# Problem Set #1

## Due: Thursday, February 16, 2006, in class

**Boldface changes added on February 10, 2006**

1. **Geodesic in Newtonian Gravitational Potential**

Dodelson, Chapter 2, exercise 3

**In part b, the sum $p^0 + m\phi$ is conserved under some condition. What is the condition? Note: more components of connection may be needed in addition to those in part (a).**

**In part c, Drop the $m$ in the expression for Newton's 2nd Law.**

2. **Robertson-Walker Metric (from Fall 2001)**

Consider the general Robertson-Walker metric, written as the invariant line element:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right]$$

Find coordinate transformations that will put the line element in the following forms:

$$ds^2 = a^2(\tau)\left[-d\tau^2 + d\chi^2 + r^2(\chi)(d\theta^2 + sin^2\theta d\phi^2)\right]$$

$$ds^2 = a^2(\tau)\left[-d\tau^2 + \frac{d\bar{r}^2 + \bar{r}^2(d\theta^2 + sin^2\theta d\phi^2)}{(1+\frac{1}{4}k\bar{r}^2)^2}\right]$$

$$ds^2 = -dt^2 + a^2(t)\frac{(dx^2+dy^2+dz^2)}{\left[1+\frac{1}{4}k(x^2+y^2+z^2)\right]^2}$$

For each case, indicate the full range of the variables. Give explicit formulae for $r(\chi)$ and $\bar{r}(r)$. (Hint: Different forms may be required for $k > 0, k < 0, \text{and} k = 0$. Note also that $a(\tau)$ is not the same function of its argument as $a(t)$.)

3. **Isotropy implies Homogeneity**

a. Use elementary thought experiments to convince yourself (as well as me), that a 3-dimensional hypersurface of constant time which is isotropic at all locations must be totally homogeneous.

b. Use the space-like components of the contracted Bianchi identities, ${G^{ij}}_{;j} = 0$, as a statement of isotropy to more formally show that the **spatial components of** ${R_\mu}^\nu$ must be uniform on the hypersurface (the gradient of curvature is zero everywhere).

4. **Rotating Flat space time**

Consider the Minkowski line element in cylindrical coordinates:

$$ds^2 = -dt^2 + dr^2 + dz^2 + r^2 d\phi^2$$

and introduce a change in variable $\phi$ to account for uniform rotation: $\phi = \phi' - \omega t$.

a. Write down the metric in the new coordinate system, both $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$.

b. Find the non-zero Christoffel symbols corresponding to this metric.

c. Calculate the corresponding Ricci Tensor, $R_{\mu\nu}$ and Scalar, $R$.

d. What do Einstein's field equations imply for the energy momentum tensor, $T_{\mu\nu}$?