MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Applications of Group Theory to the Physics of Solids—6.734J & 8.510J

PROBLEM SET #6

Issued: April 5, 2002 Due: April 12, 2002

1. Suppose that a stress is applied to fcc aluminum in a high symmetry direction such as the (100) and (111) directions, and suppose that the effect of the resulting strain is to lower the symmetry of aluminum from cubic O_h symmetry to a lower point group symmetry. The situation outlined here arises in the fabrication of very thin aluminum films on substrates.

   (a) How many independent elastic constants are there in the thin aluminum film if it is stressed along the (100) direction? The lower symmetry group in this case is D_{4h}.

   (b) What is the new symmetrized form of the stress-strain relations for the (100) strain (see Eq. 11.34 of the notes)?

   (c) What is the form of the C_{ij} tensor for the (100) strain (see Eq. 11.44 of the notes)?

   (d) Now consider the case of thin film growth along a (111) direction. How many independent elastic constants are there in the case of aluminum stressed along the (111) direction where the lower symmetry group is D_{3h}?

   (e) Compare the form of the C_{ij} tensor for stress along (111) with stress along (100) (part c).

   (f) How would you design an experiment that would be sensitive to the differences in C_{ij} for these two orientations of the thin film growth?

2. Since the magnetoconductivity tensor depends on the $\vec{v} \times \vec{B}$ term in the equations of motion, the magnetoconductivity has more independent tensorial coefficients than the conductivity tensor.

   (a) Find the number of independent coefficients in the magnetoconductivity tensor for a crystal with C_1 symmetry; in this case the zero field conductivity tensor has 6 independent components.

   (b) Find the number of independent coefficients in the magnetoconductivity tensor for GaAs (T_d symmetry).

3. (a) List the real space symmetry operations of the non-symmorphic two-dimensional square space group $p4gm$ (#12). What are the classes? What is the multiplication table for symmetry operations within the unit cell (but including the glide plane operations)?

   (b) Explain the diagrams and the point symmetry entries a, b, c, d for space group #12 ($p4gm$) in Fig. 12.20 which was taken from the International Crystallography Tables.

   (c) Show that in the diamond structure, the product of two symmetry operations involving translations τ yields a symmetry element with no translations

$$\{\alpha|\tau\}\{\beta|\tau\} = \{\gamma|0\}$$

   where $\vec{\tau} = (1, 1, 1)a/4$. What is the physical significance of this result?

4. (a) $(\mathrm{GaAs})_n/(\mathrm{AlAs})_m$ superlattices grown along the {100} direction are lattice matched. What are the space groups appropriate to this superlattice for $(n+m)$ = even integer and for $(n+m)$ = odd integer?

   (b) What are the space groups for $(\mathrm{GaAs})_n/(\mathrm{AlAs})_m$ superlattices grown along the {111} direction when $(n+m)/3 = k$ (where k = integer) and when $(n+m)/3$ is not an integer?

   (c) Describe an experiment that you would carry out to distinguish between these 4 space groups.