MASSACHUSETTS INSTITUE OF TECHNOLOGY

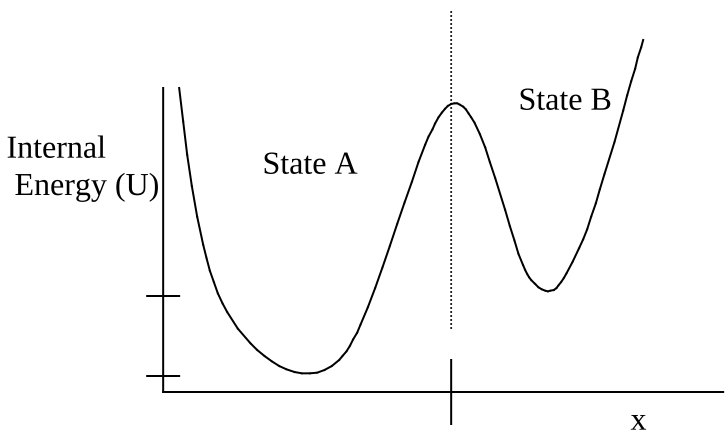
Molecular, Cellular and Tissue Biomechanics
BEH.410 / 2.978J / 6.524J / 10.537J

Problem Set #5 **Issued: 4/12/02**
Due: 4/22/02

READING ASSIGNMENT (will help out with homework!):
Read chapter 2 of Boal, Mechanics of the Cell (see link to text on BEH.410 website).
Read handout entitled "Macromolecular Mechanics".

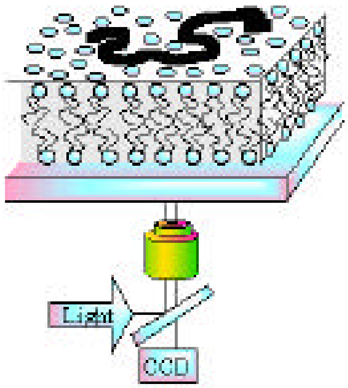
Problem #1: Probability of Conformational States

(a) Consider a protein that can exist in 2 configurations A and B as shown in the figure below. In class we mentioned that the ratio of the probability of being in state B to state A is given by $P(B)/P(A) = \exp(-\Delta G/kT)$, where $\Delta G=G_B-G_A$. Derive this relationship. (hint: start with thinking about discrete states where $P_i = 1/Z \exp(-U_i/kT)$).



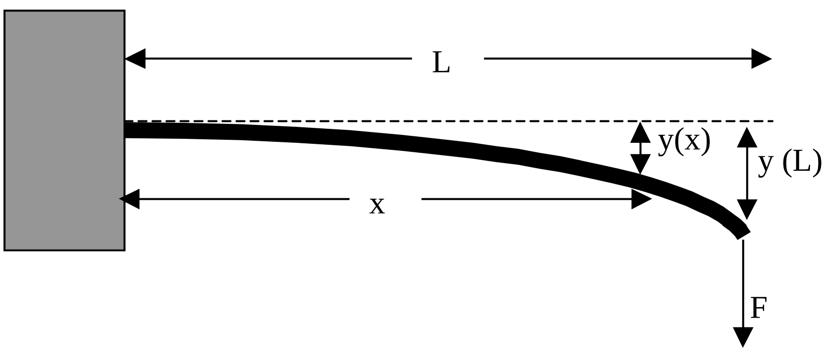
Problem #2: Biopolymer Mechanics

Recently Maier and Radler (PRL (82) 1911, 1999) have adsorbed DNA onto a lipid bilayer as shown in the figure below. The bilayer is mobile and so we can think about this system as a polymer in 2 dimensions (2-D). Suppose we try to pull on the chain with a given force to get it to 80% of the complete extension, will this require more or less force in 2-D compared to 3-D ? Why ? What is the ratio of the magnitude of the forces (note you can use the large extension limits of the force/extension relationships)?

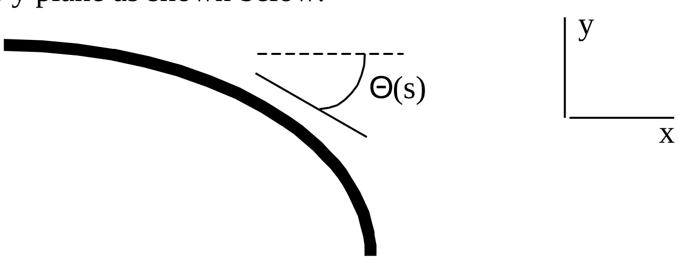


Problem #3: Stiff Filaments and Cantilevered Beams

Recall that the beam equation for a thin elastic rod is M= E I/R where M is the bending moment, E is the Young's modulus, I is the second moment of inertia and 1/R is the radius of curvature. Now consider the cantilevered beam as shown in the figure below.



Another way to write the beam equation is to consider the tangent angle θ(s) for a beam in the x-y plane as shown below.



a) Show that is the tangent angle is small then the beam equation can be written as:

(1) $$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

b) Solve (1) for the cantilevered beam shown above.
c) Look at the form of the solution for the free end (y=L). What is the effective spring force constant for this system ?
d) What magnitude of forces can you expect to measure with a glass rod of 0.25μm diameter, length of 100 μm and E=70 GPa.
e) Consider a *thermally* fluctuating microtubule which is 100 microns long and a persistence length of 60,000 μm. Calculate $<y(L)^2>$ assuming the chain forms a smooth bend. Do you expect to be able to see measure this using fluorescence microscopy (hint: resolution ~1/wavelength of light)?

Problem #4: Microscopic Description of Diffusion

Consider a small particle in a solution which is fluctuating due to thermal motion. For simplicity we will assume the particle is moving in 1 dimension. The system is described by a location x(t) at time t, a liquid viscosity μ and mass m. The particle is continually colliding with the solvent molecules which impart a random force. A force balance on the particle gives the Langevin equation

$$m\frac{d^2x}{dy^2} + \xi\frac{dx}{dt} = F(t)$$

where ξ is the drag coefficient for the particle in the solvent (6πμa for a sphere of radius a) and F is a random force with zero mean, i.e. <F>=0

a) Multiply the Langevin equation by x and average the result. And solve the resulting equation to show for $< x\frac{dx}{dt} >$ to show that $< x\frac{dx}{dt} >= C\exp(-\gamma t) + \frac{kT}{\xi}$, where C is a constant of integration and γ = ξ/m is related to the characteristic time of the system. Hint: you will need to use the equipartition theorem which states that $\frac{1}{2}m < (\frac{dx}{dt})^2 >= \frac{1}{2}kT$. Evaluate the constant of integration by assuming that the particle started at the origin.
b) Realizing that $< x\frac{dx}{dt} >= \frac{d}{dt} < x^2 >$, integrte the result from (a) above to derive an expression for the mean-squared displacement $<x^2>$.
c) Now consider two limiting cases, corresponding to $t << \gamma^{-1}$ and $t >> \gamma^{-1}$, i.e. for times that are very short and vary long compared to the characteristic time γ^{-1}. Comment on these limiting forms.