Class Project #1

Due: Thursday, March 23, 2006, in class

The Cosmic Fate of ^{7}Be

For this project, you will work in groups of 2 (or 3) to reach the goals described below. Hand in one report, with all applicable names, on the due date above. This is, to my knowledge, an unsolved (at the very least an unpublished) problem, so you may encounter unexpected difficulties.

In this project, you will compile a complete history of ^{7}Be nuclei produced during Standard Big Bang Nucleosynthesis (SBBN). In addition, you will also consider the decay products of cosmic ^{7}Be electron capture, and speculate on the possibility of detection.

1) At the conclusion of SBBN ($T_\gamma = 10\,\text{keV}$), assume that nuclear reactions have frozen out with a cosmic number ratio of $\left(\frac{^7\text{Be}}{\text{H}}\right)_{BBN} = 5 \times 10^{-10}$. The Beryllium nuclei will eventually decay to ^{7}Li via electron capture. Due to the low density of electrons when the universe has cooled below 10keV, electron capture does not occur until ^{7}Be recombines with an electron. Assume the mean lifetime of triply-ionized $^7\text{Be}^{3+}$ to electron capture the bound electron is $\tau = 1.5 \times 10^7 s$. Use the Saha equation to calculate the physical number density of $^7\text{Be}^{3+}$ as a function of photon temperature, T, for $T > 4$ eV. Assume a baryon to photon ratio, $\eta = 6 \times 10^{-10}$, a total matter density of $\Omega_m h^2 = 0.13$, and flat universe. Find a differential equation that describes the change in the number density of ^{7}Be including the decay due to electron capture. *Numerically integrate the equation starting at either the post-BBN temperature of 10keV or at the temperature corresponding to the miniumum of the equilibrium abundance of $^7Be^{3+}$ to ^7Be.* End the integration when the comoving number density of ^{7}Be has dropped to 10^{-5} of the post-BBN density. Show your results as a plot of the fractional number of ^{7}Be electron captures as a function of photon temperature. Also show the ionization fraction of $^7\text{Be}^{3+}$ to $^7\text{Be}^{4+}$ as a function of temperature. Justify the assumption to only consider the $^7\text{Be}^{3+}$ ion and its single bound electron as available for electron capture.

2) The electron capture decay has two channels:

$$89.7\% : \ ^7\text{Be} + e^- \to {}^7\text{Li} + \nu_e(861.8 \text{ keV})$$

$$10.3\% : \ ^7\text{Be} + e^- \to {}^7\text{Li}^* + \nu_e(384.3 \text{ keV})$$

$$\text{and immediately } ^7\text{Li}^* \to {}^7\text{Li} + \gamma \ (477.5 \text{ keV})$$

Use the decay rates found earlier to predict the cosmic electron neutrino spectrum of ^{7}Be electron capture observable today. Assume the ν_e is very light $m_{\nu_e} \ll 1\text{eV}$. Makes 2 plots: one showing the present differential number density of these neutrinos today as a function of energy (in units of eV): $\frac{dn_{\nu_e}}{dE}$, and the second plot the showing present day number flux: $\frac{dN_{\nu_e}}{dEdtdAd\Omega}$ also as a function of energy.

(continued)

Is this neutrino background detectable today? If not, how many orders of magnitude in sensitivity are required beyond today's experiments?

Finally, double check that if you integrate up the total number of neutrinos today that the comoving number density is equal to the original number of ^{7}Be nuclei (since each ^{7}Be only produces 1 neutrino).

3) Now determine the fate of the 477.5 keV photons produced in the electron capture process 10.3% of the time. When produced they are likely far out of equilibrium with the photon/electron plasma. The first step is to follow their evolution until decoupling at $z = 1100, T = 0.265$ eV. Between the epoch of production and decoupling, the photons undergo both redshift of expansion and Compton scattering, with a Thomson cross section of $\sigma_T = \frac{8\pi\alpha^2}{3m_e^2} = 6.6524 \times 10^{-25}\text{cm}^2$. Assume each scattering increases the wavelength of the photons by 1 Compton wavelength: $\frac{2\pi}{m_e} = 2.427 \times 10^{-10}$ cm.

Write down a differential equation for the describes the wavelength change of a photon as a function of scale factor: $\frac{d\lambda}{da}$ or $\frac{d\lambda}{dT}$ including one term for photon redshift and one term for Compton scattering. Numerically integrate to find $\Delta\lambda$ starting with the wavelength appropriate for photons produced in ^{7}Be electron capture photons to decoupling. Show a plot of $\Delta\lambda$ versus $T_{production}$ appropriate for the range of T relevant for photons produced by ^{7}Be electron capture.

4) Investigate the propagation of photons, with the range of wavelengths calculated above, from the epoch of decoupling to the present. Are there any other scattering processes that will alter the energy distribution? Plot the energy distribution as a function of wavelength of the photons today by numerically combining the results in parts 1 & 3 and show in units of Jy per unit solid angle (10^{-23} erg/s/cm^2/Hz/rad^2). Overplot a blackbody specturm of 2.728K for comparison (you may have to scale one of the plots by a constant for display purposes). Can you find a simple analytic fit to the distribution of cosmic ^{7}Be photons? Is this photon distribution detectable with current technology? If not, how many orders of magnitude gain in sensitivity is required?