

## Problem Set #3

**Due: Thursday, March 9, 2006, in class**

### 1. Neutrino Mass Densities

Consider three light (but not massless) neutrinos with degenerate masses ( $m_{\nu_e} \simeq m_{\nu_\mu} \simeq m_{\nu_\tau}$ ).

- a) Calculate the mass of the neutrinos so that the current total cosmological mass density in neutrinos is equal to  $\Omega_\nu h^2 = 0.12$ , where  $h$  is the dimensionless Hubble Constant,  $H_0 = 100 h$  km/s/Mpc.
- b) Calculate the scale factor,  $a$ , when neutrinos of this mass become non-relativistic, by equating the energy density contributed by the mass to that contributed by the neutrino momentum.
- c) How far in comoving distance do the neutrinos travel between a scale factor of  $a = 10^{-9}$  and the scale factor calculated above. You may assume a flat universe with the energy densities contributed just by photons and neutrinos.

### 2. Comoving Entropy Densities

- a) Find an expression for the comoving entropy density,  $S$ , for a non-relativistic weakly interacting gas in local thermal equilibrium with temperature,  $T$ . Show that this is negligible compared to a relativistic gas at the same temperature.
- b) Find an expression for the entropy of vacuum energy with  $\rho = -P$ . Can you explain this behavior?

### 3. Quark/anti-quark Asymmetry

The baryon-to-photon ratio,  $\eta$ , is defined as the ratio at the conclusion of Big Bang Nucleosynthesis.

Current measurements of light elements as well as data from WMAP constrain this ratio to approximately  $\eta = 6 \times 10^{-10}$ . Relate this ratio to the quark/anti-quark asymmetry (consider just up and down quarks) present before the quark/hadron transition:  $\delta_q = \frac{q}{\bar{q}} - 1$ . Assume 3 quarks per baryon, and account for relativistic particles that will contribute to the photon number density between quark/anti-quark annihilation and electron/positron annihilation.

### 4. Expansion History with Decaying Dark Matter

- a) Calculate the relic abundance of a Weakly Interacting Massive Particle (WIMP) with a rest mass of 250 GeV. Assume a thermally averaged cross-section of  $\langle \sigma v \rangle = 10^{-25}$  cm<sup>3</sup>/s and the effective relativistic degrees of freedom,  $g_* = 100$ . Express your answer as a comoving mass density,  $\rho * a^3$ .
- b) If the WIMP is unstable and decays (with end products of photons) with a rate of 0.2 Gyr<sup>-1</sup>, what is the present cosmological energy density of WIMPs? For this part, you can assume the age of the universe is 13.7 Gyr.
- c) Calculate and plot the age of the universe starting from the epoch of matter/radiation equality as a function of scale factor. You may assume a flat universe with energy contribution from WIMPs, photons and massless neutrinos, and that  $H_0 = 70$  km/s/Mpc. Include a plot of  $\rho_\gamma$  and  $\rho_{WIMP}$  as a function of scale factor.