Problem Set #2

Due: Thursday, February 23, 2006, in class

1. **Quasar Absorption Line Statistics**

Absorption systems observed along the radial line of sight to distant quasars reveal the presence of neutral hydrogen and various ions (of carbon, silicon, etc.) that may be associated with high redshift galaxies. One can express the number of absorption systems per unit redshift as:

$$\frac{d\tau}{dz} = n(z)\sigma\frac{dl}{dz}$$

where τ is the optical depth, $n(z)$ is the proper number density, σ is the physical cross section to produce the observed absorption line strength, and $\frac{dl}{dz}$ is the differential distance along the radial null geodesic. Assume a comoving density of galaxies of $10^{-3}\,\mathrm{Mpc}^{-3}$, and find the cross-section for galaxies in kpc^2 to exhibit $\frac{d\tau}{dz} = 3$ at $z = 2$. Assume a ΛCDM cosmology ($\Omega_\Lambda = 0.7, \Omega_m = 0.3, H_0 = 70\,\mathrm{km/s/Mpc}$).

2. **Direct observations of relative expansion**

Suppose you have a technique to measure the redshift for objects at a redshift, $z = 1$, to a precision of $\Delta z = 10^{-6}$, and you make a robust measurement of a particular object today. In a ΛCDM cosmology ($\Omega_\Lambda = 0.7, \Omega_m = 0.3, H_0 = 70\,\mathrm{km/s/Mpc}$), how long do you need to wait to make a second measurement to detect a difference in the object's observed redshift?

Alternatively, if you are able to measure a large sample of N objects to increase your statistical precision by $\sqrt{N}$, how many objects do you need in your sample if your time baseline is only 1 year?

You may want to read a paper on a similar measurement by Avi Loeb (1998, ApJ, 499, L111) for more details.

3. **Supernova rates**

Calculate the supernova rate (let's consider just Type II core-collapse SNe), in two cases.

a) Calculate the expected number of Type II SNe that will explode in the next hour in the present 3-d hypersurface of constant time. This rate is not observable, and is simply the product of the current volume of the comoving horizon and the present SNe rate. Assume a ΛCDM cosmology, with $\Omega_\Lambda = 0.7, \Omega_m = 0.3, H_0 = 70\,\mathrm{km/s/Mpc}$. You may use the present Type II rate from Madau et al. (1998, MNRAS, 297, L17) of $SNR_{II} = 4.6 \times 10^{-5}\,\mathrm{SNe/yr/Mpc^3}$.

b) Now calculate the SNe rate in our past light cone with the same cosmology and a redshift dependent rate of $SNR_{II} = 4.6 \times 10^{-5}\,[x^5 e^{-x}/2 + (1 - e^{-x})]\,\mathrm{SNe/yr/Mpc^3}$, where $x = 20.3\,a^{3/2}$.

c) At which scale factor is the contribution to the rate in part b) greatest?

4. **Distance Ratios**

Calculate the luminosity distances for the cosmological models described by the parameters in the table below. Assume $H_0 = 70\,\mathrm{km/s/Mpc}$, and give the distances in Mpc.

Ω_m	Ω_{DE}	w_{DE}	Ω_K	$d_L(z=0.1)$	$d_L(z=0.5)$	$\frac{d_L(z=0.5)}{d_L(z=0.1)}$
0.30	0.7	-1.0	0			
0.25	0.75	-1.0	0			
0.30	0.7	-0.9	0			
0.30	0.65	-1.0	0.05			

Once you've completed the table, briefly describe the precision required of an astrophysical standard candle to distinguish between the four models above. Express this precision in astronomical magnitudes.