Physics 8.07, Fall 1999
Homework #2

Reading for Tuesday September 21: Griffiths pp. 110–127

Reading for Thursday September 23: Griffiths pp. 38–45, 127–146

Problem Set #2

Due **Thursday September 23** by 9:30 AM in the 8.07 homework box in 4-339B.

1. A static charge distribution produces a radial electric field

$$\mathbf{E} = A\frac{e^{-br}}{r^2}\hat{r}$$

with A, b constants. Find the charge density everywhere in space. Calculate the total charge.

2. A sphere of radius R has uniform charge density ρ except for a hollow spherical region of radius r whose center is displaced from the center of the larger sphere by a vector $\mathbf{a}$ ($a < R - r$).

   (a) Find the electric field everywhere in the hollow region.

   (b) Find the potential V at the center of the hollow region.

3. Consider a parallel plate capacitor formed from two conducting plates of area A separated by a distance d. The capacitor is charged to a potential V and disconnected from the charging circuit. Ignore fringing fields in this problem.

   (a) Calculate the electrostatic energy stored in the capacitor.

   (b) Using your answer to 3a compute the work needed to move the plates together to a distance $d' < d$.

   (c) Calculate the net force on one of the plates.

   (d) Using your answer to 3c compute the work needed to move the plates together to a distance $d' < d$, and compare with 3b.

4. Griffiths problem 2.48.

5. Consider a hemisphere of radius R located in the upper half-space $z > 0$ with total charge Q uniformly distributed over the surface (as discussed in the example in class). Find the electric field everywhere on the z-axis. Check your answer by (i) verifying that your answer at $z = 0$ agrees with that calculated in class, and (ii) adding together your answer with the reflected electric field for a hemisphere in the lower half-space $z < 0$ and verifying that the total electric field is that of a charged sphere.

6. As in Problem 4 in last week's homework, three charges are placed on the x-axis in three-dimensional space:

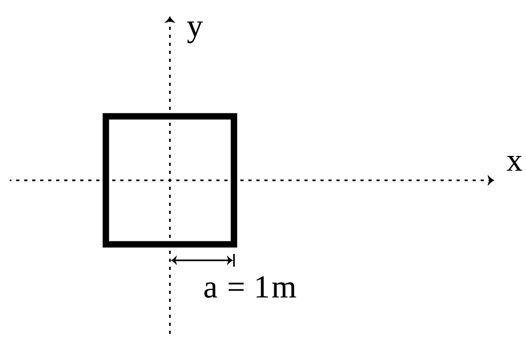
$$6\text{C at } (0, 0, 0), \quad -3\text{C at } (1\text{m}, 0, 0), \quad -3\text{C at } (-1\text{m}, 0, 0).$$

Use the method of multipoles to find the leading long-range potential V in all directions and compare to your answer from last week for the electric field on the y-axis.

7. A square of side length $L = 2a = 2$m is constructed from nonconducting charged line segments and arranged in the x-y plane as shown, with center at the origin. The charge density on the segments composing the square is given by

$$\lambda_h(x) = -\lambda_0 \cos(\pi x/a), \qquad \lambda_v(y) = -\lambda_0 \cos(\pi y/a)$$

for the horizontal and vertical segments respectively, where $\lambda_0 = 1$ C/m.



(a) Compute the monopole, dipole, and quadrupole moments of this configuration.

(b) What is the leading term in the long-range potential along the x-axis?

(c) Numerically compute the potential at the points x = 2m, 3.3m, 3.6m, 4m, 5m, 6m, 10m, 100m, 1000m along the x-axis. Compare the predicted asymptotic form of the potential with your numerical results by graphing your data points and the analytic form from 7b on a log-log plot.