Problem Set 9 Solution
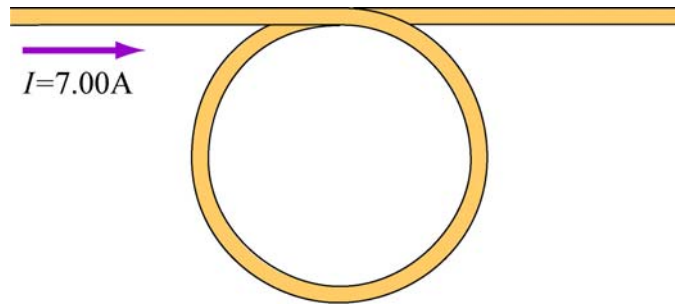
Problem 1: 30-7 and 8

A conductor consists of a circular loop of radius $R = 0.100$ m and two straight, long sections, as shown below. The wire lies in the plane of the paper and carries a current of $I = 7.00$ A. Determine the magnitude and direction of the magnetic field at the center of the loop.

$I=7.00$A

Solution:

We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\dfrac{\mu_0 I}{2\pi R}$ directed into the page) and the field due to the circular loop (having magnitude $\dfrac{\mu_0 I}{2R}$ and directed into the page). The resultant magnetic field is
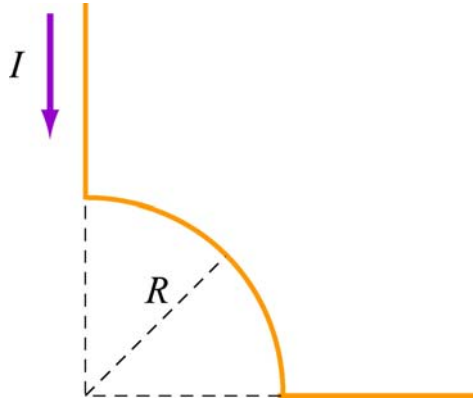
$$B = \left(1 + \frac{1}{\pi}\right)\frac{\mu_0 I}{2R} \tag{1.1}$$

which gives

$$\boxed{\vec{B} = 58.0 \ \mu\text{T directed into the page}} \tag{1.2}$$

Problem 2: 30-9

The segment of wire below carries a current of $I = 5.00$ A , where the radius of the circular arc is $R = 3.00$ cm . Determine the magnitude and direction of the magnetic field at the origin.

Solution:

For the straight sections

$$d\vec{s} \times \hat{r} = 0 \tag{2.1}$$

The quarter circle makes one-quarter of the field of a full loop. Therefore

$$\vec{B} = \frac{1}{4} \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{8R} \text{ into the paper} \tag{2.2}$$

which gives

$$\boxed{\vec{B} = 26.2 \ \mu\text{T into the paper}} \tag{2.3}$$

Problem 3: 30-24

A cylindrical conductor of radius $R = 2.50$ cm carries a current of $I = 2.50$ A along its length; this current is uniformly distributed throughout the cross-section of the conductor.

(a) Calculate the magnetic field midway along the radius of the wire (that is, at $r = \dfrac{R}{2}$)

(b) Find the distance beyond the surface of the conductor at which the magnitude of the magnetic field has the same value as the magnitude of the field at $r = \dfrac{R}{2}$

Solution:

For $r \leq a$,

$$B = \frac{\mu_0 I}{2\pi a^2} r \tag{3.1}$$

Substituting, you will get

$$\boxed{B = 10.0 \ \mu T} \tag{3.2}$$

For magnetic field beyond the conductor's surface, we have

$$r = \frac{\mu_0 I}{2\pi B} \tag{3.3}$$

Substituting, $B = 10.0 \ \mu T$, we will have

$$r = 0.0500 \text{ m} \tag{3.4}$$

which means when it is 2.50 cm beyond the conductor's surface, the magnitude of the magnetic field has the same value as the magnitude of the field at $r = \dfrac{R}{2}$

Problem 4: 31-9

A loop of wire in the shape of a rectangle of width w and length L and a long, straight wire carrying a current I lie on a tabletop as shown below.

    (a) Determine the magnetic flux through the loop due to the current I.

    (b) Suppose that the current is changing with time according to $I = a + bt$, where a and b are constants. Determine the induced emf in the loop if $b = 10.0$ A/s, $h = 1.00$ cm, $w = 10.0$ cm, and $L = 100$ cm. What is the direction of the induced current in the rectangle?

Solution:

    (a)

$$d\Phi_B = \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \frac{\mu_0 I}{2\pi x} L\, dx \tag{4.1}$$

Therefore,

$$\Phi_B = \int_h^{h+w} \frac{dx}{x} \frac{\mu_0 IL}{2\pi} = \boxed{\frac{\mu_0 IL}{2\pi} \ln\left(\frac{h+w}{h}\right)} \tag{4.2}$$

    (b)

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 IL}{2\pi}\ln\left(\frac{h+w}{h}\right)\right] = -\left[\frac{\mu_0 L}{2\pi}\ln\left(\frac{h+w}{h}\right)\right]\left(\frac{dI}{dt}\right) \tag{4.3}$$

Substituting, and knowing the $\dfrac{dI}{dt} = b$, we have

$$\boxed{\varepsilon = -4.80 \ \mu\text{V}} \tag{4.4}$$

The long wire produces magnetic flux into the page through the rectangle. As it increases, the rectangle wants to produce its own magnetic field out of the page, which it does by carrying a *counterclockwise* current.

Problem 5: 31-22

A conducting rod of length l moves on two horizontal, frictionless rails, as shown below. If a constant force of 1.00 N moves the bar at 2.00 m/s through a magnetic field **B** that is directed into the page,

   (a) What is the current through an 8.00 Ω resistor R?

   (b) What is the rate at which energy is delivered to the resistor?

   (c) What is the mechanical power delivered by the force $\mathbf{F}_{app}$?

Solution:

Since
$$F_B = IlB \tag{5.1}$$

and
$$\varepsilon = Blv \tag{5.2}$$

we have
$$I = \frac{\varepsilon}{R} = \frac{Blv}{R} \tag{5.3}$$

Thus
$$B = \frac{IR}{lv} \tag{5.4}$$

   (a) Since
$$F_B = \frac{I^2 lR}{lv} \tag{5.5}$$

   therefore, we have
$$I = \sqrt{\frac{F_B v}{R}} \tag{5.6}$$

   which gives
$$\boxed{I = 0.500 \text{ A}} \tag{5.7}$$

   (b)
$$P = I^2 R = \boxed{2.00 \text{ W}} \tag{5.8}$$

(c) For constant force,

$$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = (1.00 \text{ N})(2.00 \text{ m/s}) \quad (5.9)$$

which gives

$$\boxed{P = 2.00 \text{ W}} \quad (5.10)$$

Problem 6: 31-31

Two parallel rails with negligible resistance are 10.0 cm apart and are connected by a 5.00Ω resistor. The circuit also contains two metal rods having resistances of 10.0Ω and 15.0Ω sliding along the rails. The rods are pulled away from the resistor at constant speeds 4.00 m/s and 2.00 m/s, respectively. A uniform magnetic field of magnitude 0.0100 T is applied perpendicular to the plane of the rails. Determine the current in the 5.00Ω resistor.
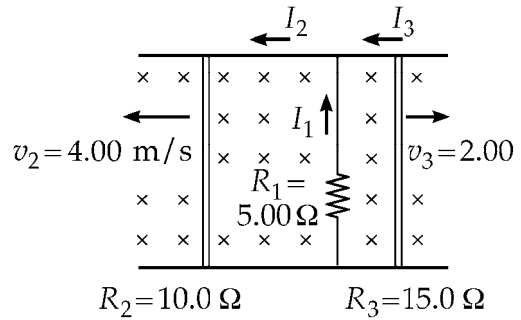
Solution:

Name the currents as shown in the diagram.

Left loop:

$$+Bdv_2 - I_2R_2 - I_1R_1 = 0 \qquad (6.1)$$

Right loop:

$$+Bdv_3 - I_3R_3 + I_1R_1 = 0 \qquad (6.2)$$

At the junction:

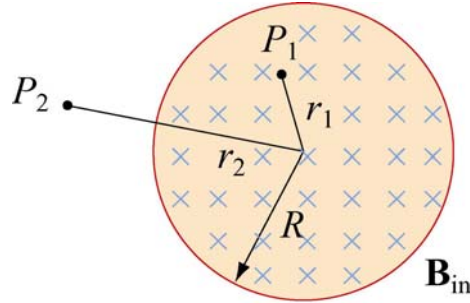$$I_2 = I_1 + I_3 \qquad (6.3)$$

Solving for I_1, we will get

$$I_1 = Bd\left(\frac{v_2R_3 - v_3R_2}{R_1R_2 + R_1R_3 + R_2R_3}\right) \text{ upward} \qquad (6.4)$$

Substituting the known values, we will get

$$\boxed{I_1 = 145 \ \mu\text{A upward}} \qquad (6.5)$$

Problem 7- 31.32

For the situation described in the figure below,

the magnetic field changes with time according to the expression

$$B = \left(2.00t^3 - 4.00t^2 + 0.800\right) \text{ T} \tag{7.1}$$

and $r_2 = 2R = 5.00$ cm.

(a) Calculate the magnitude and direction of the force exerted on an electron located at point P_2 when $t = 2.00$ s.

(b) At what time is the force equal to zero?

Solution:

(a) We know that

$$\frac{dB}{dt} = 6.00t^2 - 8.00t \tag{7.2}$$

and

$$|\varepsilon| = \frac{d\Phi_B}{dt} \tag{7.3}$$

Therefore, at $t = 2.00$ s

$$E = \frac{\pi R^2}{2\pi r_2} \left(\frac{dB}{dt}\right) \tag{7.4}$$

Since $F = qE$, then after substitution, we have

$$\boxed{F = 8.00 \times 10^{-21} \text{ N clockwise for the electron}} \tag{7.5}$$

(b) That happens when $\dfrac{dB}{dt} = 0$, therefore

$$\boxed{t = 1.33 \text{ s}}$$

(7.6)

Problem 8: 31.60

A conductor rod moves with a constant velocity **v** perpendicular to a long, sraight wire carrying a current *I* as shown below. Show that the magnitude of the EMF generated between the ends of the rod is

$$|\varepsilon| = \frac{\mu_0 vI}{2\pi r} l \tag{8.1}$$

Solution:

Since we know that

$$\varepsilon = Blv \tag{8.2}$$

At a distance *r* from wire

$$B = \frac{\mu_0 I}{2\pi r} \tag{8.3}$$

then the result is immediate.

Problem 9: 31.66

A conducting rod of length l moves with velocity $\mathbf{v}$ parallel to a long wire carrying a steady current I. The axis of the rod is maintained perpendicular to the wire with the near end a distance r away. Show that the magnitude of the emf induced in the rod is

$$|\varepsilon| = \frac{\mu_0 I}{2\pi} v \ln\left(1 + \frac{l}{r}\right) \tag{9.1}$$

Solution:

Find an expression for the flux through a rectangular area "swept out" by the bar in time t. The magnetic field at a distance x from wire is

$$B = \frac{\mu_0 I}{2\pi x} \tag{9.2}$$
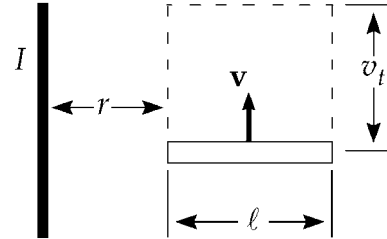
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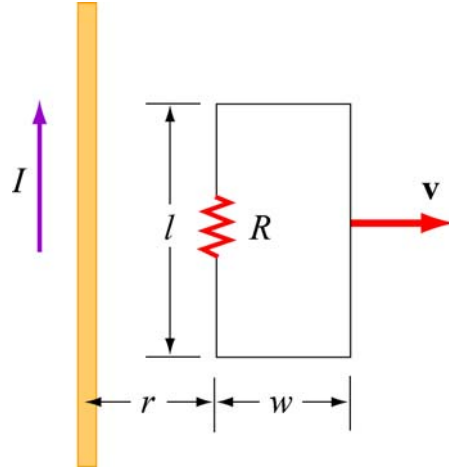
$$\Phi_B = \int B \, dA \tag{9.3}$$

therefore

$$\Phi_B = \frac{\mu_0 I}{2\pi} vt \int_r^{r+l} \frac{dx}{x} \tag{9.4}$$

where vt is the distance the bar has moved in time t. therefore

$$\boxed{|\varepsilon| = \frac{d\Phi_B}{dt} = \frac{\mu_0 I v}{2\pi} \ln\left(1 + \frac{l}{r}\right)} \tag{9.5}$$

Problem 10: 31.67

A rectangle loop of dimensions l and w moves with a constant velocity $\mathbf{v}$ away from a long wire that carries a current I in the plane of the loop.

The total resistance of the loop is R. Derive an expression that gives the current in the loop t the instant the near side is a distance r from the wire.

Solution:

The magnetic field at a distance x from a long wire is

$$B = \frac{\mu_0 I}{2\pi x} \tag{10.1}$$

Therefore

$$d\Phi_B = \frac{\mu_0 I}{2\pi x}\left(l\, dx\right) \tag{10.2}$$

So

$$\Phi_B = \frac{\mu_0 Il}{2\pi}\int_r^{r+w}\frac{dx}{x} = \frac{\mu_0 Il}{2\pi}\ln\left(1+\frac{w}{r}\right) \tag{10.3}$$

Therefore

$$\varepsilon = -\frac{d\Phi_B}{dt} = \frac{\mu_0 Ilv}{2\pi r}\frac{w}{r+w} \tag{10.4}$$

and

$$\boxed{I = \frac{\varepsilon}{R} = \frac{\mu_0 Ilv}{2\pi rR}\frac{w}{r+w}} \tag{10.5}$$