# The Evolution of Behavior

Andrew W. Lo

*Course 9.S915: What Is Intelligence?*

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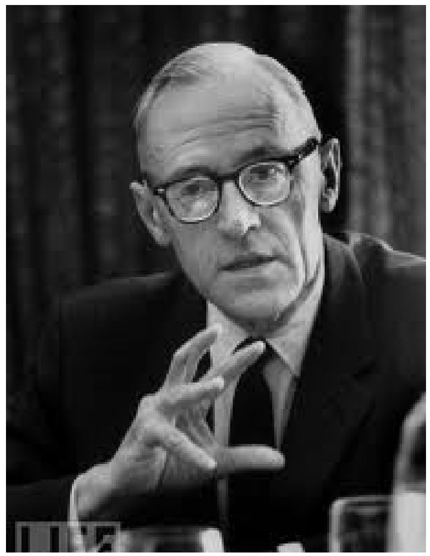
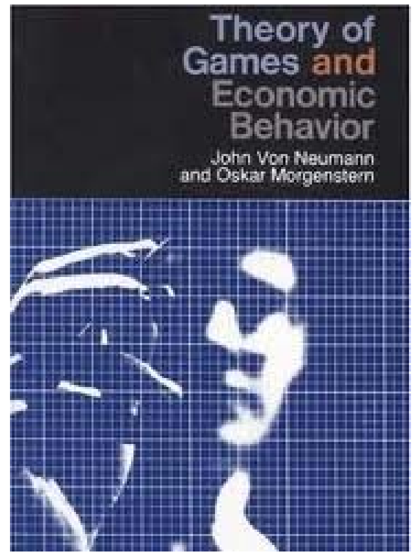
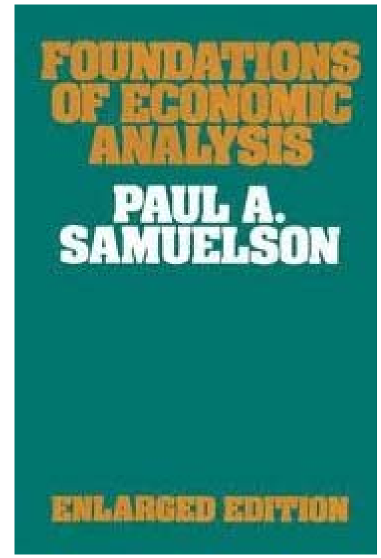
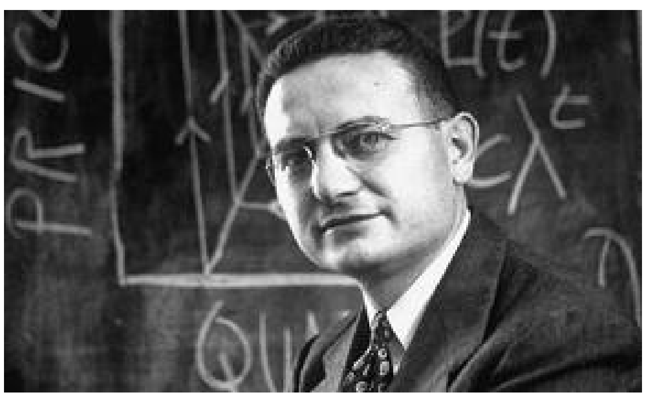
# Motivation

## Theory of Economic Behavior





### Utility Theory

$$\underset{C}{\text{Max}} \quad U(C) \quad \text{s.t.} \quad C \in \mathcal{B}$$

$$U'(\cdot) > 0 \quad , \quad U''(\cdot) < 0$$

# Motivation

## Cognitive and Behavioral Biases

- **Loss Aversion**
- **Probability Matching**
- Anchoring
- Framing
- Overconfidence
- Overreaction
- Herding
- Mental Accounting
- etc.

# Motivation

## Urn A Contains 100 Balls:

- 50 Red, 50 Black
- Pick A Color, Then Draw A Ball
- If You Draw Your Color, $10,000 Prize
- What Color Would You Prefer?
- How Much Would You Pay To Play?

# Motivation



**Urn B Contains 100 Balls:**

- Proportion Unknown
- Pick A Color, Then Draw A Ball
- If You Draw Your Color, $10,000 Prize
- What Color Would You Prefer?
- How Much Would You Pay To Play?

**Knight's (1921) Dichotomy of Risk vs. Uncertainty**

- Infinite-order Theory of Mind

# Loss Aversion



- A: $240,000
- B: $1,000,000 With 25% Probability
  $0 With 75% Probability

**Which Would You <u>Prefer</u>?**

# Loss Aversion



- C: – $ 750,000
- D: – $1,000,000 With 75% Probability
  $0 With 25% Probability

**Which Would You Prefer?**

# Loss Aversion



- A+D: $240,000 With 25% Probability
  – $760,000 With 75% Probability

- B+C: $250,000 With 25% Probability
  – $750,000 With 75% Probability

**Now Which Would You Prefer?**

# Probability Matching

**Consider Repeated Coin-Toss Guessing Game:**

- If you're correct, you get $1, otherwise –$1
- Suppose coin is biased (75% H, 25% T)
- Profit-maximizing strategy: HHHHHHHHHHHH
- Actual behavior: HHHTHHHHTTHHHTHHHHTH
- Common to ants, fish, pigeons, primates, etc.
- Why? Is it irrational or adaptive?

# Literature Review



- Behavorial economics and finance
  - Thaler, Shefrin, Statman, Shiller
- Psychology and cognitive sciences
  - Simon, Tversky, Kahneman
- Evolutionary psychology and sociobiology
  - Wilson, Hamilton, Trivers, Cosmides, Tooby, Gigerenzer
- Evolutionary game theory and economics
  - Malthus, Schumpeter, von Hayek, Maynard Smith, Nowak, Robson, L. Samuelson
- Behavioral ecology and evolutionary biology
  - Darwin, Levin, Clarke

# Summary of Brennan and Lo (2011)



**Our Contribution: Evolutionary Origin of Behavior**

- How did certain behaviors come to be?
- If they are irrational, why do they persist?
- Are all behaviors created equal?
- Simple framework for answering these questions
  - We **derive** risk aversion, loss aversion, probability matching, and randomization from evolution!
  - Some behaviors are "rational" from the population perspective, not from the individual's perspective

# Summary of Brennan and Lo (2011)



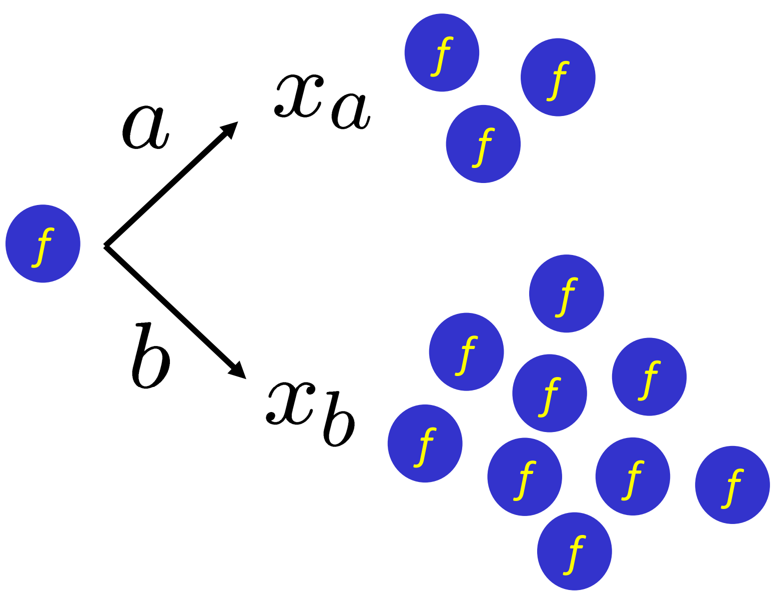
**Our Contribution: Evolutionary Origin of Behavior**

- Behavioral "biases" exist for a reason (hard-wired)
- They may not be advantageous in all environments
- Understanding their mechanisms is critical for reconciling efficient markets with behavioral finance (Adaptive Markets Hypothesis)
- Also critical for implementing regulatory reform
- Has implications for intelligence and learning

# Binary Choice Model

## Consider "Asexual Semelparous" Individuals

- Individual lives one period, makes one decision, *a* or *b*
- Generates offspring $x = x_a$ or $x_b$ , then dies
- Offspring behaves exactly like parent




### Individual Behavior:

$$x_i^f \;=\; I_i^f\, x_a \;+\; (1 - I_i^f)\, x_b$$

$$I_i^f \;\equiv\; \begin{cases} 1 & \text{with probability } f \\ 0 & \text{with probability } 1-f \end{cases}$$

$$f \in [0, 1]$$

# Binary Choice Model



**Consider "Asexual Semelparous" Individuals**

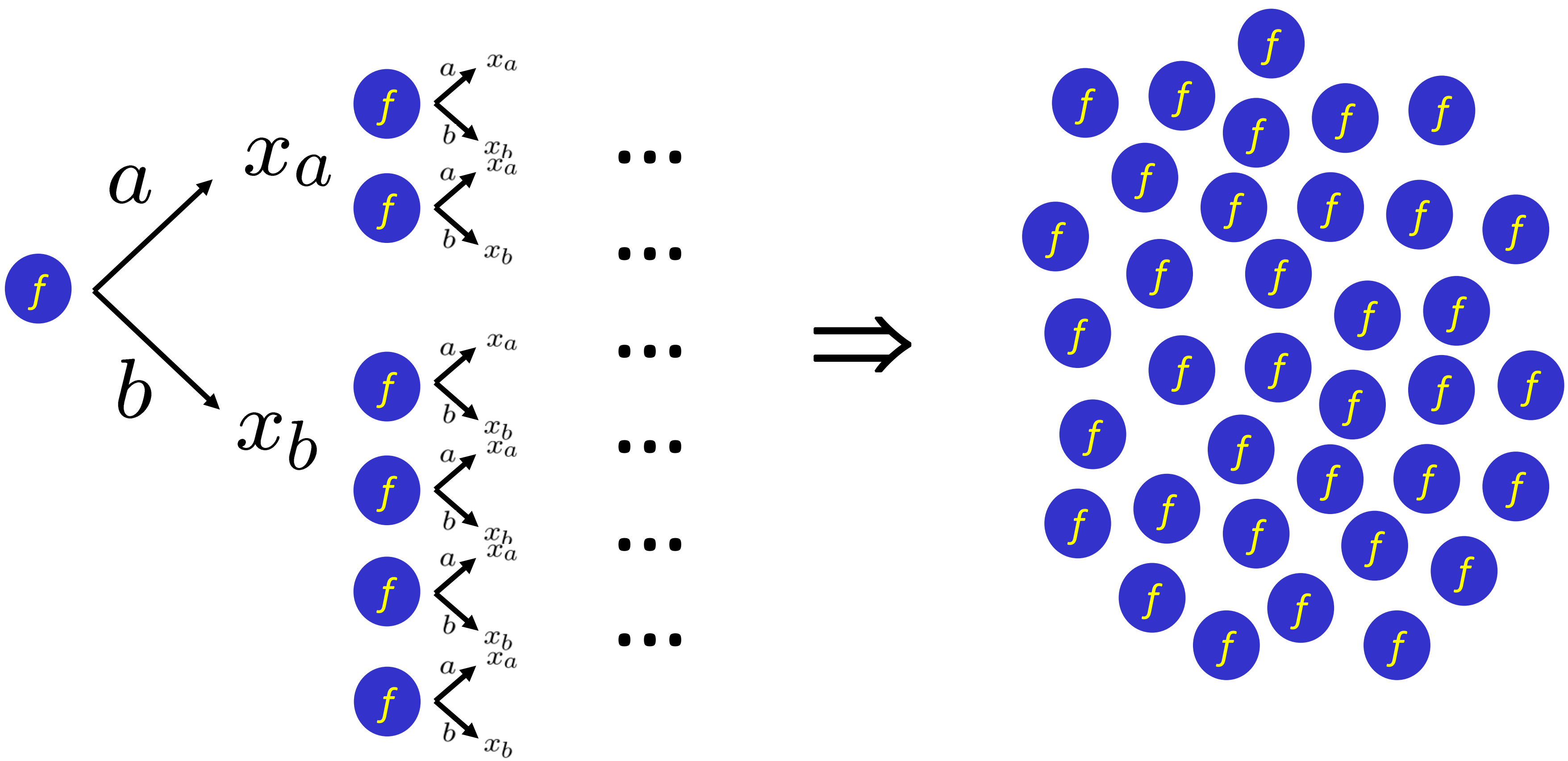
- If $f = 1$, individual always chooses *a* (offspring too)
- If $f = 0$, individual always chooses *b* (offspring too)
- If $0 < f < 1$, individual randomizes with prob. $f$ and offspring also randomizes with same $f$
- Impact of behavior on reproductive success: $\Phi(x_a, x_b)$
  - Summarizes environment and behavioral impact on fitness
  - Links behavior directly to reproductive success
  - Contains all genetic considerations
  - Biological "reduced form"

# Binary Choice Model



## Consider "Asexual Semelparous" Individuals

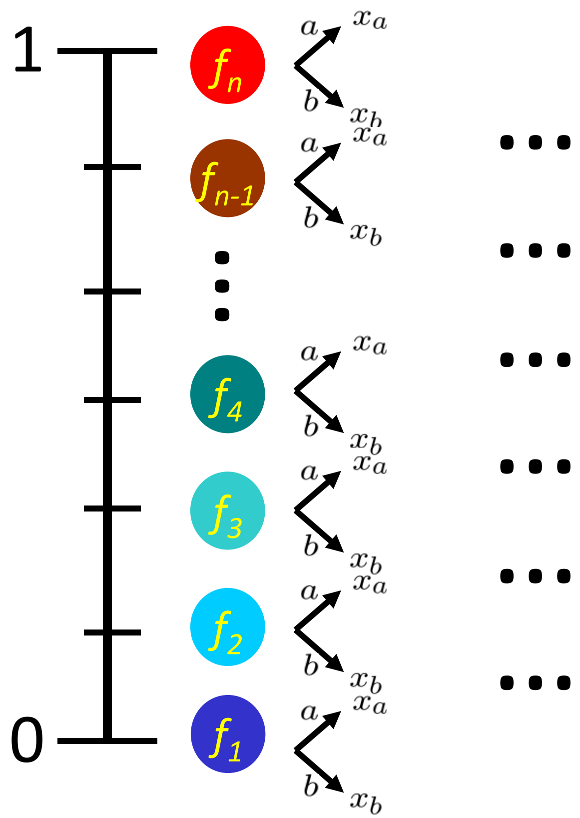
- This is repeated over many generations



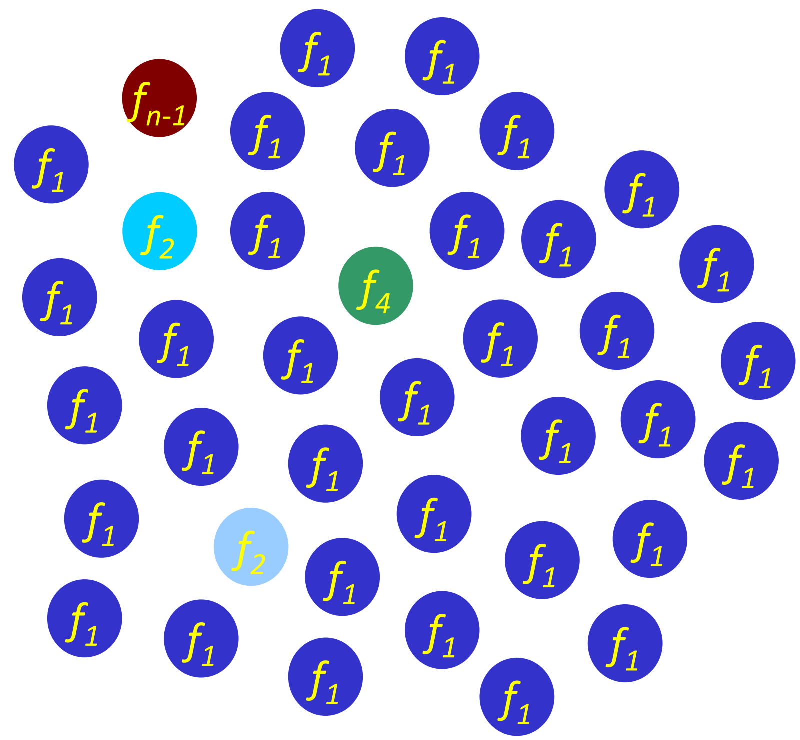
# Binary Choice Model

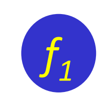
## Consider "Asexual Semelparous" Individuals

- Initial population is uniformly distributed on [0,1]




$\Rightarrow$




**Which behavior dominates?** 

# Binary Choice Model



**Consider "Asexual Semelparous" Individuals**

- Assume $\Phi(x_a, x_b)$ is **identical** across individuals
- Assume $\Phi(x_a, x_b)$ is **IID** across time
- Individuals are "mindless", **not** strategic optimizers
- Which *f* survives over many generations?
- In other words, what kind of behavior evolves?
- Evolution is the "process of elimination" (E. Mayr)
- Mathematics: find the *f* that maximizes growth rate
- This *f** will be the behavior that **survives** and **flourishes**

# Population Arithmetic



$$n_t^f = \sum_{i=1}^{n_{t-1}^f} x_{i,t}^f = \Big(\sum_{i=1}^{n_{t-1}^f} I_{i,t}^f\Big) x_{a,t} + \Big(\sum_{i=1}^{n_{t-1}^f} (1 - I_{i,t}^f)\Big) x_{b,t}$$

$$n_t^f \overset{p}{=} n_{t-1}^f \big( f x_{a,t} + (1-f) x_{b,t} \big)$$

**LLN**

$$n_T^f \overset{p}{=} \prod_{t=1}^{T} \big( f x_{a,t} + (1-f) x_{b,t} \big)$$

$$= \exp\Big( \sum_{t=1}^{T} \log\big( f x_{a,t} + (1-f) x_{b,t} \big) \Big)$$

$$\frac{1}{T} \log n_T^f \overset{p}{=} \frac{1}{T} \sum_{t=1}^{T} \log\big( f x_{a,t} + (1-f) x_{b,t} \big)$$

$$\overset{p}{\to} \mu(f) \equiv \mathrm{E}[\log(f x_a + (1-f) x_b)]$$

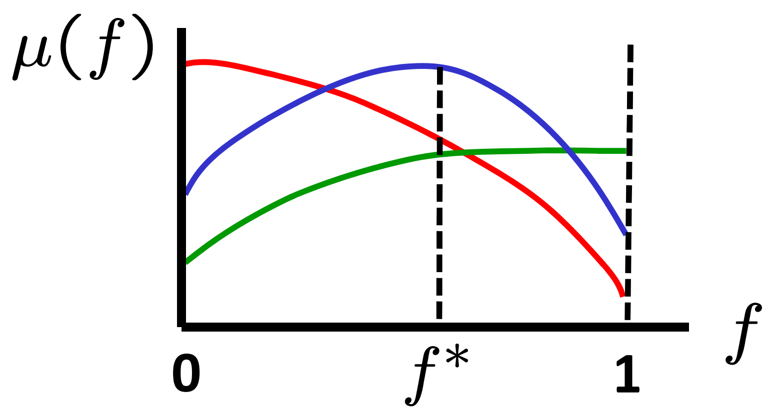
# Population Arithmetic

- Which type of individuals will grow fastest?
- The $f^*$ that maximizes $\mu(f)$ on [0,1]:

$$f^* \equiv \operatorname*{argmax}_f \mu(f) = \operatorname*{argmax}_f \mathrm{E}[\log(f x_a + (1-f) x_b)]$$

$$\mu''(f) = -\mathrm{E}\left[\frac{(x_a - x_b)^2}{\log^2(f x_a + (1-f) x_b)}\right] < 0$$

- $\mu(f)$ is strictly concave on [0,1]; unique maximum
- Three possibilities:



# Population Arithmetic



- **Growth-optimal** $f^*$ given by:

$$f^* = \begin{cases} 1 & \text{if } \mathrm{E}[x_a/x_b] > 1 \text{ and } \mathrm{E}[x_b/x_a] < 1 \\ \text{solution to (1)} & \text{if } \mathrm{E}[x_a/x_b] \geq 1 \text{ and } \mathrm{E}[x_b/x_a] \geq 1 \\ 0 & \text{if } \mathrm{E}[x_a/x_b] < 1 \text{ and } \mathrm{E}[x_b/x_a] > 1 \end{cases}$$

$$0 = \mathrm{E}\left[\frac{x_a - x_b}{f^* x_a + (1-f^*)x_b}\right] \qquad (1)$$

$$\mathrm{E}\left[\frac{x_a}{f^* x_a + (1-f^*)x_b}\right] = \mathrm{E}\left[\frac{x_b}{f^* x_a + (1-f^*)x_b}\right]$$

# Population Arithmetic



## How Does *f** Persist?  By Natural Selection:

$$\left(\frac{n_T^{f'}}{n_T^{f^*}}\right)^{1/T} \overset{p}{=} \exp\Big(\big[\mu(f') - \mu(f^*)\big]\Big) \overset{p}{\to} 0$$

$$\Rightarrow \quad \frac{n_T^{f'}}{n_T^{f^*}} \overset{p}{\to} 0$$

- *f** type takes over exponentially fast
- Behavior *f** is optimal for the **population**
- Behavior *f** is not necessarily optimal for the **individual**
- This requires no intention, deliberation, or intelligence
- Contrast this behavior with utility maximization!

# Probability Matching Explained



**Consider Special Case For $\Phi(x_a, x_b)$ :**

| Action | State 1 (prob. $p$) | State 2 (prob. $1-p$) |
|---|---|---|
| $a$ | $x_a = m$ | $x_a = 0$ |
| $b$ | $x_b = 0$ | $x_b = m$ |

- Outcomes $x_a$ and $x_b$ are perfectly out of phase

$$\mu(f) = \log m + p \log f + (1-p)\log(1-f)$$

$$f^* = p$$

- Probability matching!
- This behavior will dominate the population (eventually)

# Probability Matching Explained



## What About the "Optimal" Strategy for the Individual?

- Suppose p > ½ ; then $\hat{f} = 1$
- The first time $x_a = 0$, all individuals of this type vanish
- This behavior cannot persist; $f^*$ persists
- $f^*$ may be interpreted as a primitive version of altruism

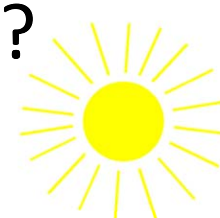
## When Is Probability Matching Advantageous?

- When two choices are highly negatively correlated
- Diversification improves likelihood of survival
- "Nature Abhors An Undiversified Bet"

# Probability Matching Explained

## Consider A Simple Ecology with Rain/Shine:

- Decision: build nest in a or b?
- Optimize or **randomize**?



$$(p = 0.75) \qquad (1 - p = 0.25)$$

*f* → **a:**

$$x_a = 3 \qquad x_a = 0$$

*1-f* → **b:**

$$x_b = 0 \qquad x_b = 3$$

# Probability Matching Explained



$p$ = 0.75
$m$ = 3

| Generation | $f$ = 0.20 | $f$ = 0.50 | $f^*$= 0.75 | $f$ = 0.90 | $f$ = 1 |
|---|---|---|---|---|---|
| 1 | 21 | 6 | 12 | 24 | 30 |
| 2 | 12 | 6 | 6 | 57 | 90 |
| 3 | 6 | 12 | 12 | 144 | 270 |
| 4 | 18 | 9 | 24 | 387 | 810 |
| 5 | 45 | 18 | 48 | 1,020 | 2,430 |
| 6 | 96 | 21 | 108 | 2,766 | 7,290 |
| 7 | 60 | 42 | 240 | 834 | 21,870 |
| 8 | 45 | 54 | 528 | 2,292 | 65,610 |
| 9 | 18 | 87 | 1,233 | 690 | 196,830 |
| 10 | 9 | 138 | 2,712 | 204 | 590,490 |
| 11 | 12 | 204 | 6,123 | 555 | 1,771,470 |
| 12 | 36 | 294 | 13,824 | 159 | 5,314,410 |
| 13 | 87 | 462 | 31,149 | 435 | 15,943,230 |
| 14 | 42 | 768 | 69,954 | 1,155 | 0 |
| 15 | 27 | 1,161 | 157,122 | 3,114 | 0 |
| 16 | 15 | 1,668 | 353,712 | 8,448 | 0 |
| 17 | 3 | 2,451 | 795,171 | 22,860 | 0 |
| 18 | 3 | 3,648 | 1,787,613 | 61,734 | 0 |
| 19 | 9 | 5,469 | 4,020,045 | 166,878 | 0 |
| 20 | 21 | 8,022 | 9,047,583 | 450,672 | 0 |
| 21 | 6 | 12,213 | 6,786,657 | 1,215,723 | 0 |
| 22 | 0 | 18,306 | 15,272,328 | 366,051 | 0 |
| 23 | 0 | 27,429 | 34,366,023 | 987,813 | 0 |
| 24 | 0 | 41,019 | 77,323,623 | 2,667,984 | 0 |
| 25 | 0 | 61,131 | 173,996,290 | 7,203,495 | 0 |

# Probability Matching Explained



## Now Consider a More General $\Phi(x_a, x_b)$

$$\text{Prob}(x_a = c_{a1}, x_b = c_{b1}) \;=\; p \;\in\; [0,1]$$

$$\text{Prob}(x_a = c_{a2}, x_b = c_{b2}) \;=\; 1 - p \;\equiv\; q$$

$$0 \;\leq\; c_{ij} \;\;,\;\; i = a, b \;\;,\;\; j = 1, 2$$

$$0 \;\neq\; c_{aj} \;+\; c_{bj}$$

- Then the growth-optimal behavior *f** depends only on

$$r_j \;\equiv\; c_{aj}/c_{bj} \;\;,\;\; j = 1, 2$$

# Probability Matching Explained

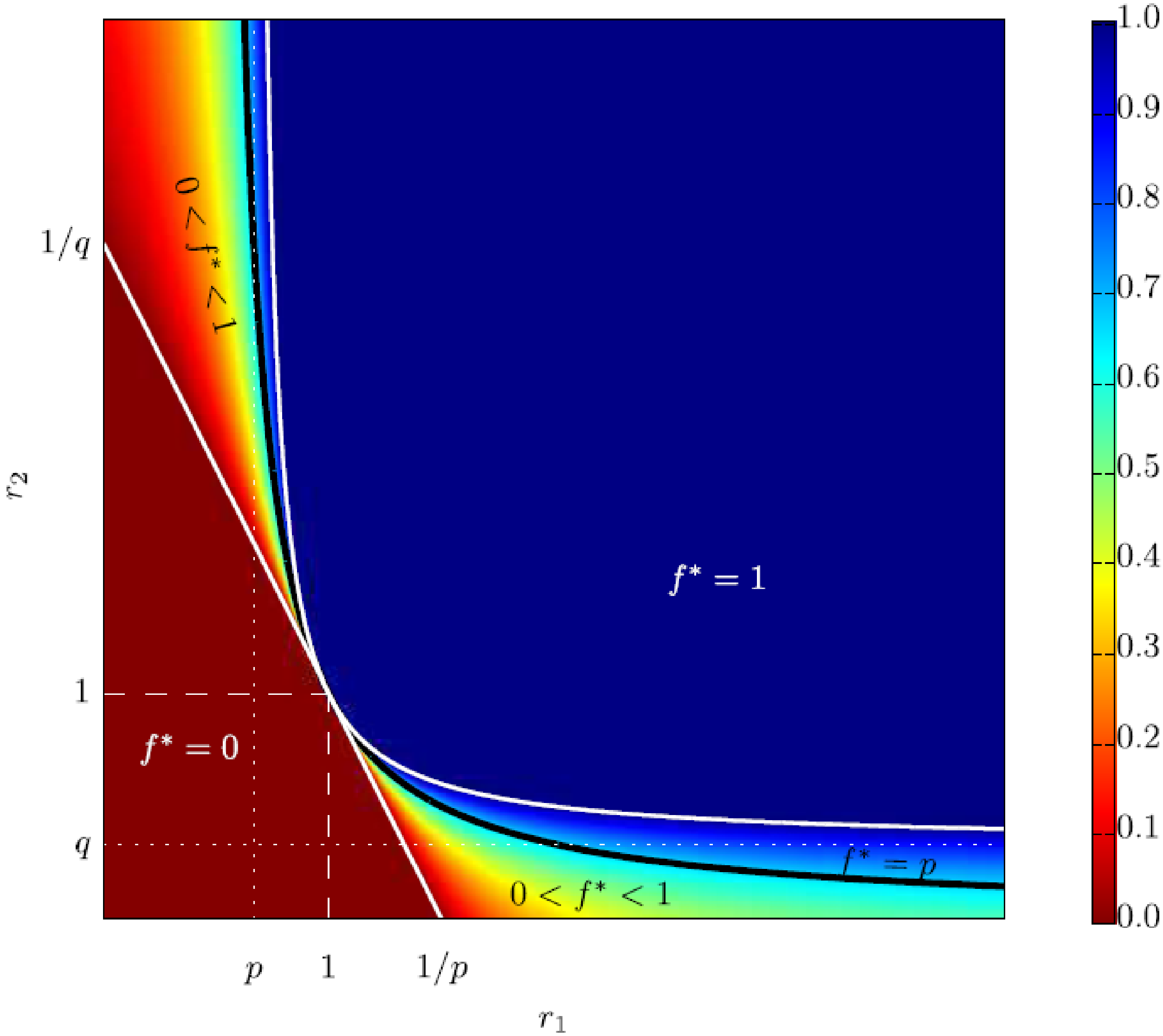
- Growth-optimal behavior *f** given by:

$$f^* = \begin{cases} 1 & \text{if } r_2 \in [q + \frac{pq}{r_1 - p}, \infty) \text{ and } r_1 > p \\ \frac{p}{1-r_2} + \frac{q}{1-r_1} & \text{if } \begin{cases} r_2 \in \left(\frac{1}{q} - \frac{p}{q} r_1, q + \frac{pq}{r_1-p}\right) \text{ and } r_1 > p, \text{ or} \\ r_2 \in \left(\frac{1}{q} - \frac{p}{q} r_1, \infty\right) \text{ and } r_1 \leq p \end{cases} \\ 0 & \text{if } r_2 \in [0, \frac{1}{q} - \frac{p}{q} r_1] \end{cases}$$

$$\begin{aligned} f^* &= p\left(1 + O(1/r_1) + O(r_2)\right) \\ &\approx p \quad \text{if } r_1 \gg 0, \ r_2 \ll 1 \end{aligned}$$

# Probability Matching Explained






**Exact probability matching condition:**

$$0 = p\frac{r_2}{1-r_2} + q\frac{1}{1-r_1}$$

# Risk Preferences



## To Study Risk Preferences, Let *b* Be "Riskless"

$$\text{Prob}(x_a = c_{a1}, x_b = c_b) = p \in [0,1]$$
$$\text{Prob}(x_a = c_{a2}, x_b = c_b) = 1 - p \equiv q$$
$$\text{and} \quad c_{a1} < c_b < c_{a2}$$

- Parametrize *b* as a convex combination of *a* outcomes

$$c_b = \theta c_{a1} + (1-\theta) c_{a2} \quad , \quad \theta \in (0,1)$$

$$\theta \approx 0 \Rightarrow \text{sure thing close to best risky outcome}$$

$$\theta \approx 1 \Rightarrow \text{sure thing close to worst risky outcome}$$

# Risk Preferences



- Growth-optimal behavior *f**:

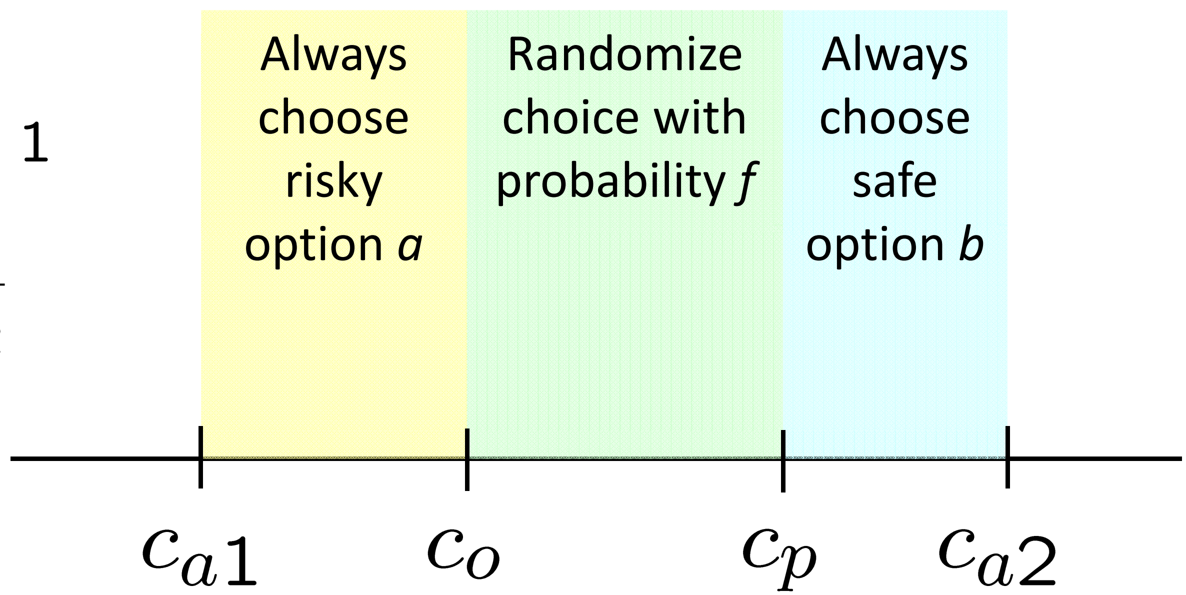
$$f^* = \begin{cases} 1 & \text{if } c_b \in [c_{a1}, c_o) \\ \left(1 - \frac{p}{\theta}\right)\left(1 + \frac{1}{(1-\theta)(s-1)}\right) & \text{if } c_b \in (c_o, c_p) \\ 0 & \text{if } c_b \in (c_p, c_{a2}] \end{cases}$$

$$\theta_o \equiv ps/(ps+q)$$

$$s \equiv c_{a2}/c_{a1} > 1$$

$$c_o \equiv \frac{1}{p/c_{a1} + q/c_{a2}}$$

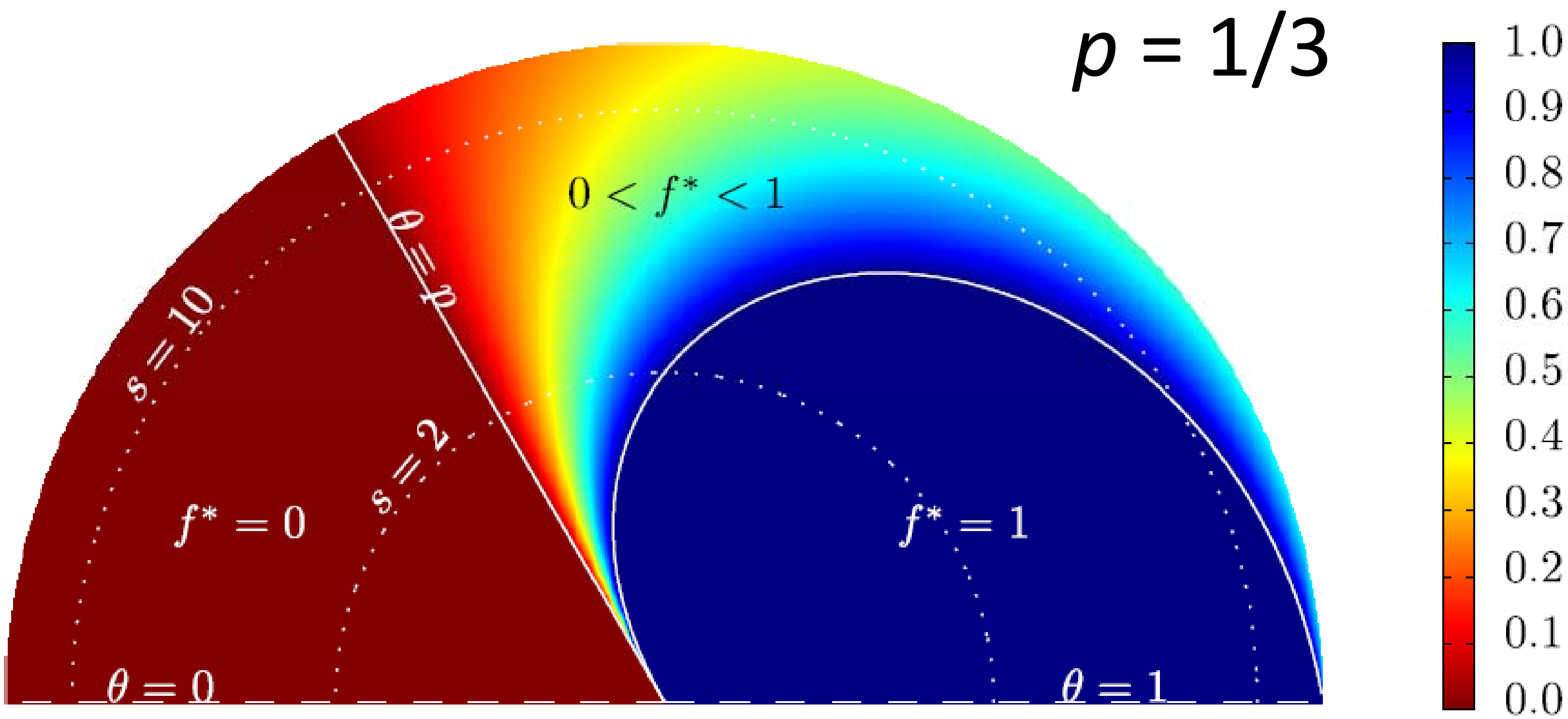
$$c_p \equiv pc_{a1} + qc_{a2} = \mathrm{E}[c_a]$$



# Risk Preferences



- Growth-optimal behavior *f**:




- For s $\approx$ 1, choice is deterministic, but not when s >> 1

# Risk Aversion

- Define risky outcomes relative to riskless outcome

$$c_{a1} \equiv c_b - d$$
$$c_{a2} \equiv c_b + u \quad , \quad u, d > 0$$

- Suppose *p* = ½ and *f** = ½ (indifferent between *a* and *b*)

$$u = d + \frac{d^2}{c_b - d}$$
$$\pi \equiv u - d = \frac{d^2}{c_b - d}$$

# Risk Aversion



- $\pi$ can be viewed as an evolutionary risk premium
- Due to Jensen's Inequality:

$$\exp(\mathrm{E}[\log(x)]) \leq \mathrm{E}[x]$$

- Risk aversion is "hard-wired" into survivors
- Equilibrium is not necessary to determine $\pi$

# Loss Aversion

**Two Key Observations:**

1. Must translate fecundity into financial wealth
2. Total wealth is what matters, not increments

- Define a reproduction function *c*(*w*) that maps financial wealth *w* into number of offspring *c*(*w*):

(A3) $c(w)$ is a continuous non-decreasing function of wealth $w$.

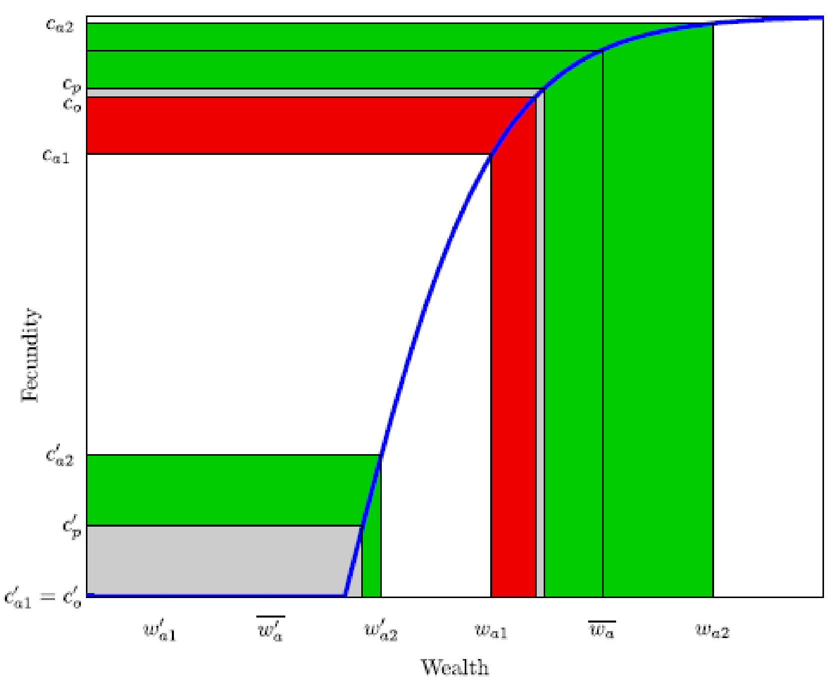
(A4) $c(w) = 0$ for all levels of wealth $w$ below a subsistence level $w_o$.

(A5) $c(w)$ is bounded above by some finite number $\bar{c} > 0$.

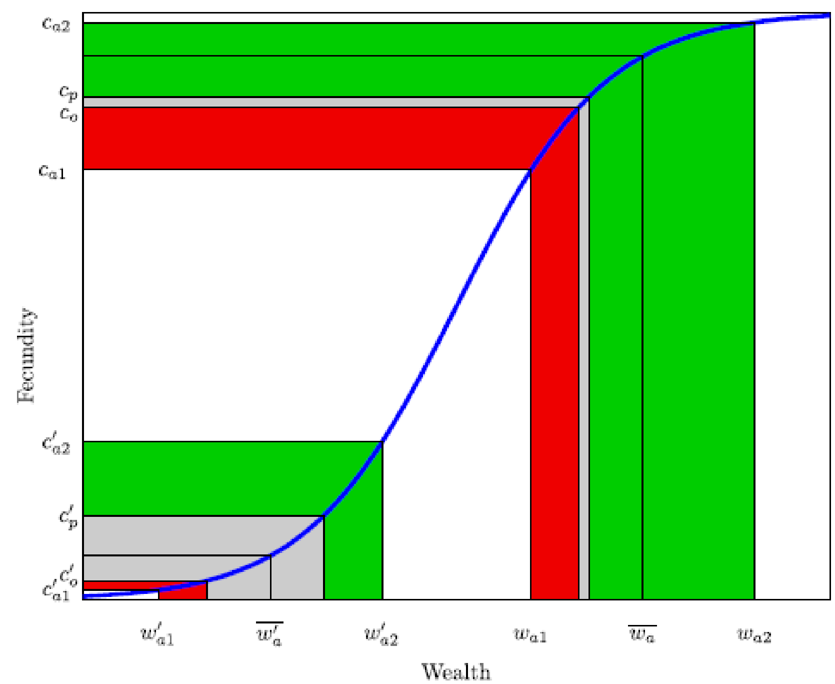
# Loss Aversion



**Proposition 5** *If $c(w)$ satisfies (A1)–(A5) and is twice continuously differentiable, then $c(w)$ is concave for sufficiently large values of $w$ and convex for sufficiently small values of $w$. If $c(w)$ is not continuously differentiable, then a slightly weaker result holds.*




(a) Kinked Subsistence Threshold

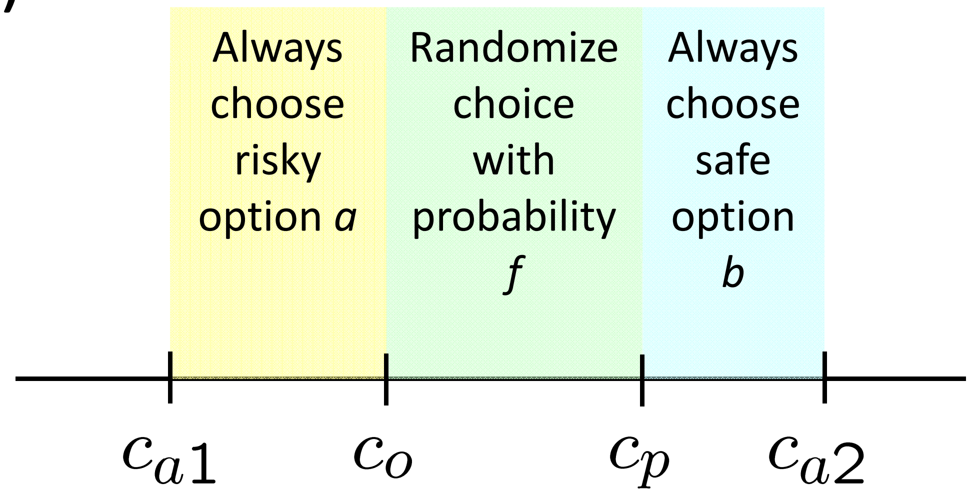



(b) Smooth Subsistence Threshold

# Loss Aversion

## But What About the "Reference Point"?

- Has to do with **incremental** vs. absolute reward
  - Experimenter offers incremental payoff
  - Preferences shaped by absolute payoff (*x*)
- Explains why experiments yield inconsisent findings
- For simplicity, suppose $c(w) = w$



# Systematic vs. Idiosyncratic Risk



**Natural Selection Yields The Following Behaviors:**

- Probability matching ("Herrnstein's Law")
- Randomization
- Risk aversion and risk-sensitive foraging behavior
- Loss aversion, anchoring, framing

**What If ($x_a$,$x_b$) Are Not Identical Across Individuals?**

- Suppose fecundity is IID across individuals and time

$$x_i^f = I_i^f x_{a,i} + (1-I_i^f)\, x_{b,i} \ , \quad I_i^f \equiv \begin{cases} 1 & \text{with probability } f \\ 0 & \text{with probability } 1-f \end{cases}$$

# Systematic vs. Idiosyncratic Risk

$$n_t^f = \sum_{i=1}^{n_{t-1}^f} x_{i,t}^f \neq \Big(\sum_{i=1}^{n_{t-1}^f} I_{i,t}^f\Big) x_{a,t} + \Big(\sum_{i=1}^{n_{t-1}^f} (1 - I_{i,t}^f)\Big) x_{b,t}$$

$$n_t^f = \sum_{i=1}^{n_{t-1}^f} x_{i,t}^f = \Big(\sum_{i=1}^{n_{t-1}^f} I_{i,t}^f x_{a,i,t}\Big) + \Big(\sum_{i=1}^{n_{t-1}^f} (1 - I_{i,t}^f) x_{b,i,t}\Big)$$

$$n_t^f \stackrel{p}{=} n_{t-1}^f \big( f\mu_a + (1-f)\mu_b \big)$$

$$n_T^f \stackrel{p}{=} \prod_{t=1}^{T} (f\mu_a + (1-f)\mu_b) = \exp\Big(\sum_{t=1}^{T} \log(f\mu_a + (1-f)\mu_b)\Big)$$

$$\frac{1}{T}\log n_T^f \stackrel{p}{=} \frac{1}{T}\sum_{t=1}^{T} \log(f\mu_a + (1-f)\mu_b)$$

$$\stackrel{p}{\to} \mu(f) \equiv \not{E}[\log(f\mu_a + (1-f)\mu_b)]$$

**Non-stochastic!**

# Systematic vs. Idiosyncratic Risk



**Growth-Optimal Behavior With Idiosyncratic Risk:**

$$f^* \;=\; \begin{cases} 1 & \text{if } \mu_a > \mu_b \\ 0 & \text{if } \mu_a \leq \mu_b \end{cases}$$

- In this case, no difference between individually optimal and growth-optimal behavior
- No "behavioral biases"; no risk aversion; everyone behaves "rationally" (*Homo economicus*)
- Behavior can be identical because environment is not
- If environment is identical, behavior cannot be
- "Nature abhors an undiversified bet"!

# Bounded Rationality and Intelligence



## Simon's Notion of "Satisficing":

- Heuristics, not optimization
- Develop mental models to simplify decisions
- Impact on AI, but not on economics
- How do we know what is "good enough"?

## Answer $\Rightarrow$ We Don't! Our Heuristics Evolve

- In our framework, let state variable *z* be correlated to *x* and observable at some cost *c*

# Bounded Rationality and Intelligence



**Consider Getting Dressed:**

- 5 Jackets, 10 Pants, 20 Ties, 10 Shirts, 10 Pairs of Socks, 4 Pairs of Shoes, 5 Belts
- 2,000,000 Possible Outfits!
- Takes 1 Second To Evaluate Each Outfit
- How Long To Get Dressed?
- 23.1 Days!

**How Do We Get Dressed So Quickly?**

$\Rightarrow$ **Evolution of Heuristics**

# Bounded Rationality and Intelligence

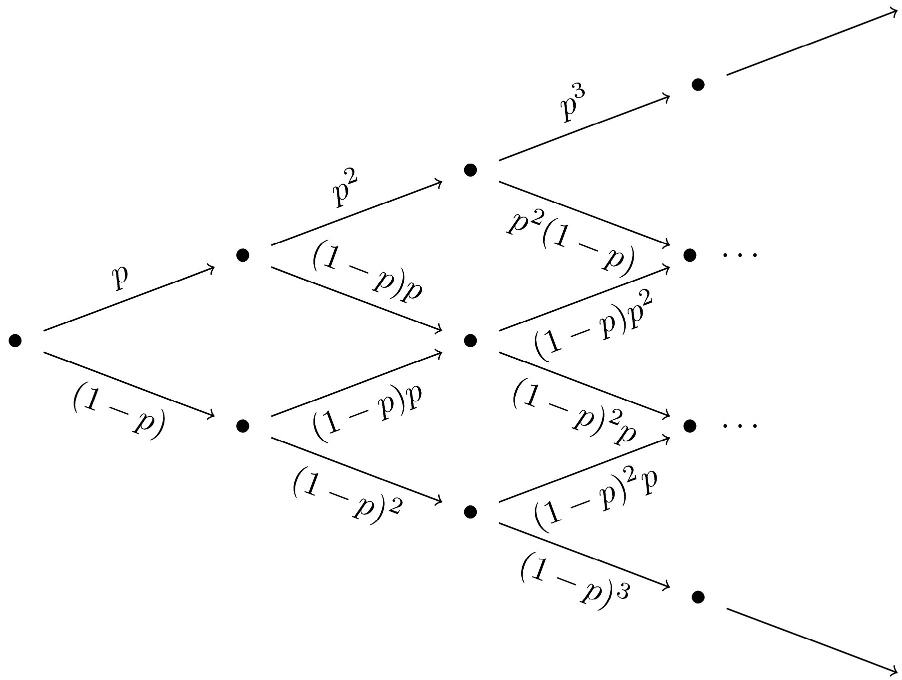
## What Is Intelligence? An Evolutionary Definition:

- Behavior that confers reproductive advantage
- Hawkins memory/prediction model fits this definition

## What Is Stupidity?

- Behavior that is counterproductive to survival

# Bounded Rationality and Intelligence

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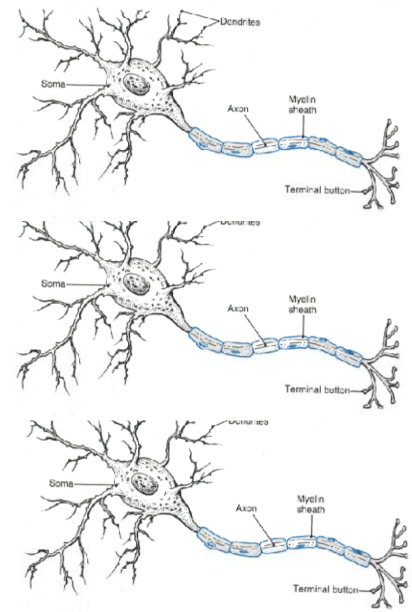



## Evolution At The Speed of Thought

- Behavioral plasticity
- Hierarchy of behaviors (Herrnstein vs. Heimlich)
- Implications for neurophysiology

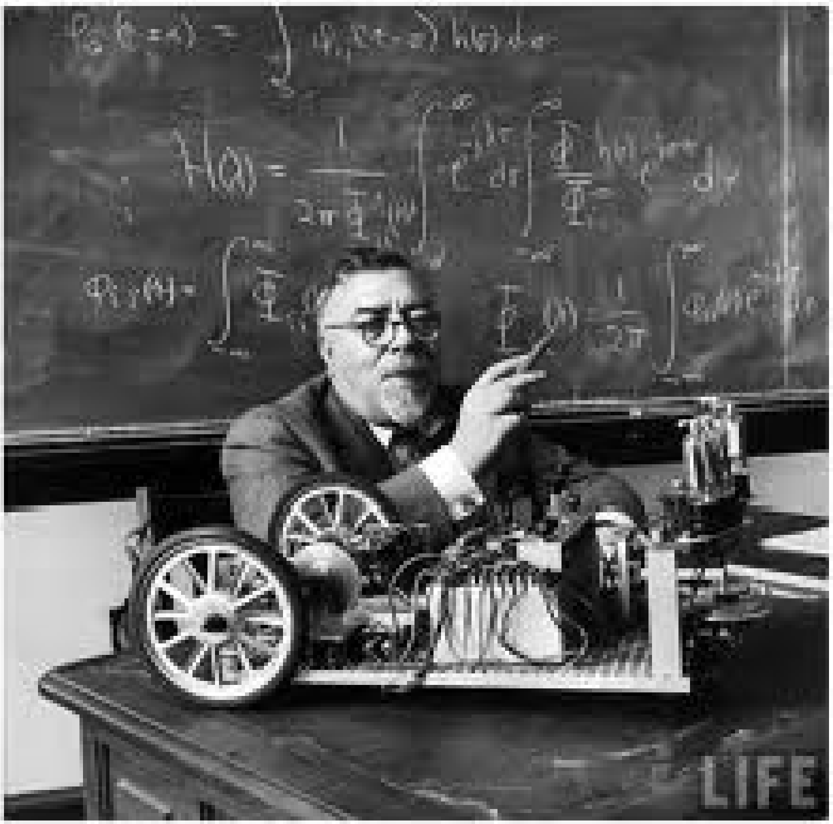
# Other Extensions

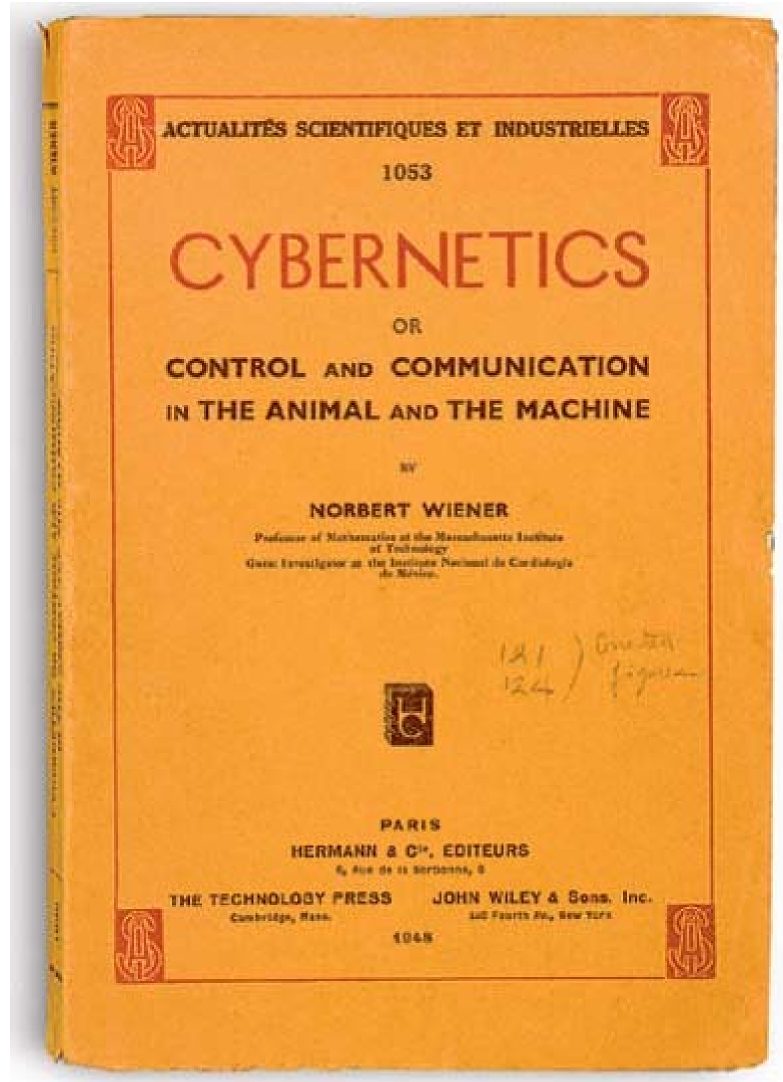


- Sexual reproduction
- Iteroparity
- Multivariate multi-stage choice problems
- Resource constraints, strategic interactions, population equilibrium
- Time-varying and nonstationary $\Phi(x_a,x_b)$
  - Environmental shocks yield punctuated equilibria
  - Group selection
  - "Complex adaptive systems"

# The Challenge

## Can We Construct a Complete Theory of Human Behavior?



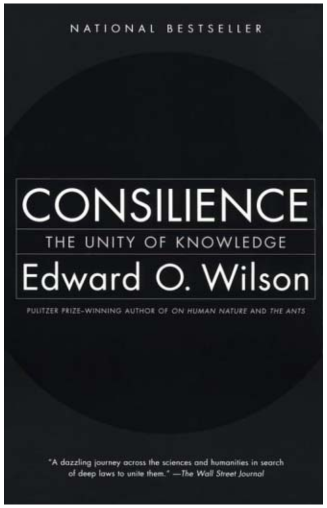



# The Challenge

## Consilience (E.O. Wilson, 1998):

The Consilience of Inductions takes place when an Induction, obtained from one class of facts, coincides with an Induction, obtained from another different class. This Consilience is a test of the truth of the Theory in which it occurs.

— William Whewell, 1840, *Philosophy of the Inductive Sciences*, 1840.



# The Challenge



- Framework for modeling the evolution of behavior
  - Abstracts from underlying genetics
  - Biological "reduced form" model
- Simplicity implies behaviors are primitive and ancient
- Mathematical basis of the Adaptive Markets Hypothesis
  - Evolution determines individual behavior
  - Evolution also determines aggregate dynamics
  - Efficiency and irrationality are both adaptive
  - The key is how environment is related to behavior

# The Challenge



## Instead of:

"It's the economy stupid!"

## We Should Say:

"It's the **environment**, stupid!"

# Thank You!

# Motivation

## Preferences Under Certainty:

- Non-Satiation
- Transitivity
- Completeness
- Diminishing Marginal Utility

## Finance Theory Is Complete

# Motivation



**Preferences Under Uncertainty:**

- Utility of a random variable
- Difficult to evaluate
- Requires strong assumptions
- Von Neumann and Morganstern
- Expected Utility Theory (EUT)

**Modern Economics and Finance Are Built on EUT**

# Motivation



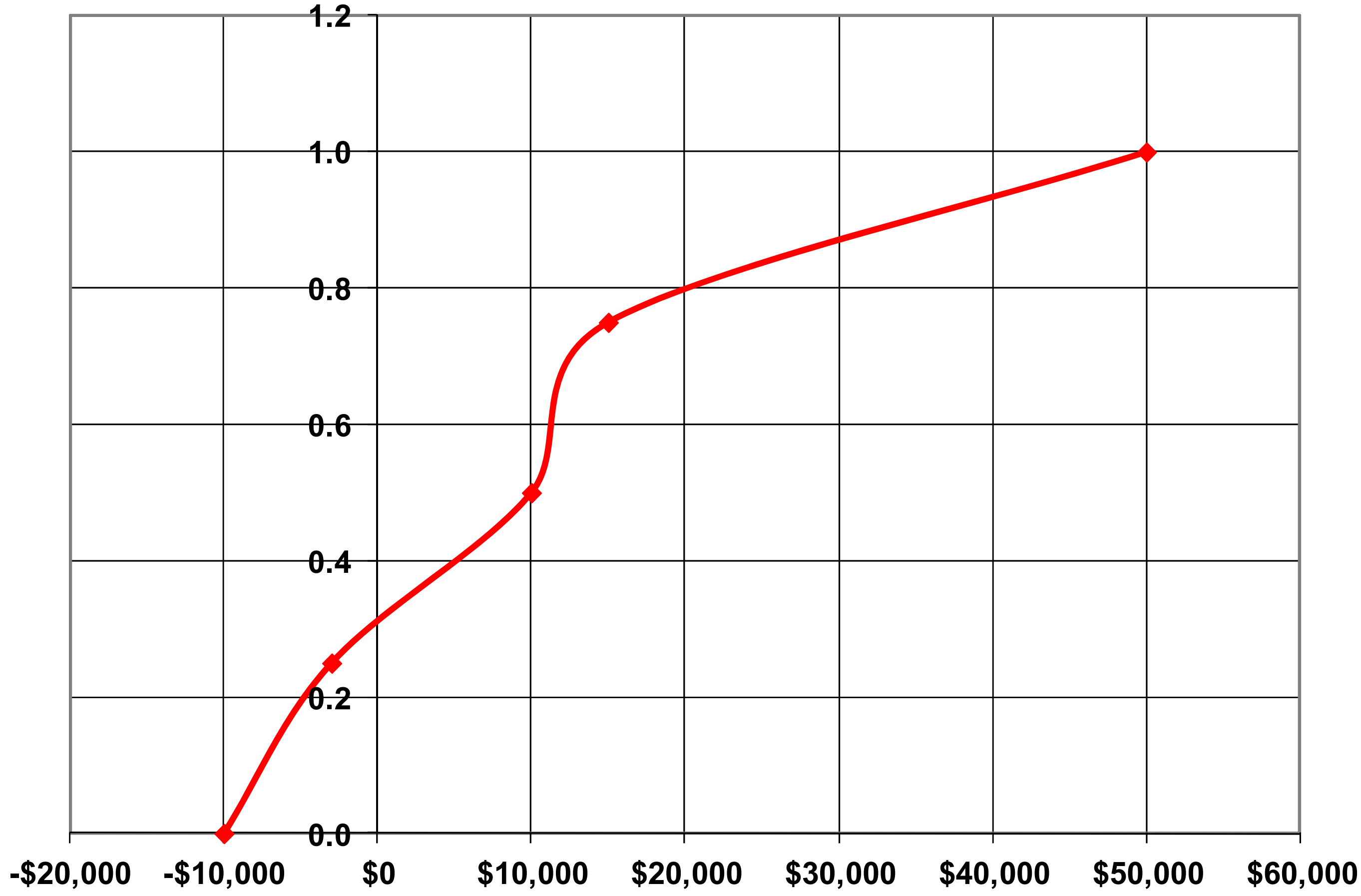
## G2 and G3:

| | | | |
|---|---|---|---|
| Probability | G1 | G2 | G3 |
| 50% | $50,000 | $50,000 | ??? |
| 50% | ($10,000) | ??? | ($10,000) |
| | | | |
| Certainty | X1 | X2 | X3 |
| Equivalent: | ??? | ??? | ??? |

# Motivation



## Estimated Utility Function



# Motivation



## Denote By *U(x)* Your Utility Function

- $U(\$50{,}000) = 1$, $U(-\$10{,}000) = 0$
- Consider three gambles, G1, G2, G3:

| | | |
|---|---|---|
| G1: | \$50,000 | With 50% Probability |
| | –\$10,000 | With 50% Probability |

What is the <u>most</u> you would pay for G1?