MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics of Solids II —6.732

Quiz #3 December 7, 1998

1. (45 points)

Consider the six ellipsoidal conduction band carrier pockets in silicon with longitudinal and transverse effective masses of $m_l/m_0 = 1.0$ and $m_t/m_0 = 0.2$, respectively. Suppose that a magnetic field is applied in the (100) direction. The magnetic energy levels for an electron associated with one of these ellipsoidal constant energy surfaces (neglecting the spin of the electron) is given by

$$E_n(k_B) = \hbar^2 k_B^2/2m_\parallel^* + \hbar\omega_c^*(n + 1/2)$$

where k_B and $m_\parallel^*$ are, respectively, the wavevector and the effective mass component along the magnetic field, and ω_c^* is the cyclotron frequency corresponding to motion in the plane perpendicular to the magnetic field. Assume that the spin splitting of the Landau levels can be neglected.

(a) Suppose that there are a total of 10^{17} electrons/cm^3 in the conduction band of a silicon specimen. For a magnetic field along the (100) direction, find the magnetic field value B_0 for which all of the carriers have been emptied out of the light cyclotron mass carrier pockets.

(b) For the magnetic field B_0 in part (a), how many Landau levels are occupied in the heavy cyclotron mass pockets, and what is the occupation for each? Some useful formulae may be:

$$n = \frac{\sqrt{2}}{\pi^2\lambda^3}\sum_{l=0}^{l_F}(l'_F - l)^{1/2}$$

where $l'_F = E_F/(\hbar\omega_c^*) - 1/2$ and $\lambda = (\hbar c/eB)^{1/2}$.

(c) What is the shift in Fermi level for part (a) relative to the Fermi level at zero magnetic field?

(d) Estimate the temperature range where you would have to conduct your experiments to probe the effects calculated in parts (a), (b), and (c).

(e) Explain physically the dependence of the low temperature resistivity normal to the magnetic field for B in the range $0.9B_0 \leq B \leq 1.1B_0$. (No detailed calculation is expected.)

(f) If we prepare the n-type Si (carrier concentration 10^{17}/cm^3) in a quantum well of 50 Å thickness with quantum confinement along the (100), at what magnetic field B_2 along (100) does the $n = 2$ Landau level pass through the Fermi level for this 2D electron gas? Explain physically the dependence of the low temperature resistivity normal to the magnetic field B, for B in the range $0.9B_2 \leq B \leq 1.1B_2$.

(g) If a thin Si wire of 50 nm diameter with its axis oriented along (100) is prepared, estimate the magnetic field value above which boundary scattering by the wire boundary would be greatly reduced.

2. (30 points)

Consider that the rare earth erbium impurity (which is used for fiber optics amplifiers) enters a crystal lattice substitutionally as an Er^{3+} ion with an electronic configuration $4f^{11}5s^2 5p^6$.

(a) Using Hund's rule, find s, ℓ, j.

(b) Using the formula for the Landé g-factor

$$g = \frac{(3/2)j(j+1) - (1/2)\ell(\ell+1) + (1/2)s(s+1)}{j(j+1)}$$

what is the expected magnetic moment in units of Bohr magnetons for each Er^{3+} ion in its ground state?

(c) Suppose that the Er^{3+} ion is put into a low magnetic field. What is the expected energy separation between adjacent magnetic levels associated with the ground state multiplet? What condition is satisfied in the low magnetic field region?

(d) What is the magnetic level spacing at high magnetic fields? What condition must be satisfied to be in the high magnetic field region?

(e) Write an expression for the spin-orbit splitting of the Er^{3+} level.

(f) What is the spin-orbit splitting for gadolinium Gd^{3+} in an electronic configuration $4f^7 5s^2 5p^6$?

3. (25 points)

(a) Is liquid helium ($1s^2$ atomic configuration) paramagnetic or diamagnetic? Why?

(b) EuTe is an antiferromagnetic semiconductor with a Néel temperature of 10 K. The main carrier scattering mechanisms are due to departures from the symmetry of the ideal crystal structure arising from phonons, impurities and spin disorder. Indicate features in the temperature dependence of the resistivity associated with each of these mechanisms.

(c) To reduce the size of the magnetic particles used for information storage, would you look for materials with low or high magnetic anisotropy energy, or with low or high exchange energy?

(d) Atomic Ni is in a $3d^8 4s^2$ configuration and forms compounds such as $NiCl_2$ where the nickel ion is in a $3d^8$ configuration, while the nickel in $Ni(CO)_4$ is in a $3d^{10}$ configuration. What is the temperature dependence of the magnetic susceptibility expected for each of the compounds $NiCl_2$ and $Ni(CO)_4$?

(e) Gadolinium metal forms a ferromagnet with a magnetic ordering Curie temperature of 292 K. Write an expression for the saturation magnetization at $T = 0$, and estimate the number of Bohr magnetons per Gd atom.