Class Project #2

Due: Thursday, May 11, 2006, in class

The Angular Power Spectra from WMAP

Again, you will work in groups of 2 or 3 to step through parts detailed below. You are welcome to work in the same groups as before, but let me know if you would like to rearrange. Hand in one report, with all applicable names, on the due date above. These data have been thoroughly investigated and you can find the full set of analysis papers at:

`http://lambda.gsfc.nasa.gov/product/map/current/map_bibliography.cfm`

In this project, I want you to analyze one data subset of the cosmic microwave background anisotropies recently made available in the 3-year data release by the WMAP science team. I've chosen a single band (V-band at 60 GHz) and a dataset which has been processed to remove many of the galactic foregrounds. I have a mirror image of the the original HEALPIX ".fits" file available at:

`http://web.mit.edu/~burles/www/WMAP/wmap_band_forered_iqumap_r9_3yr_V_v2.fits`

You can also find a usable version in gzipped Ascii format:

`http://web.mit.edu/~burles/www/WMAP/WMAP_Vband_gt30.asc.gz`

The columns of the ascii file are: (1) Galactic Latitude (deg), (2) Galactic Longitude (deg), (3) Temperature Difference (mK) (4) Q Polarization (mK) (5) U Polarization (mK) (6) Effective Number of Observations (a weight)

You should look at the WMAP data papers for more details. The Ascii file contains only measurement above the galactic plane (Galactic latitude $b > 30\,\text{deg}$). This is approximately 1/4 of the full sky, and contains 785408 pixels. Each pixel is approximately 0.01313 deg^2 in angular size. I'll refer to the Temperature data as a vector $\vec{T}$, and the Polarization data as the vectors $\vec{Q}, \vec{U}$.

1) **Variance in the maps** Find the mean and variance in the temperature and polarization sets (variances are $\sigma_T^2, \sigma_Q^2, \sigma_U^2$ respectively). How many pixels in the dataset have fall outside of 5-sigma from 0mk in either the $\vec{T}, \vec{Q}, \vec{U}$ data? If you exclude these pixels, what are the corrected values of $(\sigma_T^2)_{corr}, (\sigma_Q^2)_{corr}, (\sigma_U^2)_{corr}$? Keeping these pixels masked, what is the correlation coefficient between $\vec{T}$ and the vectors $\vec{Q}$ and $\vec{U}$: i.e. $C_{TQ} = \frac{<\vec{T}\cdot\vec{Q}>}{\sigma_T\, \sigma_U}$

2) **Projection Maps** Make two maps of the northern galactic hemisphere, with the galactic pole at the center. You can choose either a stereographic projection or an equal area projection (like Lambert or Hammer-Aitoff). One map should show the temperature anisotropies with a scale range of -5σ to +5σ. The other map should be a map of total polarization: $\vec{P} = \sqrt{\vec{Q}^2 + \vec{U}^2}$, and should scale from 0 to mean($\vec{P}$) + 5σ_P.

3) **Spherical Harmonic Coefficients of $\vec{T}$:** Assume that the temperature map can be represented by $\vec{T} = \sum_{l,m} a_{l,m} Y_{l,m}$, where $a_{l,m}$ are the complex coefficients and $Y_{l,m}$ are spherical harmonics. Use the definition $Y_{l,m} \equiv P_l^m(\mu) e^{im\phi}$, where $P_l^m(\mu)$ are associated Legendre polynomials (see Abramowitz and Stegun, 1964), $\mu = \cos(\theta)$, $\theta = 90 - b$, and $\phi =$ Galactic longitude. Each multipole, l, has $2l + 1$ harmonics with $-l \leq m \leq +l$. To calculate the complex coefficients, use the following relation

$$a_{l,m} = \frac{\sum_{pixels} T\, Y_{l,m}}{\sum_{pixels} Y_{l,m} Y_{l,m}^*} \tag{1}$$

where $Y_{l,m}^*$ is the complex conjugate of $Y_{l,m}$. You can decide which pixels you want to include, for instance you may want to exclude pixels which you identified as outliers in the previous part.

Calculate the complex coefficients for $l = 60, 120, 220, 280$ and for each l plot the $2l + 1$ coefficients as points in the complex plane. In each case, calculate the average squared value: $C_l = \frac{\sum_m |a_{l,m}|^2}{2l+1}$. Show $\sqrt{C_l}$ in each of the 4 plots as a circle in the complex plane.

4) **Angular power spectrum $C_l l(l+1)$:** Now compute C_l for angular multipoles from $2 < l < 300$. Make a log-linear plot of $\frac{C_l l(l+1)}{2\pi}$ versus multipole l. This will be quite noisy, so make a second plot averaging multipoles in consecutive bins of 20. Identify, if possible, the center of a peak near $l = 220$. Translate this angular scale into a comoving distance assuming a ΛCDM cosmology ($\Omega_m = 0.3, \Omega_\Lambda = 0.7, H_0 = 70$km/s/Mpc).