MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics of Solids II —6.732

Quiz #1 October 16, 1998

1. 60 points

Suppose that you have just discovered a new cubic compound semiconductor called novelite which crystallizes in the zincblende structure, a common crystal structure for III-V compound semiconductors such as GaAs. Suppose that the valence band maxima are at the Γ point ($k = 0$) are associated with constant energy surfaces that are approximately spherical and consist of a heavy hole band ($m^*_{hh} = m_0$), a degenerate light hole band ($m^*_{lh} = m_0/3$), and a split-off band ($m^*_{soh} = m_0/5$) which is 0.1 eV below the valence band maximum. Suppose that the conduction band minima are at all Σ points crystallographically equivalent to $(0.85,0.85,0)\pi/a$. These equivalent points are all located along all the equivalent $\{110\}$ directions (ΓK).

Assume the following dispersion relations for the electrons

$$E_c(\vec{k}) = \frac{\hbar^2}{2}\left(\frac{k_1^2}{m_1^*} + \frac{k_2^2}{m_2^*} + \frac{k_3^2}{m_3^*}\right)$$

where the $\vec{k}_1$ and $\vec{k}_2$ vectors are, respectively, along the (110) and $(1\bar{1}0)$ directions, and $\vec{k}_3$ is along the (001) direction. Assume that along the longitudinal (110) direction $m_1^* = m_0$, and along the directions transverse to this direction take $m_2^* = m_0/3$ and $m_3^* = m_0/9$.

(a) If each electron carrier pocket has n electrons, find an expression for the minimum k_F for a single electron ellipsoid in terms of n and the effective mass components.

(b) What is the length of k_F along the (111) direction relative to the minimum k_F?

(c) How many equivalent electron ellipsoids are there?

(d) Find an expression for the density of states for the electrons.

(e) Find an expression for the density of states for the holes.

(f) Find an expression for the Fermi energy at temperature T if no dopants are added and all the carriers are generated thermally.

(g) Assuming equal relaxation times for all the holes and electrons, find the relative contributions of each electron and hole carrier type to the electrical conductivity (assume $\vec{E} \parallel (110)$).

(h) Which carrier pocket makes the largest contribution to the transverse magnetoresistance?

2. 40 points

Suppose that you have an fcc semimetal with 2 atoms per unit cell. Suppose that the electrons are at the L points, $\pi/a(1,1,1)$, ($m_l^* = 0.3m_0$ and $m_t^* = 0.1m_0$) in the Brillouin zone and the holes are in a single carrier pocket at the Γ point ($k = 0$) with $m_h^* = 0.3m_0$, and assume that the energy overlap for this semimetal is 10 meV.

(a) Find the position of the Fermi level for the 3 dimensional semimetal at $T = 0$.

(b) What is the smallest phonon wavevector required for inter-pocket scattering between the electron pockets for this 3D semimetal?

(c) What is the smallest vector for inter-pocket scattering if umklapp processes were to occur?

(d) Suppose that the semimetal is now prepared as a thin layer (quantum well) between alkali halide insulating barriers with the (001) crystalline direction normal to the thin layer of the semimetal (layer thickness = 50 Å). Find an expression for the energy of the lowest subband of the L point electrons as a function of the semimetal layer thickness.

(e) At what layer thickness does the thin film experience a semimetal-semiconductor transition?

(f) Suppose that the semimetal thin film is grown along a (111) direction. Is the layer thickness for the semimetal-semiconductor transition larger, smaller or the same as in (d)? Why?

(g) Design an experiment that will tell you that the semimetal-semiconductor transition has occurred.