac{1}{m_e} + \frac{1}{m_{hh}} \right] \cdot \frac{1}{L_z^2}$

(d)

$$\varepsilon = \varepsilon_{core} + \frac{4\pi i}{\omega} \cdot \frac{n e^2 \tau}{m(1 - i\omega\tau)}$$

$$\varepsilon_{Free\ Carrier} = \frac{4\pi i}{\omega} \cdot \frac{n_0 e^2 \tau_0}{m_e(1 - i\omega\tau_0)}$$

But this is true only for $\varepsilon_{x,y}$.

For ε_z there is no contribution from free carriers.

(e) $\alpha_{abs} = \frac{\omega k}{c} = \frac{\omega\, Im(\sqrt{\varepsilon})}{c}$

$$\varepsilon = \frac{4\pi n_0 e^2 \tau_0}{m_e \omega (1 + \omega^2 \tau_0^2)} (i - \omega\tau_0)$$

$$\alpha_{abs} = \sqrt{\frac{4\pi n_0 e^2 \omega \tau_0}{m_e c^2 (1 + \omega^2 \tau_0^2)}}\, Im\left(\sqrt{i - \omega\tau_0}\right)$$

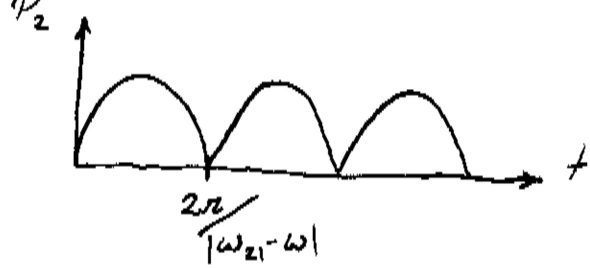
where $\sqrt{x + iy} = r^{1/2} e^{i\theta/2}$

$$\begin{cases} r = (x^2 + y^2)^{1/2} \\ \theta = tg^{-1}(y/x) \end{cases}$$

$$\alpha_{abs} \sim \sqrt{\omega} \quad \text{for } \omega\tau \ll 1$$

(3) (a) The treatment presented in Appendix A leads to the result that the transition probability dependence on frequency and time is:

$$P \propto I_0 \, |\langle \Psi_2 | \bar{p} | \Psi_1 \rangle|^2 \cdot \frac{\sin^2\left(\frac{E_2 - E_1 \pm \hbar\omega}{2\hbar} t\right)}{(E_2 - E_1 \pm \hbar\omega)^2}$$

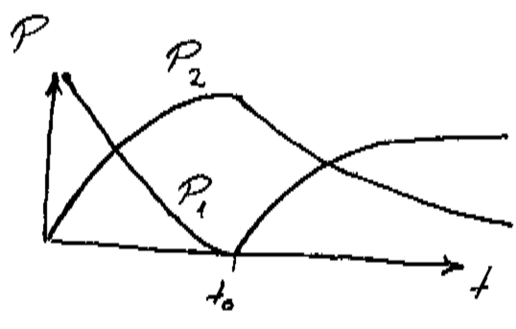


(b) The population does not change in the absence of the perturbation because E_2 is an eigenstate of H_0

In practise the system will relax back to E_1

$$P = e^{-\frac{t - t_0}{\tau}}$$ (1st order kinetics)

(c)



④ ⓐ $\varphi_{nk}(k) = \varphi_{nk_0} + K\, \vec{\nabla}_k\, \varphi_{nk_0}$

$$\langle v|p|c\rangle = \underbrace{\langle \varphi_{vk_0}|p|\varphi_{ck_0}\rangle}_{0} + K\,\langle \vec{\nabla}_k \varphi_{vk_0}|\bar{p}|\varphi_{ck_0}\rangle + K\,\langle \varphi_{vk_0}|\bar{p}|\vec{\nabla}_k \varphi_{ck_0}\rangle$$

The operator $\vec{\nabla}_k$ is related to $\bar{r}$, therefore the brackets are non-vanishing integrals of even functions.

ⓑ $\alpha = \frac{c}{\omega}|\langle v|p|c\rangle|^2 \rho_{cv}(\omega) = \frac{c}{\omega}K^2 \cdot (\hbar\omega - E_g)^{1/2}$

$\hbar\omega = E_g + f(m)\cdot K^2$

$\Downarrow$

$\alpha = \frac{1}{\omega}(\hbar\omega - E_g)^{3/2}$

ⓒ Direct : $\frac{1}{\omega}(h\omega - E_g)^{1/2}$

Indirect : $\frac{1}{\omega}(\hbar\omega - E_g \pm \hbar\omega_{ph})^2$

ⓓ For 2D $\rho_{cv} = \text{const.}$

Direct : $\alpha = \frac{1}{\omega}$ $(\hbar\omega \geq E_g)$

Forbidden : $\alpha = \frac{1}{\omega}(\hbar\omega - E_g)$