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Molecular, Cellular & Tissue Biomechanics Spring 2002

Problem Set #4

Issued: Wed., 3/13/02
Due: Wed., 3/20/02

READING ASSIGNMENT: Sections 8.5 and 8.7 in the Grodzinsky-Chapter 8 notes.

Problem 1 Oscillatory Compression of Poroelastic Tissue

This problem is modeled after the poroelastic stress relaxation response derived in the previous Problem Set [3.2, parts (c) and (e)]. Using the same confined compression geometry of Problem 3.2, we now wish to find an expression for the displacement $\hat{u}(z, \omega)$ that results from an applied "sinusoidal steady state" displacement $u(z = 0, t) = u_o \, cos\omega t$ at the surface $z = 0$.

(a) From the poroelastic diffusion equation for $u(z, t)$, let the solution $u(z, t)$ have the form

$$u(z,t) = ReAe^{j\alpha(L-z)}e^{j\omega t}$$

which represents a "diffusion wave" at frequency ω decaying in the z-direction with the space constant α. Inserting this form into the diffusion equation along with $(\partial/\partial t \rightarrow j\omega)$, show that

$$\alpha = \pm(1-j)\sqrt{\omega/2Hk}$$

(b) Since the complex diffusion equation is second order in z, the solution is the superposition of two solutions corresponding to +α and -α. Using the boundary conditions on $\hat{u}(z, \omega)$ at z=0 and z=L, show that the solution has the form:

$$\hat{u}(z,\omega) = \frac{u_o sinh\gamma(L-z)}{sinh\gamma L}$$

Find γ in terms of material properties and frequency, and interpret the poroelastic "penetration depth" γ by sketching your solution at an instant in time, but identifying the "envelope" $sinh\gamma L$.

Problem 2 Electromechanical Interactions in Charged Poroelastic Tissue

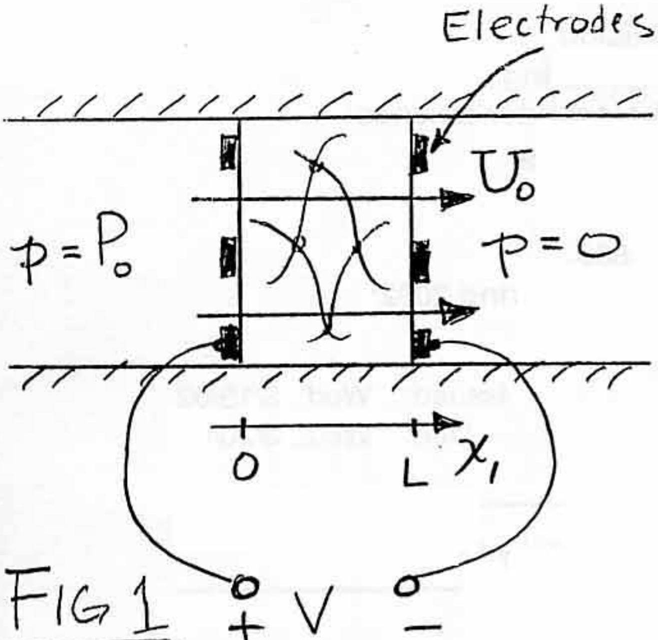

FIG 1

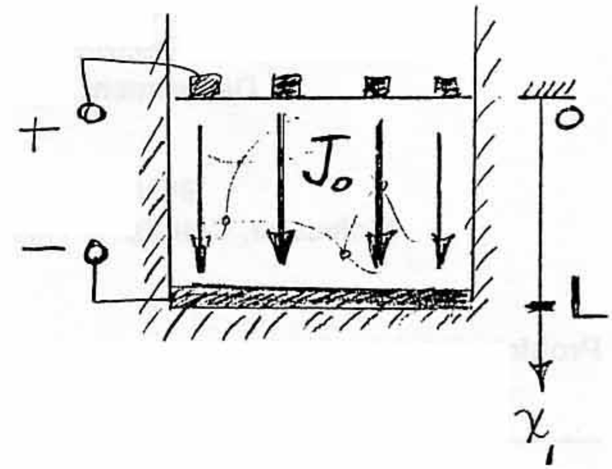

FIG 2

A tissue specimen in the form of a cylindrical disk of thickness L is placed in the radially confining geometries of Fig 1 and Fig 2, in a buffer of 0.15 Molar NaCl. Assume that the tissue can be modeled as a hydrated, linear, isotropic, homogeneous matrix having negative fixed charge groups.

(a) **Write the 1-dimensional poroelastic equations of motion** that describe the response of the tissue to mechanical or electrical stimuli in any experimental configuration. These equations include (i) conservation of momentum, (ii) conservation of mass, (iii) a stress-strain constitutive law, (iv-v) constitutive laws for relative fluid velocity and electrical current density caused by pressure and voltage gradients (expressed in terms of the coupling coefficients k_{ij}, and (vi) conservation of current. You can assume that the k_{ij} are known positive-definite constants obtained from independent measurements. (Do NOT combine the equations yet).

(b) Using the configuration of Figure 1, a steady state pressure drop is applied across the tissue, resulting in steady fluid velocity U_o. The porous electrodes on both sides of the tissue allow fluid to move freely across the tissue and hold the tissue in place. Assume that the fluid flow U_o does NOT affect the tissue's hydraulic permeability.

(i) With the <u>electrodes shorted together</u> such that the voltage drop across the tissue is zero ($V = 0$), **find** an expression for the tissue's hydraulic permeability in terms of one or more of the k_{ij}.

(ii) With the <u>electrodes left "open circuited"</u> so that the current density J across the tissue is zero, **find** a new expression for the tissue's effective hydraulic permeability in terms of one or more of the k_{ij}.

(c) Now combine the equations of motion from part (a) to obtain a single differential equation for the displacement $u_1(x_1, t)$ in terms of material parameters (modulus, k_{ij}), and including terms that account for the possibility of a constant fluid velocity U_o and a constant current density J_o flowing across the tissue in steady state.

(d) In **Figure 2**, the porous electrode platen at $x_1 = 0$ is glued to the tissue and held fixed in space such that $u_1 = 0$ at the boundary $x_1 = 0$. In addition, $u_1 = 0$ at $x_1 = L$. A steady current density J_o is applied across the tissue by means of the electrodes. **Find** an expression for the steady state displacement $u_1(x_1)$ within the tissue caused by the applied current. From your answer, **find** an expression for the "current generated stress" τ_{11} at $x_1 = 0$.

Sketch the displacement $u_1(x_1)$ versus x_1, and briefly describe how the current causes this displacement profile.