MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Applications of Group Theory to the Physics of Solids—6.734J & 8.510J

PROBLEM SET # 1

Issued: February 8, 2002 Due: February 15, 12002

1. (a) Given a set of matrices that represent the group G, denoted by $D(R)$ (for all R in G), show that the matrices obtainable by a similarity transformation $UD(R)U^{-1}$ also are a representation of G.

   (b) Multiply the two left cosets of subgroup (E, A): (B, F) and (C, D), referring to §1.5 of the class notes. Is the result another coset or do the cosets join to one another?

   (c) Prove that in order to form a normal or self-conjugate subgroup, it is necessary to include entire classes in this subgroup.

   (d) Demonstrate that the normal subgroup of $P(3)$ includes entire classes.

2. (a) What are the symmetry operations for the molecule AB_4, where the B atoms lie at the corners of a square and the A atom is at the center and is not coplanar with the B atoms.

   (b) Find the multiplication table.

   (c) List the subgroups. Which subgroups are self-conjugate?

   (d) List the classes.

   (e) Find the multiplication table for the factor group for the self-conjugate subgroup(s) of (c).

3. The group defined by the permutations of 4 objects, $P(4)$, is isomorphic with the group of symmetry operations of a regular tetrahedron (T_d). The symmetry operations of this group are sufficiently complex so that the power of group theoretical methods can be appreciated. For notational convenience, the elements of this group are listed below.

$$\begin{array}{llll} e = (1234) & g = (3124) & m = (1423) & s = (4213) \\ a = (1243) & h = (3142) & n = (1432) & t = (4231) \\ b = (2134) & i = (2314) & o = (4123) & u = (3412) \\ c = (2143) & j = (2341) & p = (4132) & v = (3421) \\ d = (1324) & k = (3214) & q = (2413) & w = (4312) \\ f = (1342) & l = (3241) & r = (2431) & y = (4321) \end{array}$$

Here we have used a shorthand notation to denote the elements: for example $j = (2341)$ denotes

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

that is, the permutation which takes objects in the order 1234 and leaves them in the order 2341.

(a) What is the product vw? wv?

(b) List the subgroups of this group which correspond to the symmetry operations on an equilateral triangle.

(c) List the right and left cosets of the subgroup (e, a, k, l, s, t). Are they the same?

(d) List all the symmetry classes for $P(4)$, and relate them to symmetry operations on a regular tetrahedron.

(e) Find the factor group and multiplication table formed from the self-conjugate sub-group (e, c, u, y). Is this factor group isomorphic to $P(3)$?