

## Cheat Sheet for BEH.410 Homework #5

### PROBLEM #1

Consider A to be all the configurations to the left of the dotted line and B is all the configurations to the right. These 'states' do not necessarily correspond to a single energy value but are micro-ensembles, as shown in the plot.

Statistical Mechanics formulas that will help us out are:

$$\text{The partition function: } Z = \sum_{i=1}^N \exp(-U_i / kT)$$

$$\text{Entropy: } S = -k \sum_{i=1}^N P_i \ln P_i$$

$$\text{Boltzmann's factor: } P_i = \frac{1}{Z} \exp \frac{-U_i}{kT}$$

Free energy:  $G_a = \langle U \rangle_a - TS_a$ , where here we consider the free energy of 'A' and  $\langle U \rangle_a$  is the ensemble average of the internal energy in state A.

It's fairly easy to show that  $G_a = -kT \ln Z_a$  where  $Z_a$  is the partition function for A (we stated this in class). To progress further you need to consider what is the probability in being in the state A and then state B....that's all the hints for now!

### PROBLEM #2

You have two choices on how to proceed on this problem:

- 1) compare the equations for the wormlike chain we showed in class (3-D) to Maha's notes (2-D)....pretty easy.
- 2) consider the end-to-end distribution function for a freely-jointed chain in 2-D and 3-D at equilibrium (linear limit). It will help if you look at the chapter in Boal about polymers...

Make sure to explain physically what is going on here....remember our friend entropy is important !

### PROBLEM #3

- a) consider relations such as  $\frac{dx}{ds} = \cos(\theta)$ ,  $\frac{dy}{ds} = \sin(\theta)$ ,  $\frac{d\theta}{ds} = \frac{1}{R}$ . Also then simplify further by assuming small angles...
- b) For the non-MechE's,  $M(x) = F(L-x)$ .
- c) Form of a linear spring force law:  $F(y) = k y$ , where  $k$  is the spring constant.
- d) Put in the numbers.....
- e) For a smooth bend of the rod (some arc with radius of curvature  $R$ ) we have  $y(L) = L^2/2R$ . We can then think about the energy of this curved rod in terms of  $E(y)$  and use Boltzmann's factor to get an average value of any quantity of interest,  $\langle y^2 \rangle \dots$

### PROBLEM #4

- a)  $F$  is a random force so it is not correlated with position, i.e.  $\langle Fx \rangle = \langle F \rangle \langle x \rangle$ .
- b) use the correct chain rule  $\left\langle x \frac{dx}{dt} \right\rangle = \frac{1}{2} \frac{d}{dt} \langle x^2 \rangle$