MASSACHUSETTS INSTITUTE OF TECHNOLOGY
ESG Physics

8.02 with Kai | Spring 2003

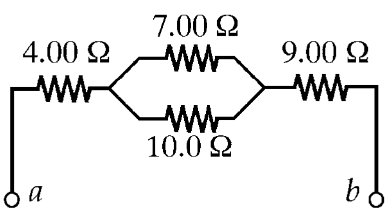
Problem Set 6 Solution

Problem 1: 28.6

(a) Find the equivalent resistance between points *a* and *b* in Figure P28.6.
(b) Calculate the current in each resistor if a potential difference of 34.0 V is applied between points *a* and *b*.

Solution:

(a)



$$R_p = \frac{1}{\frac{1}{7.00} + \frac{1}{10.0}} = 4.12\Omega \tag{1.1}$$

$$R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00 = \boxed{17.1\,\Omega} \tag{1.2}$$

(b) For the $4.00\,\Omega$ and $9.00\,\Omega$ resistors, since $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{34.0}{17.1} = \boxed{1.99\text{ A}} \tag{1.3}$$

For the 7.00 Ω resistor, we have

$$I = \frac{8.18}{7.00} = \boxed{1.17\text{ A}} \tag{1.4}$$

For the 10.0 Ω resistor, we have

$$I = \frac{8.18}{10.0} = \boxed{0.818\text{ A}} \tag{1.5}$$

Problem 2: 28.16.
Two resistors connected in series have an equivalent resistance of 690 Ω. When they are connected in parallel, their equivalent resistance is 150 Ω. Find the resistance of each resistor.

Solution:
Denoting the two resistors as *x* and *y*,

$$x + y = 690 \tag{2.1}$$

and

$$\frac{1}{150} = \frac{1}{x} + \frac{1}{y} \tag{2.2}$$

Substituting equation (2.1) to (2.2), we have

$$\frac{1}{150} = \frac{1}{x} + \frac{1}{690 - x} = \frac{(690 - x) + x}{x(690 - x)} \tag{2.3}$$

which gives

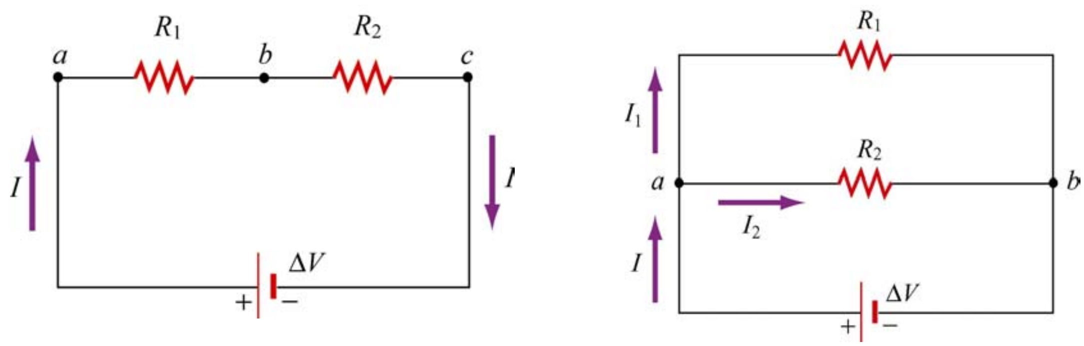
$$x^2 - 690x + 103{,}500 = 0 \tag{2.4}$$

By solving this quadratic equation, we have

$$\boxed{x = 470\ \Omega} \text{ and } \boxed{y = 220\ \Omega} \tag{2.5}$$

Problem 3: 28.17

In Figures 28.4 and 28.5, let $R_1 = 11.0\,\Omega$, let $R_2 = 22.0\,\Omega$, and let the battery have a terminal voltage of 33.0 V.



(a) in the parallel circuit shown in Figure 28.5, which resistor uses more power?

(b) Verify that the sum of the power (I^2R) used by each resistor equals the power supplied by the battery $(I\Delta V)$.

(c) In the series circuit, which resistor uses more power?

(d) Verify that the sum of the power (I^2R) used by each resistor equals the power supplied by the battery $(P = I\Delta V)$.

(e) Which circuit configuration uses more power?

Solution:

(a)

$$\Delta V = IR \tag{3.1}$$

Therefore

$$33.0\text{ V} = I_1(11.0\,\Omega) \text{ and } 33.0\text{ V} = I_2(22.0\,\Omega) \tag{3.2}$$

Therefore,

$$I_1 = 3.00\text{ A} \text{ and } I_2 = 1.50\text{ A} \tag{3.3}$$

Since $P = I^2R$, we have

$$P_1 = (3.00\text{ A})^2(11.0\,\Omega) \text{ and } P_2 = (1.50\text{ A})^2(22.0\,\Omega) \tag{3.4}$$

which gives

$$P_1 = 99.0\text{ W} \text{ and } P_2 = 49.5\text{ W} \tag{3.5}$$

Therefore, the 11.0-Ω resistor uses more power.

(b) Since $P_1 + P_2 = \boxed{148\text{ W}}$, we have

$$P = I(\Delta V) = (4.50)(33.0) = 148\text{ W} \tag{3.6}$$

(c)

$$R_s = R_1 + R_2 = 11.0 + 22.0 = \boxed{33.0\,\Omega} \tag{3.7}$$

Since $\Delta V = IR$,

$$I = \frac{V}{P} = \frac{33.0\ \Omega}{33.0\text{ V}} = \boxed{1.00\text{ A}} \tag{3.8}$$

Since $P = I^2R$, we have

$$P_1 = (1.00\text{ A})^2(11.0\,\Omega) \text{ and } P_2 = (1.00\text{ A})^2(22.0\,\Omega) \tag{3.9}$$

and you will probably get

$$P_1 = 110\text{ W} \text{ and } P_2 = 22.0\text{ W} \tag{3.10}$$

Therefore the 22.0-Ω resistor uses more power.

(d)

$$P_1 + P_2 = I^2(R_1 + R_2) = (1.00\text{ A})^2(33.0\,\Omega) = \boxed{33.0\text{ W}} \tag{3.11}$$

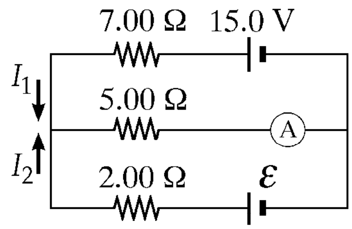
$$P = I(\Delta V) = (1.00\text{ A})(33.0\text{ V}) = 33.0\text{ W} \tag{3.12}$$

(e) The parallel configuration uses more power.

Problem 4: 28.18

The ammeter shown in Figure P28.18 reads 2.00 A. Find I_1, I_2 and ε.

Solution:



Applying Kirchhoff's Rule in the upper loop, we get

$$15.0-(7.00)I_1-(2.00)(5.00)=0 \tag{4.1}$$

which gives

$$\boxed{I_1=0.714\,\mathrm{A}} \tag{4.2}$$

Since

$$I_1+I_2=2\,\mathrm{A} \tag{4.3}$$

therefore

$$\boxed{I_2=1.29\,\mathrm{A}} \tag{4.4}$$

Applying Kirchhoff's Rule in the bottom loop, we get

$$+\varepsilon-2.00(1.29)-(5.00)(2.00)=0 \tag{4.5}$$

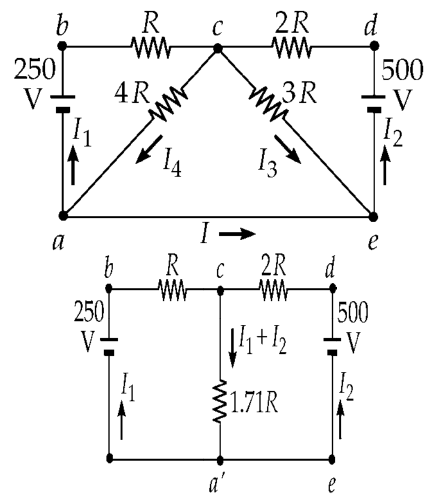
which gives

$$\boxed{\varepsilon=12.6\,\mathrm{V}} \tag{4.6}$$

Problem 5: 28.23

If $R = 1.00\ \text{k}\Omega$ and $\varepsilon = 250\ \text{V}$ in Figure P28.23, determine the direction and magnitude of the current in the horizontal wire between *a* and *e*.

Solution:



Label the currents in the branches as shown in the first figure. Reduce the circuit by combining the two parallel resistors as shown in the second figure.

Apply Kirchhoff's looop rule to both loops in Figure (b) to obtain

$$(2.71R)I_1 + (1.71R)I_2 = 250 \qquad (5.1)$$

$$(1.71R)I_1 + (3.71R)I_2 = 500 \qquad (5.2)$$

With $R = 1000\,\Omega$, simultaneous solution of these equations yields

$$I_1 = 10.0\,\text{mA} \quad \text{and} \quad I_2 = 130.0\,\text{mA} \qquad (5.3)$$

From Figure (b),

$$V_c - V_a = (I_1 + I_2)(1.71R) = 240\,\text{V} \qquad (5.4)$$

Therefore, from Figure (a),

$$I_4 = \frac{V_c - V_a}{4R} = \frac{240\,\text{V}}{4000\,\Omega} \qquad (5.5)$$

which gives

$$I_4 = 60.0\,\text{mA} \qquad (5.6)$$

Finally, applying Kirchhoff's point rule at point *a* in Figure (a) gives

$$I = I_4 - I_1 = 60.0 - 10.0 = 50.0\,\text{mA} \qquad (5.7)$$

or

$$\boxed{I = 50.0\,\text{mA flowing from point } a \text{ to point } e} \qquad (5.8)$$

Problem 6: 28.34

A 4.00-$\mathrm{M}\Omega$ resistor and 3.00-$\mu\mathrm{F}$ capacitor are connected in series with a 12.0-V power supply.

(a) What is the time constant for the circuit?

(b) Express the current in the circuit and the charge on the capacitor as functions of time.

Solution:

(a)

$$\tau = RC = \left(4.00\times10^{-6}\,\Omega\right)\left(3.00\times10^{-6}\,\mathrm{F}\right) \tag{6.1}$$

which gives

$$\boxed{\tau = 12.0\,\mathrm{s}} \tag{6.2}$$

(b) sdfldskfj

$$I = \frac{\varepsilon}{R}e^{-\frac{t}{\tau}} = \frac{12.0}{4.00\times10^{-6}}e^{-\frac{t}{12.0}} \tag{6.3}$$

$$q = C\varepsilon\left[1-e^{-\frac{t}{\tau}}\right] = 3.00\times10^{-6}\left(12.0\right)\left(1-e^{-\frac{t}{12.0}}\right) \tag{6.4}$$

After simplifying, we have

$$\boxed{q = 36.0\mu C\left(1-e^{-\frac{t}{12.0}}\right)} \text{ and } \boxed{I = 3.00\mu A e^{-\frac{t}{12.0}}} \tag{6.5}$$

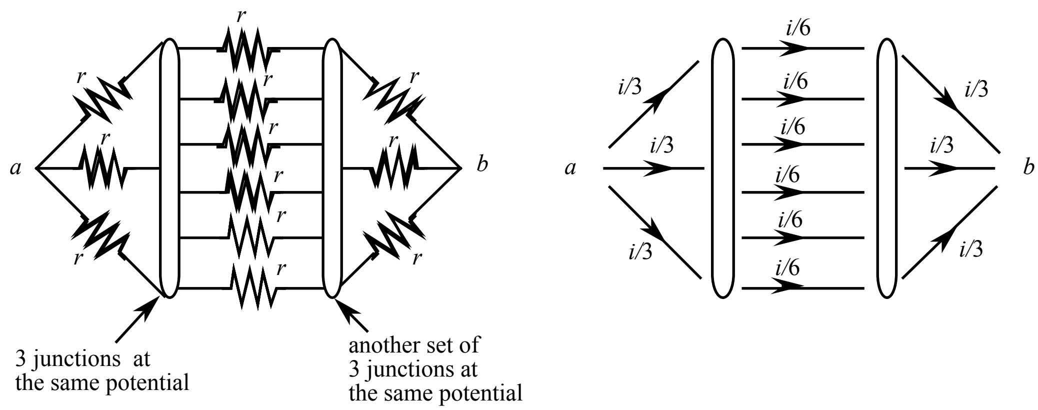
Problem 7: 28.69

(a) Using symmetry arguments, show that the current through any resistor in the configuration of Figure P28.69 is either $\frac{I}{3}$ of $\frac{I}{6}$1 All resistors have the same resistance r.

(b) Show that the equivalent resistance between points a and b is $\frac{5}{6}r$.

Solution:

(a)



(b)