Physics 8.07, Fall 1999

Homework #1

Reading for Thursday September 9: Griffiths pp. 1–24, 58–64

Reading for Tuesday September 14: Griffiths pp. 24–38, 45–57, 65–96

Reading for Thursday September 16: Griffiths pp. 96–109, 110–21, 146–55

Written problem set #1
Due Thursday September 16 by 9:30 AM in the 8.07 homework box in 4-339B.

1. Given the vectors

$$\begin{aligned}\mathbf{A} &= (1,1,1)\\ \mathbf{B} &= (1,2,3)\\ \mathbf{C} &= (3,4,12)\end{aligned}$$

calculate the following quantities:

(a) $|\mathbf{A}|, |\mathbf{B}|, |\mathbf{C}|$
(b) $\widehat{A}, \widehat{B}, \widehat{C}$
(c) $3\mathbf{A} + 2\mathbf{B} + \mathbf{C}$
(d) $\mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C} + \mathbf{B} \cdot \mathbf{C}$
(e) $\mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C} + \mathbf{B} \times \mathbf{C}$

2. Given the vector fields

$$\begin{aligned}\mathbf{u}(\mathbf{r}) &= r^5(\widehat{r} \times \widehat{z})\\ \mathbf{v}(\mathbf{r}) &= e^{-r^2}\widehat{r}\\ \mathbf{w}(\mathbf{r}) &= xye^{-r^3}\widehat{y}\end{aligned}$$

calculate the following expressions:

(a) $\nabla \cdot \mathbf{u}(\mathbf{r})$
(b) $\mathbf{w}(\mathbf{r}) \cdot \mathbf{u}(\mathbf{r})$
(c) $\nabla \times \mathbf{v}(\mathbf{r})$
(d) $\iiint |\mathbf{v}(\mathbf{r})|^2 d^3\mathbf{r}$
(e) $\iiint |\mathbf{w}(\mathbf{r})|^2 d^3\mathbf{r}$

For part (e) do the integral numerically. Use your favorite math package (e.g., Mathematica, MATLAB, etc.) or programming language (e.g., C, C++, Basic, etc.) to find a numerical approximation to the answer to within 1%. Estimate the accuracy of your approximation. Please include a printout of the input lines or program you used to do the approximation.

3. Prove that for any smooth vector field $\mathbf{v}(\mathbf{r})$ the following identity holds

$$\nabla \cdot (\nabla \times \mathbf{v}(\mathbf{r})) = 0$$

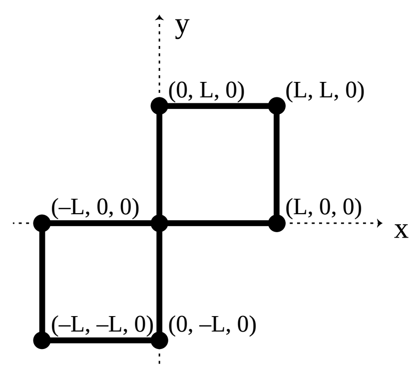
(A smooth vector field is one which is continuous and all of whose derivatives are also continuous.)

4. The following charges are placed on the x-axis in three-dimensional space:

$$6\text{C at } (0,\, 0,\, 0), \qquad -3\text{C at } (1\text{m},\, 0,\, 0), \qquad -3\text{C at } (-1\text{m},\, 0,\, 0).$$

(a) Find $\mathbf{E}(\mathbf{r})$ everywhere in space.
(b) Find the force on each of the three charges.
(c) Find $\nabla \times \mathbf{E}(\mathbf{r})$ everywhere in space.
(d) Calculate the leading term in the long-range field $\mathbf{E}(0, y, 0)$ as $y \to \infty$. (i.e., what is the leading term in an expansion of $\mathbf{E}$ in $1/y$ for large y?)

5. A configuration of charged line segments lies in the plane as shown, describing a pair of squares touching at the origin.



The segments carry a uniform charge λ per unit length. Find the total electric field at the point $(0, 0, z)$. [Hint: Griffiths, pp. 62–3.]

6. Which of the following electric fields can arise from a static charge distribution?

(a) $\mathbf{E}(\mathbf{r}) = e^{-r^2}\mathbf{r}$
(b) $\mathbf{E}(\mathbf{r}) = y^2 z\widehat{x} + xyz\widehat{y} + xy^2\widehat{z}$
(c) $\mathbf{E}(\mathbf{r}) = y^2 z\widehat{x} + 2xyz\widehat{y} + xy^2\widehat{z}$
(d) $\mathbf{E}(\mathbf{r}) = A\frac{e^{-br}}{r}\widehat{r}$, with A, b arbitrary constants