MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics of Solids II —6.732

Quiz #2 November 13, 1998

1. 40 points

Suppose that we have a heavily doped cubic semiconductor with $10^{19}/\text{cm}^3$ donor impurities that give rise to electron occupation of the lowest available conduction band states at the X-point in the Brillouin zone. Suppose that the sample is mounted in a pressure cell with an optical window and suppose that stress is applied along one of the (100) directions to lower the energy of that X-point pocket relative to that of the other X-point pockets by an amount δ proportional to the applied stress level. Suppose that the effective mass components of the X-point carriers are m_ℓ and m_t, the core dielectric constant is $\varepsilon_{\text{core}}$, and the light is incident parallel to the stress direction.

(a) Give an expression for the plasma frequency in the absence of stress.

(b) What is the corresponding expression for the plasma frequency at a high stress level when all the carriers are in the lowest lying carrier pocket?

(c) Find an expression for the absorption coefficient due to the free carriers in cases (a) and (b).

(d) Is the absorption coefficient anisotropic in cases (a) or (b) and why?

2. 30 points

(a) Find an expression for the joint density of states for a 1D electron gas (e.g., a quantum wire of 100Å diameter) for each of the pertinent types of singularities ($M_0, \ldots$). How many types of such singularities are there in 1D?

(b) What is the selection rule for allowed interband transitions in (a)?

(c) Is the binding energy for an exciton in a semiconducting wire of 100Å diameter increased or decreased relative to bulk values if the Bohr radius of the exciton for this material in the bulk is 30Å? Under which conditions would the effect of confining the exciton within a small diameter wire be large? What is the reason for your answer?

(d) Suppose that you have a 2D superlattice sample of $Si_{1-x}Ge_x/Si$ with a width of 10Å for both the $Si_{1-x}Ge_x$ quantum wells and the Si barriers on a $Si_{1-x}Ge_x$ substrate. Would you expect the Si–Si Raman frequency to be upshifted or down shifted relative to bulk Si? Why?

3. 30 points

The electromagnetic interaction is given by

$$\mathcal{H}'_{em} = -\frac{e}{mc}\vec{p}\cdot\vec{A}$$

where $\vec{A} = \vec{A}_0 \exp[i(\vec{K}\cdot\vec{r} - \omega t)]$.

(a) By considering the matrix element of $\mathcal{H}'_{em}$ between Bloch states show that in the visible range ($\hbar\omega = 2$ eV), direct interband transitions occur between wave vectors $\vec{k}$ for the initial state to $\vec{k}'$ for the final state such that $\vec{k} \approx \vec{k}'$.

(b) As the photon energy is increased, the difference between the incident and scattered wavevectors increases. At what photon energy are interband transitions from the zone center to the zone edge possible, assuming a continuum of final states to sufficiently high energies. (Assume a simple cubic material with a lattice constant of 2Å.)

(c) If the bandgap of a semiconductor decreases upon application of pressure, would you expect the photon energy threshold for interband transitions to increase or decrease with increasing temperature? Give the reasons for your conclusion.

(d) As the quantum number of a shallow donor impurity level in a semiconductor becomes smaller, the effective Bohr radius becomes smaller. What is the effect of this decreased effective Bohr radius on the approximation of a constant dielectric constant ε in the Coulomb Hamiltonian $\mathcal{H}' = -e^2/(\varepsilon r)$? Because of this effect (called the central cell correction), is the energy of the observed $1s \rightarrow 2p$ donor level transition greater or smaller than that predicted for the hydrogenic shallow impurity level model? Why? Does the hydrogenic model underestimate or overestimate the effective Bohr radius for the $n = 1$ impurity level? Why?