Quiz #2 - Solutions

1. ⓐ $E_F^h = kT \ln\left(\frac{N_c}{N_A}\right)$

$$N_c = 2\left(\frac{m_o kT}{2\pi\hbar^2}\right)^{3/2}\left[\left(\frac{m_{hh}}{m_o}\right)^{3/2} + \left(\frac{m_{lh}}{m_o}\right)^{3/2}\right]$$

$$= 2 \cdot 1.27 \cdot 10^{25} \cdot 0.483 = 1.23 \cdot 10^{25}\ m^{-3}$$

$$E_F^h = 26 \cdot \ln\left(\frac{1.23 \cdot 10^{25}}{1 \cdot 10^{24}}\right) = 65\ meV$$

ALTERNATIVELY:

$$N_A = \frac{2}{(2\pi)^3}\int_0^{k(E_F)} 4\pi k^2 dk$$

$$= \frac{1}{3\pi^2}\left(k_F^3(lh) + k_F^3(hh)\right) = \frac{1}{3\pi^2}\left(\frac{2m_o E_F}{\hbar^2}\right)^{3/2}\left[\left(\frac{m_{hh}}{m_o}\right)^{3/2} + \left(\frac{m_{lh}}{m_o}\right)^{3/2}\right]$$

$$= \frac{1}{3.4 \cdot 10^{-2}} \cdot 2.12 \cdot 10^{57} \cdot 0.483 \cdot E_F^{3/2} = 10^{24}$$

$$E_F = \left(3.3 \cdot 10^{-35}\right)^{2/3} J = 1 \cdot 10^{-23} J = 0.06\ meV$$

ⓑ $n \approx 0\ ,\ p \approx 10^{18}\ cm^{-3}$

ⓒ $\varepsilon_1(\omega_p) = \varepsilon_{core} - \frac{4\pi e^2 \tau^2}{(1+\omega_p^2\tau^2)} \cdot \left[\frac{n_{lh}}{m_{lh}} + \frac{n_{hh}}{m_{hh}}\right] = 0$

$$\omega_p^2 = \frac{4\pi e^2}{\varepsilon_{core}} \cdot \left[\frac{n_{lh}}{m_{lh}} + \frac{n_{hh}}{m_{lh}}\right]$$

$$n_{lh} = (n_{hh} + n_{lh}) \cdot \frac{m_{lh}^{3/2}}{m_{hh}^{3/2} + m_{lh}^{3/2}}$$

$$\omega_p^2 = \frac{4\pi e^2 p}{\varepsilon_{core}} \cdot \left(\frac{m_{lh}^{1/2} + m_{hh}^{1/2}}{m_{lh}^{3/2} + m_{hh}^{3/2}}\right) = 8.36 \cdot 10^{27} \cdot 2.15\ sec^{-2}$$

$$= 1.8 \cdot 10^{28}\ sec^{-2}$$

$$\omega_p = 1.34 \cdot 10^{14}\ Hz$$

(d) $\Delta E_g = \frac{\hbar^2 \pi^2}{2 m_e L^2} + \frac{\hbar^2 \pi^2}{2 m_{hh} L^2} = 3.8 \cdot 10^{-20}\,J = 0.238\,eV$

(e) The shift in energy due to quantum confinement is greater for the light holes. Therefore, the density of heavy holes will increase.

Since $\omega_p \propto m^{-1}$, and $m_{effective}$ increases,

- ω_p decreases.

(f) $\alpha(\omega) \propto \frac{\rho_{cv}(\hbar\omega)}{\omega}$

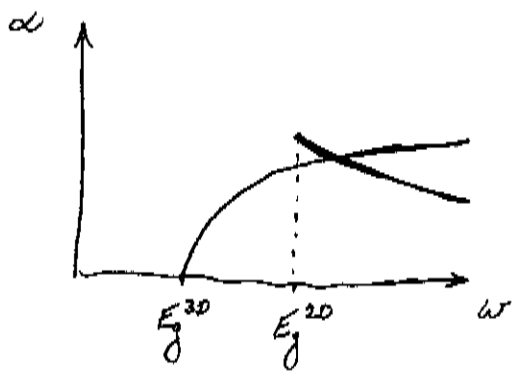
$$\rho_{cv}(\hbar\omega) = \frac{2}{(2\pi)^3} \int \frac{dS}{\nabla_k (E_c - E_v)|}_{E_c - E_v = \hbar\omega}$$

$$\rho_{cv}[3D] = \frac{2}{(2\pi)^3} \cdot \frac{4\pi k^2}{\left(\frac{\hbar^2 k}{2m^*}\right)}$$

$$\rho_{cv}[2D] = \frac{2}{(2\pi)^2} \cdot \frac{2\pi k}{\left(\frac{\hbar^2 k}{2m^*}\right)}$$

$\alpha^{3D}(\omega) \propto \frac{k}{\omega} \sim \frac{\sqrt{\hbar\omega - E_g}}{\omega}$ $\quad \hbar\omega > E_g$

$\alpha^{2D}(\omega) \propto \frac{1}{\omega}$ $\quad \hbar\omega > E_g$



(g) If the quantum well width is smaller than the exciton diameter, the charges will be forced to be closer together in space, increasing their binding energy.

(2) (a) Silicon has a large absorption coefficient at frequencies above the band gap due to the band-tracking. Large α, means large ε_2, and large contribution to the integral.

(b) The phonon frequency depends on the lattice force constant and masses $\omega = \sqrt{\frac{k}{m}}$

therefore $\omega_C > \omega_{GaAs}$

The LO-TO phonon splitting indicates a polarizable substance. GaAs will show a large splitting, but carbon not.

(c) Quantum confinement effects increase the band gap. Increase of the lattice period most commonly decreases the band gap (less perturbation).