Fragment Grammars: Productivity and Reuse in Language

Timothy J. O'Donnell

-ness

-ness

- *circuitousness, grandness, orderliness, pretentiousness, cheapness, coolness, warmness, ...*

-ness

- *circuitousness*, *grandness*, *orderliness*, *pretentiousness*, *cheapness*, *coolness*, *warmness*, ...
- Adj>N

-ness

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- Adj>N
- *grand* + -ness

-ness

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- Adj>N
- *grand* + -ness
- *pine-scentedness*

-ity

-ity

- *verticality*, *tractability*, *severity*, *seniority*, *inanity*, *electricity*, ...

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- Stress change (e.g., *normalness* v. *normality*), vowel laxing (e.g., *inane* v. *inanity*)

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- *The red lantern indicated the ethnicity/ ethnicness of the restaurant*

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- *The red lantern indicated the ethnicity/ ethnicness of the restaurant*
- **pine-scentedity*

-ity

-ity

- But ...

-ity

- But ...
  - -ile/-al/-able/-ic/-(i)an

-ity

- But ...
  - -ile/-al/-able/-ic/-(i)an
  - *Bayesable*

-ity

- But ...
  - -ile/-al/-able/-ic/-(i)an
  - *Bayesable*
    - *Bayesability*

-ity

- But ...
  - -ile/-al/-able/-ic/-(i)an
  - *Bayesable*
    - *Bayesability*
  - *Coolity is not trying* (from *Huffington Post*)

-th

-th

- *warmth*, *width*, *truth*, *depth*, ...

-th

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- Adj>N

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- Adj>N
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- *roomth*, *greenth*

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- *roomth*, *greenth*

Many enjoy the warmth, Vikings prefer the ***coolth***

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- How are such differences in productivity represented by the adult language user?
- How are such differences learned by the child?

Outline

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1. The Proposal.

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2. Five Models of Productivity and Reuse.

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3. English Derivational Morphology

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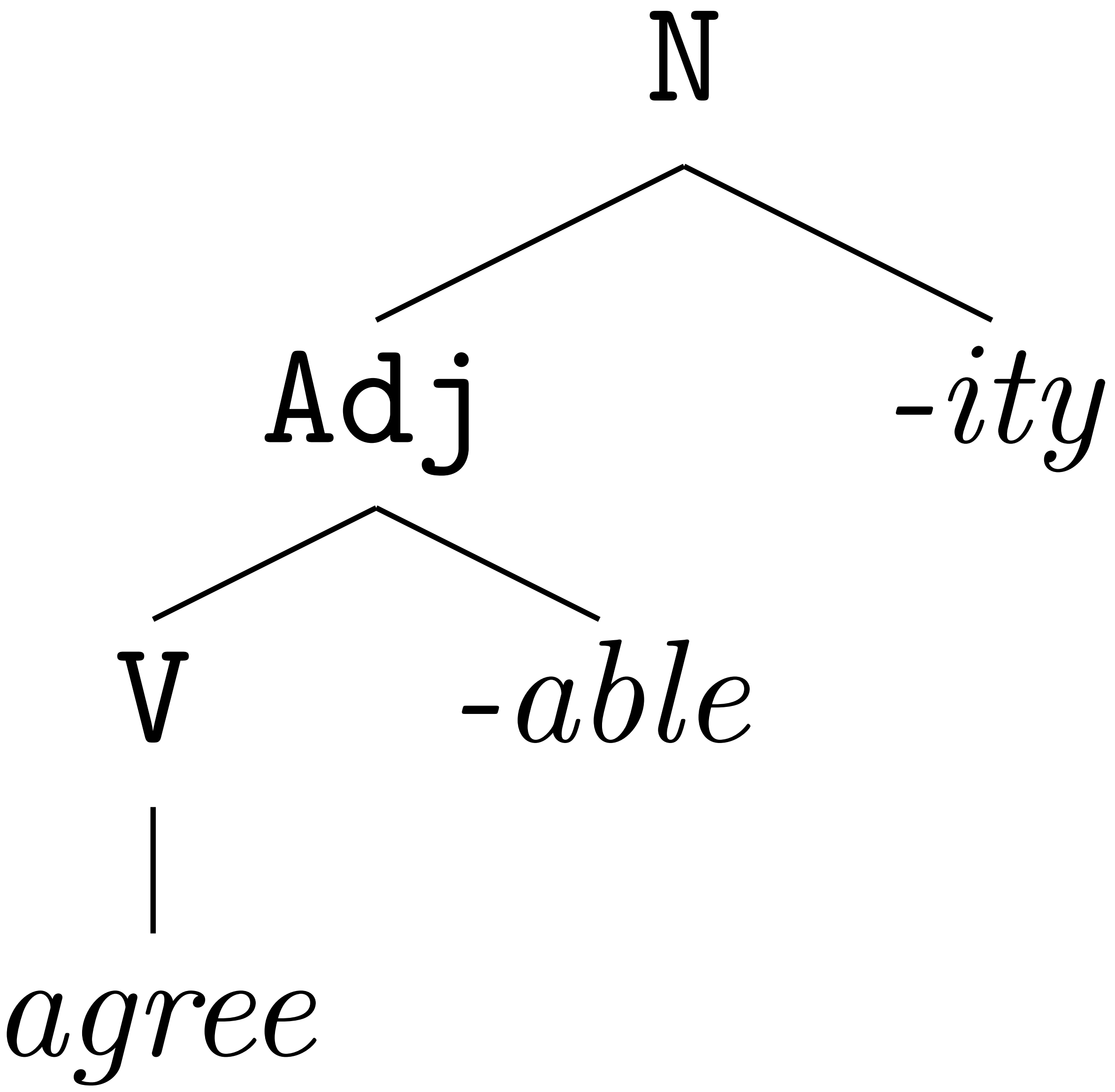
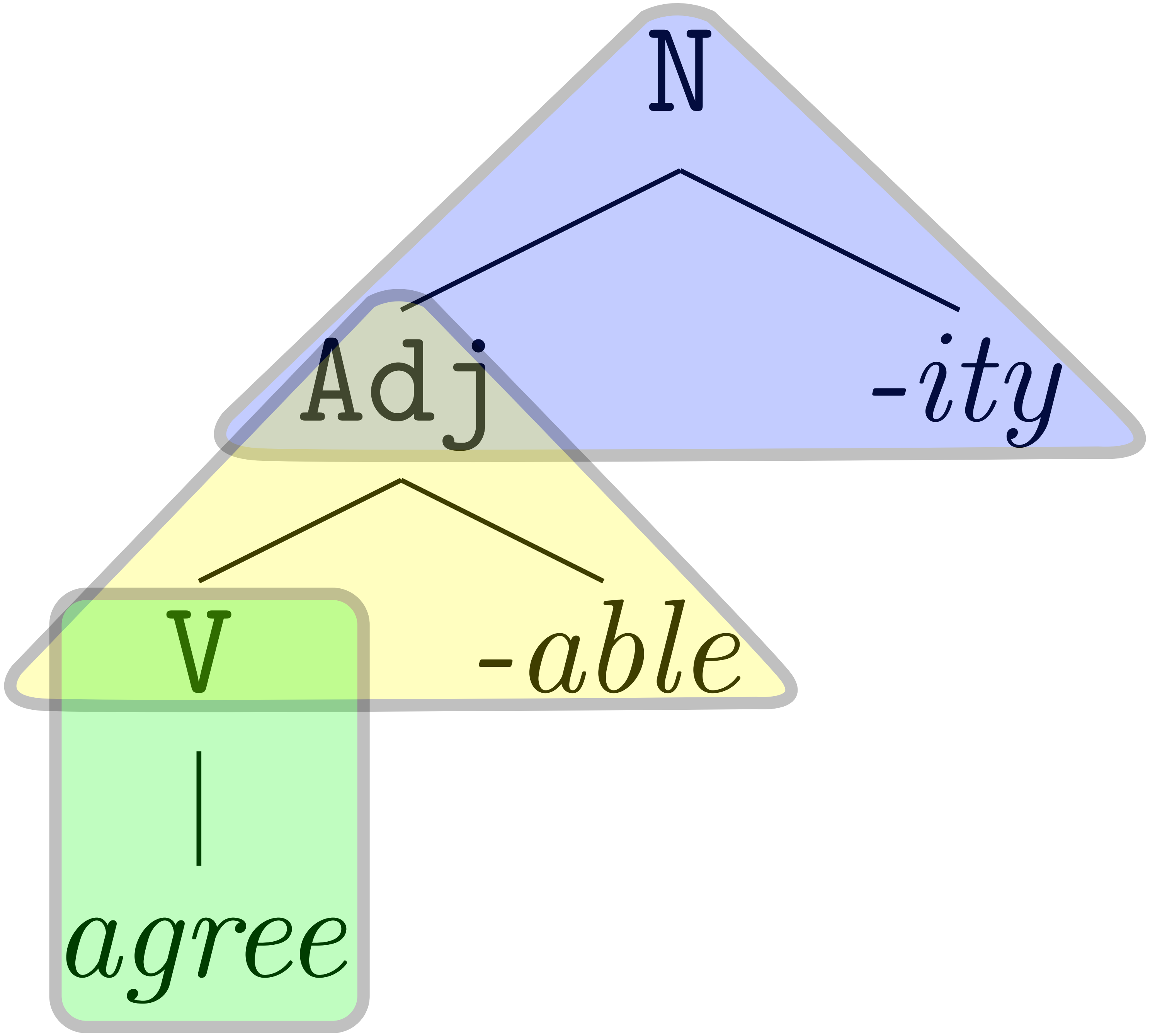
Starting Computational System

W	⟶	N	
W	⟶	V	
W	⟶	Adj	
W	⟶	Adv	
N	⟶	Adj	*-ness*
N	⟶	Adj	*-ity*
N	⟶	*electro-*	N
N	⟶	*magnet*	
N	⟶	*dog*	
...			
V	⟶	N	*-ify*
V	⟶	Adj	*-ize*
V	⟶	*re-*	V
V	⟶	*agree*	
V	⟶	*count*	
...			
Adj	⟶	*dis-*	Adj
Adj	⟶	V	*-able*
Adj	⟶	N	*-ic*
Adj	⟶	N	*-al*
Adj	⟶	*tall*	
...			
Adv	⟶	Adj	*-ly*
Adv	⟶	*today*	
...			



Subcomputations



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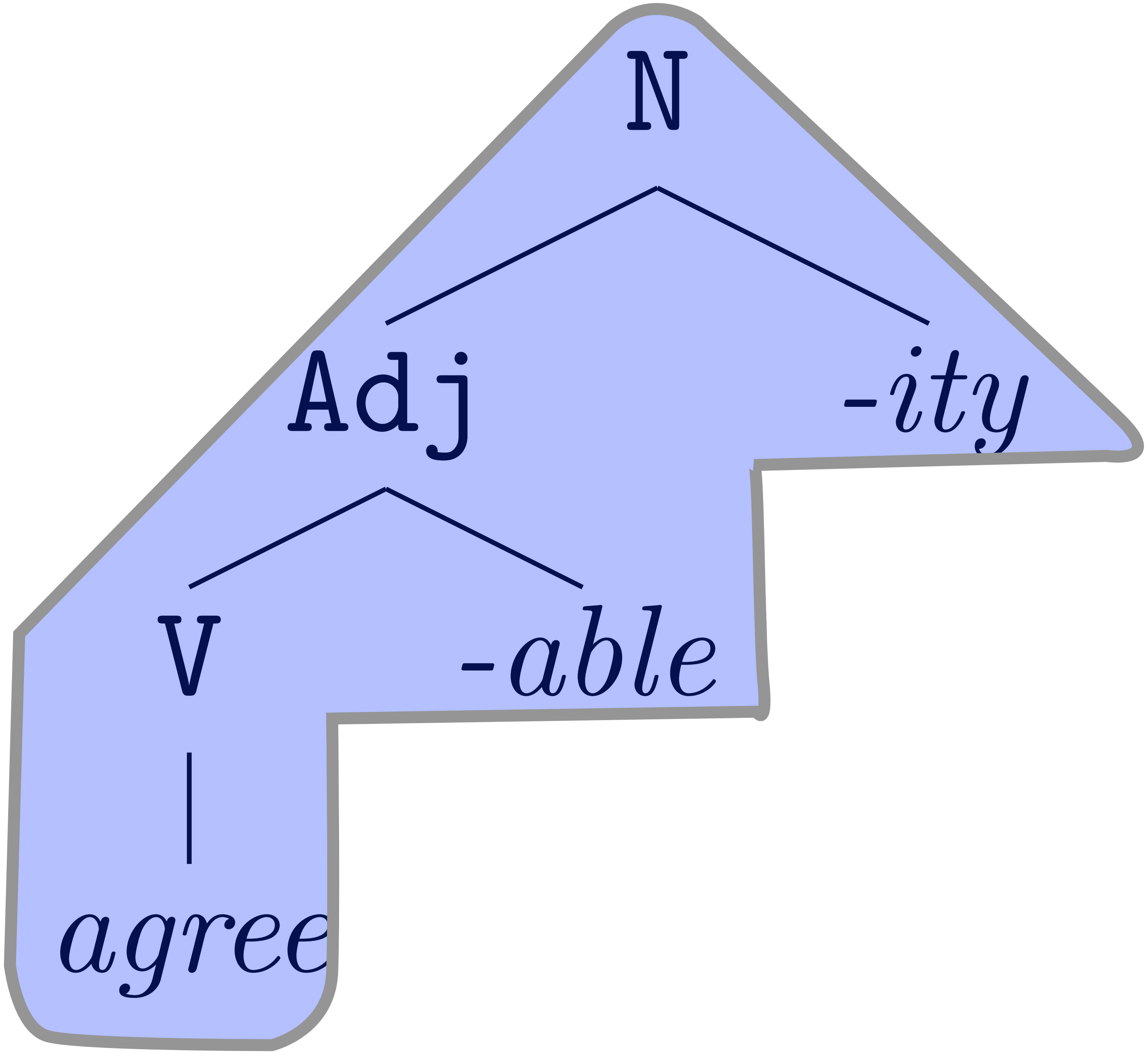
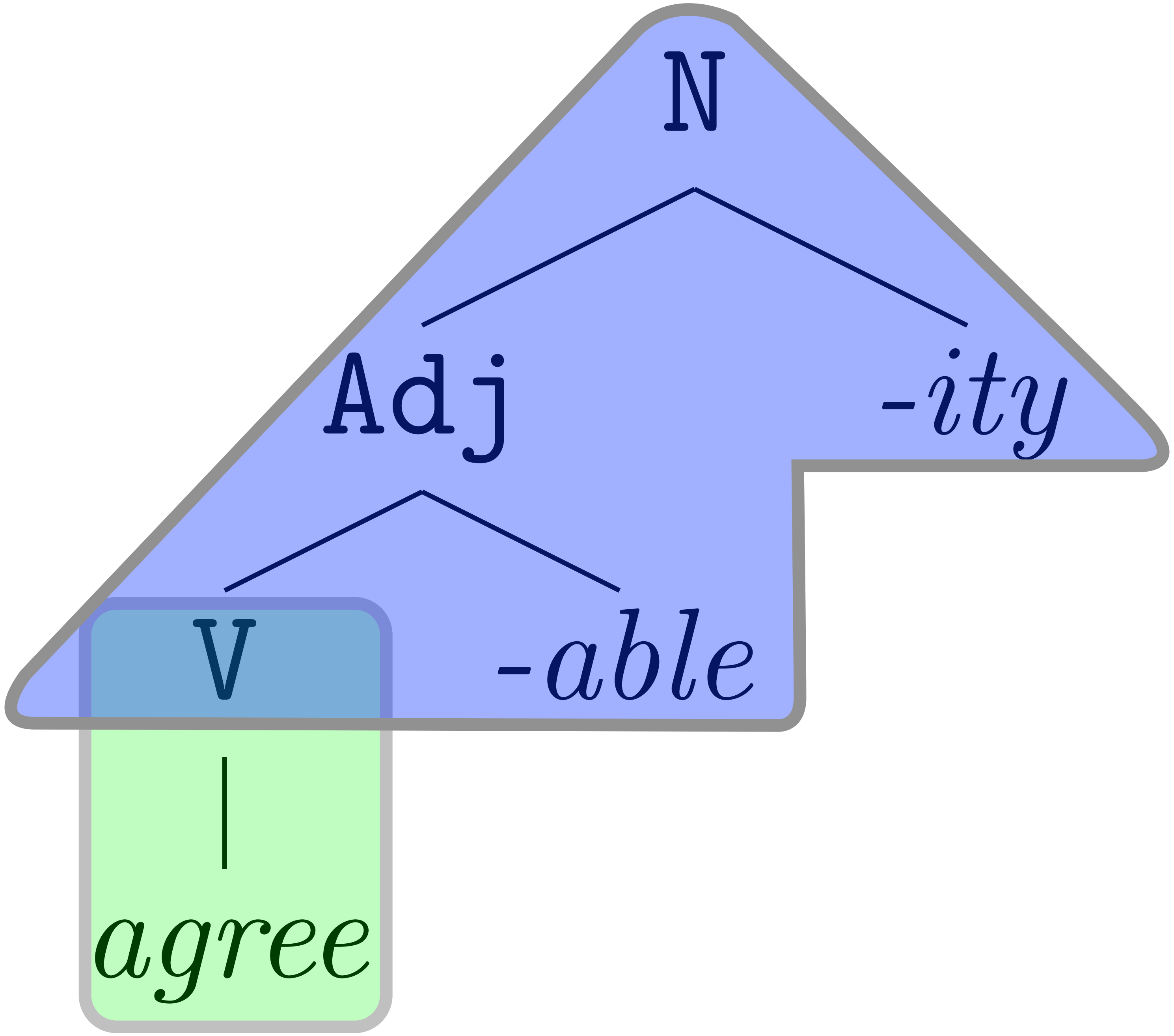


Subcomputations



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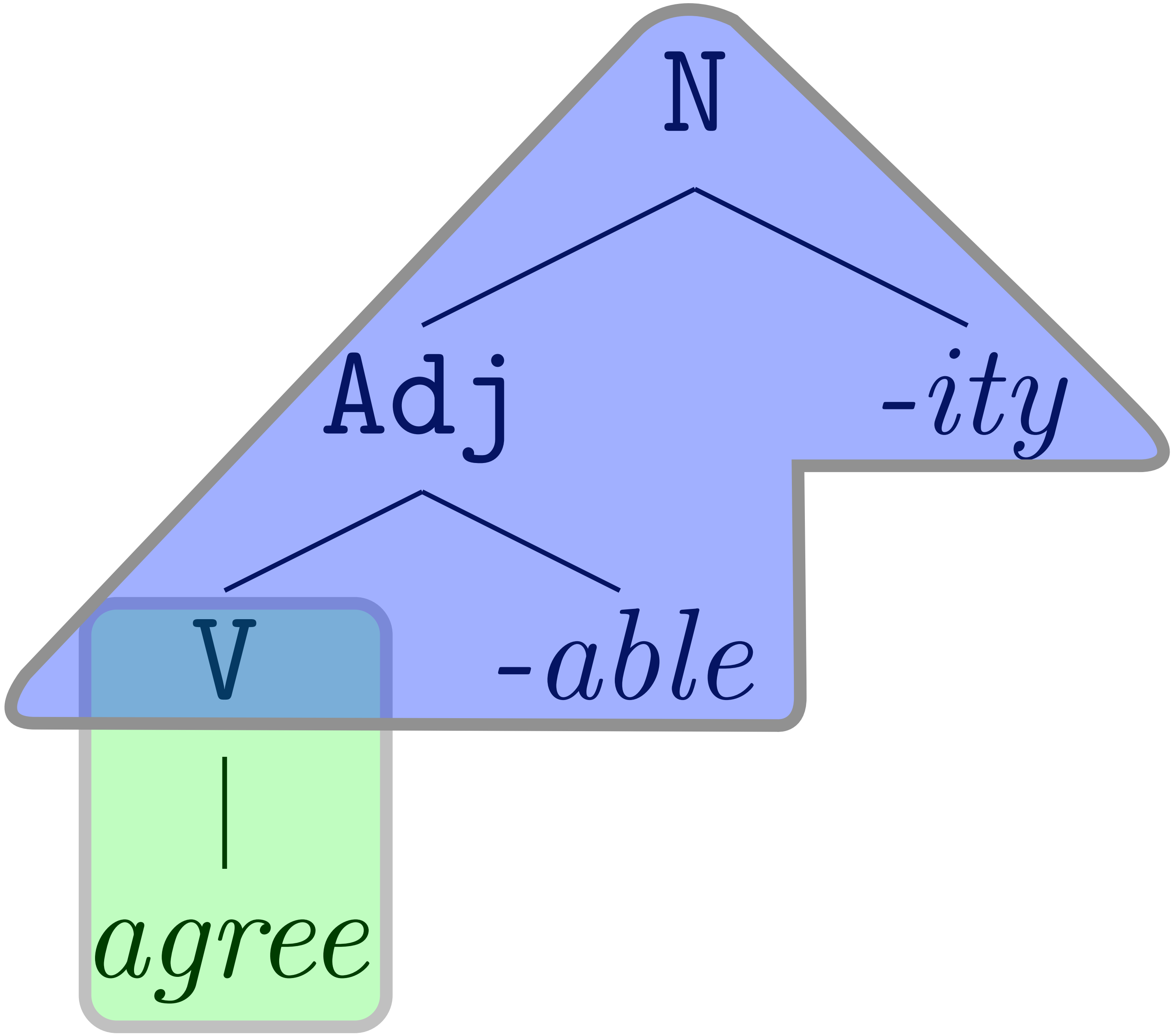
Bayesian Rational Analysis (Anderson, 1992)

- Find subcomputations which provide best explanation for the data.
- What *evidence* is available to the learner?
  - Which patterns give rise to productivity, which patterns imply reuse?

Subcomputations as Predictions



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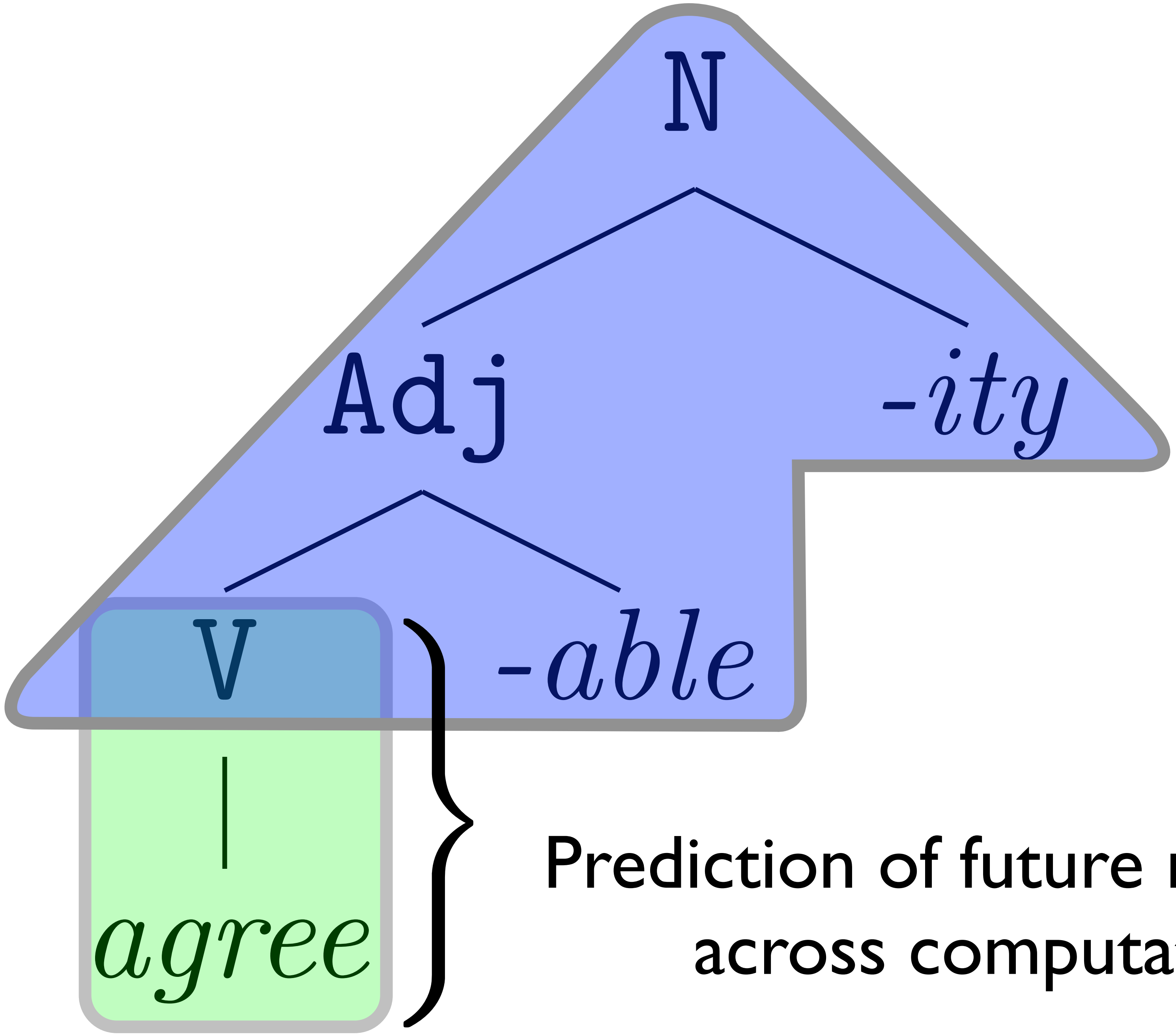
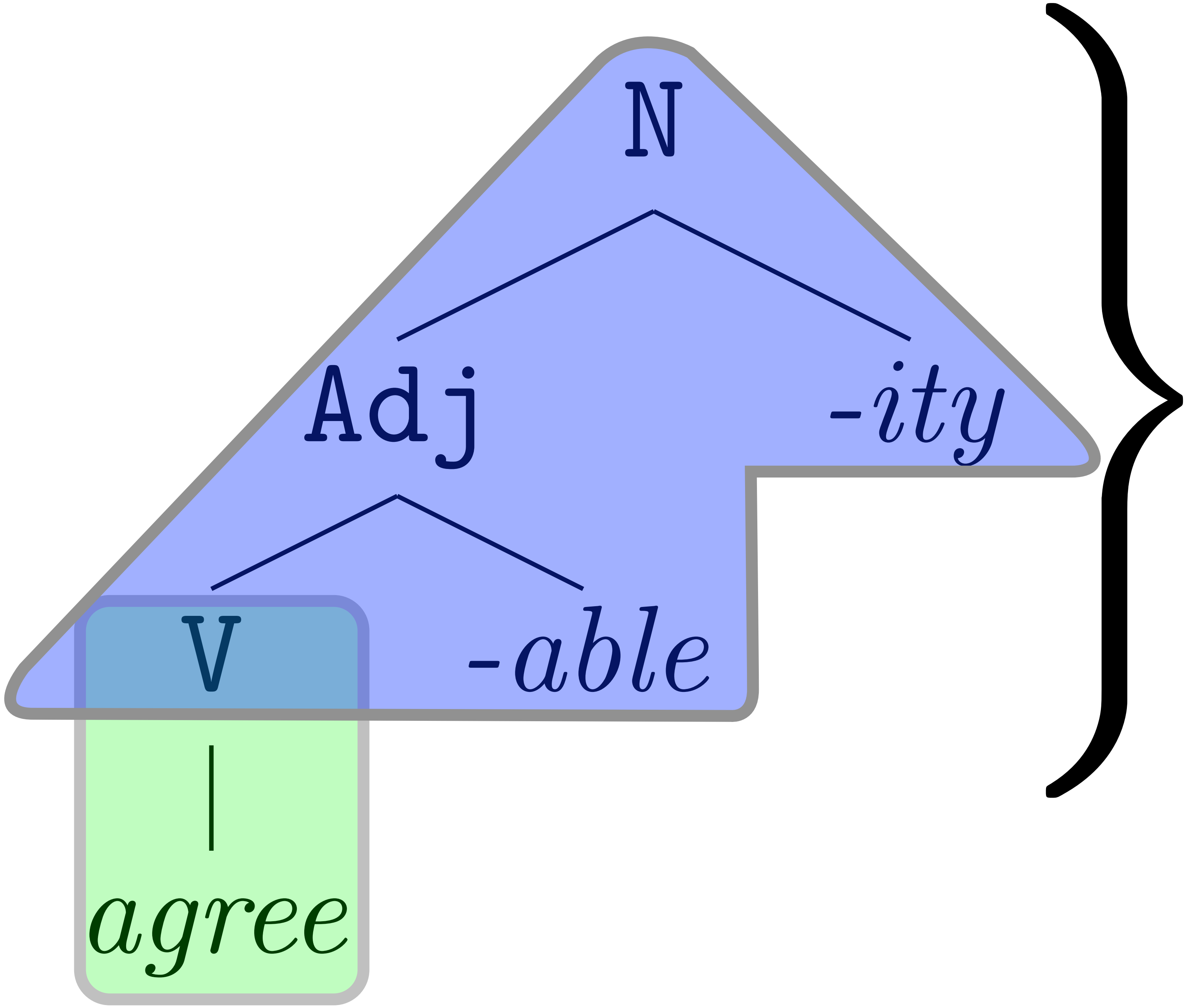


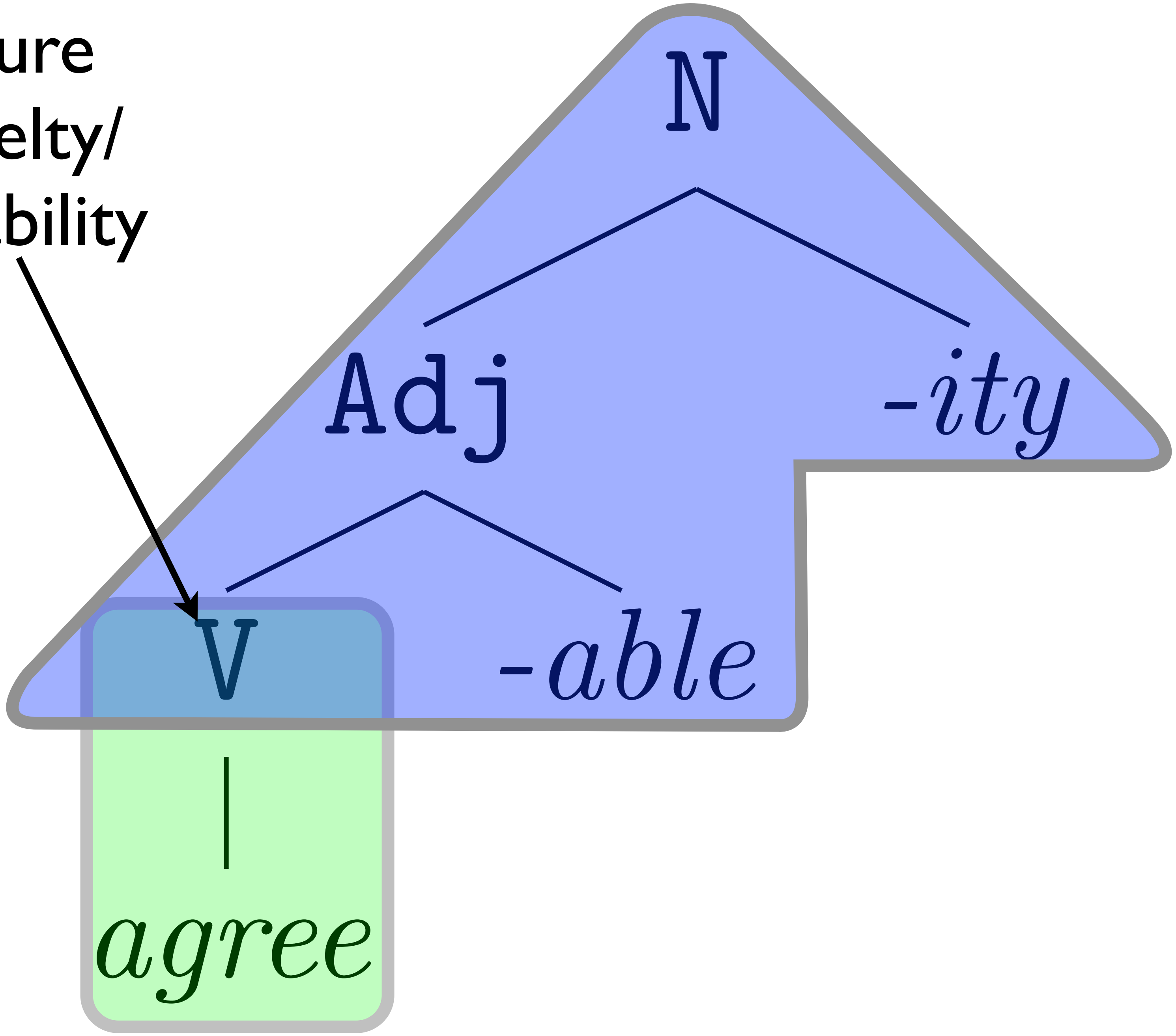
Prediction of future reusability across computations

Subcomputations as Predictions



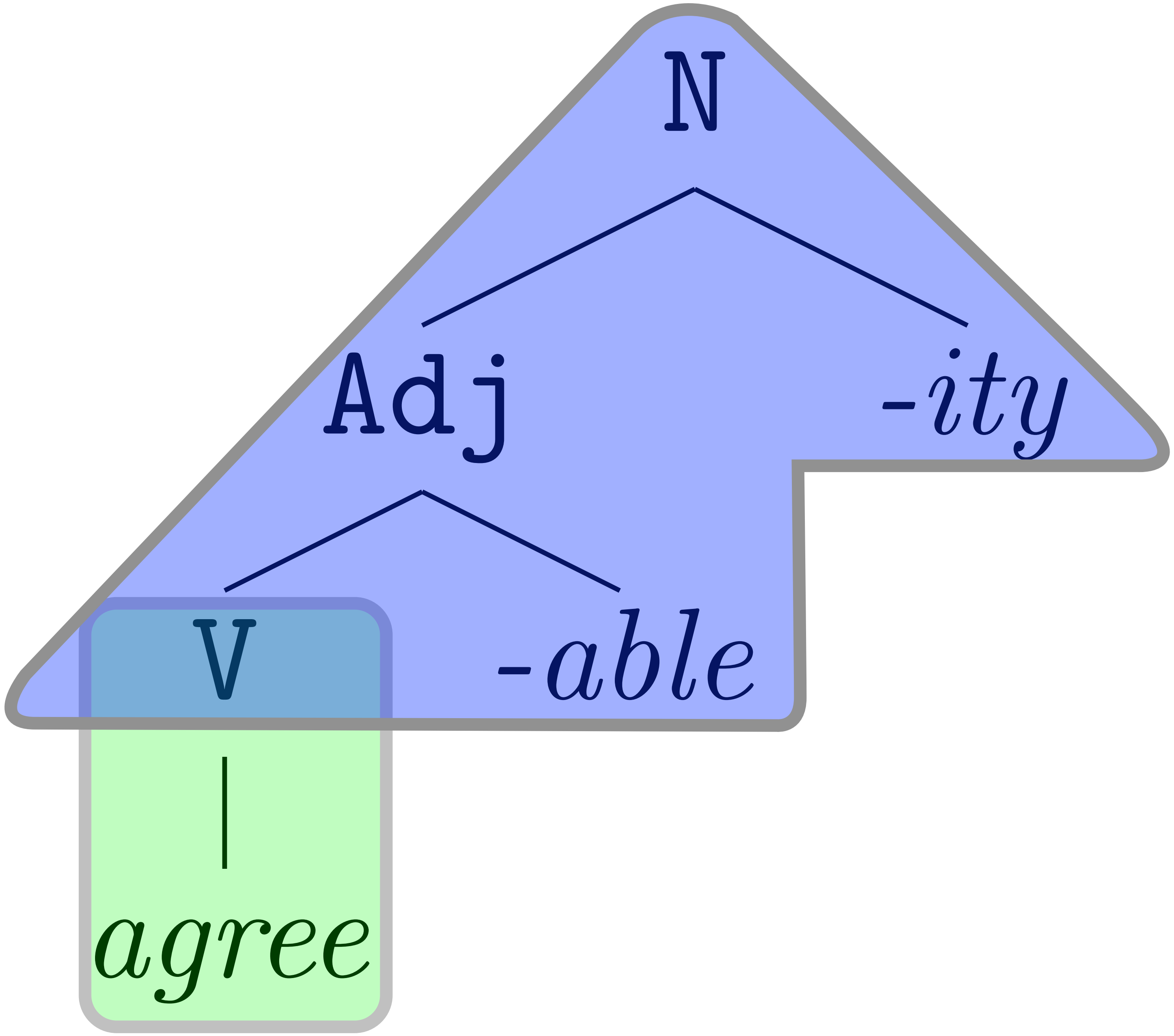
Prediction of future reusability of <u>*combination*</u>

Subcomputations as Predictions



Prediction of future novelty/ variability

Subcomputations as Predictions



Tradeoff between productivity and reuse

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- Generalization of *Adaptor Grammars* (Johnson et al., 2007).
- Bayesian non-parametric distributions (*Pitman-Yor*).
- Notion of *compiling* subcomputations via tools from probabilistic programming (Church language; Goodman et al., 2008).
    - Stochastic memoization (Johnson et al., 2007) of stochastically lazy/ eager programs.

Languages for probability

- Purposes of a language:
  - Makes writing down models easier.
  - Makes reasoning about models clearer.
  - Supports efficient inference.
  - Gives ideas about mental representation.

λ calculus

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```
(double 3)
```

```
=> 6
```

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```
(define repeat
  (λ (f) (λ (x) (f (f x)))))
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```
((repeat double) 3)
```

=> 12

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(define double
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```

```
(double 3)     => 6
```

```
(define repeat
   (λ (f) (λ (x) (f (f x)))))
```

```
((repeat double) 3)    => 12
```

```
(define 2nd-derivative (repeat derivative))
```

ψλ-calculus

- How can we use these ideas to describe probabilities?
- ψλ-calculus: a stochastic variant.
  - We introduce a random primitive `flip`, such that `(flip)` reduces to a random sample `t`/`f`.
  - The usual evaluation rules now result in *sampled* values. This induces *distributions*.
- This calculus, plus primitive operators and data types, gives the probabilistic programming language Church.

Church

Random primitives:

```
(define a (flip 0.3))
(define b (flip 0.3))
(define c (flip 0.3))
(+ a b c)
```

Goodman, Mansinghka, Roy,
Bonawitz, Tenenabum (2008)

Church

Random primitives:

```
(define a (flip 0.3)) => 1
(define b (flip 0.3))
(define c (flip 0.3))
(+ a b c)
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Church

Random primitives:

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(define a (flip 0.3)) => 1
(define b (flip 0.3)) => 0
(define c (flip 0.3))
(+ a b c)
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Random primitives:

```
(define a (flip 0.3)) => 1
(define b (flip 0.3)) => 0
(define c (flip 0.3)) => 1
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Church

Random primitives:

```
(define a (flip 0.3)) => 1
(define b (flip 0.3)) => 0
(define c (flip 0.3)) => 1
(+ a b c)             => 2
```

Goodman, Mansinghka, Roy,
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Church

Random primitives:

```
(define a (flip 0.3)) => 1 0
(define b (flip 0.3)) => 0 0
(define c (flip 0.3)) => 1 0
(+ a b c)             => 2 0
```

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Random primitives:

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(define a (flip 0.3)) => 1 0 0
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(define a (flip 0.3)) => 1 0 0
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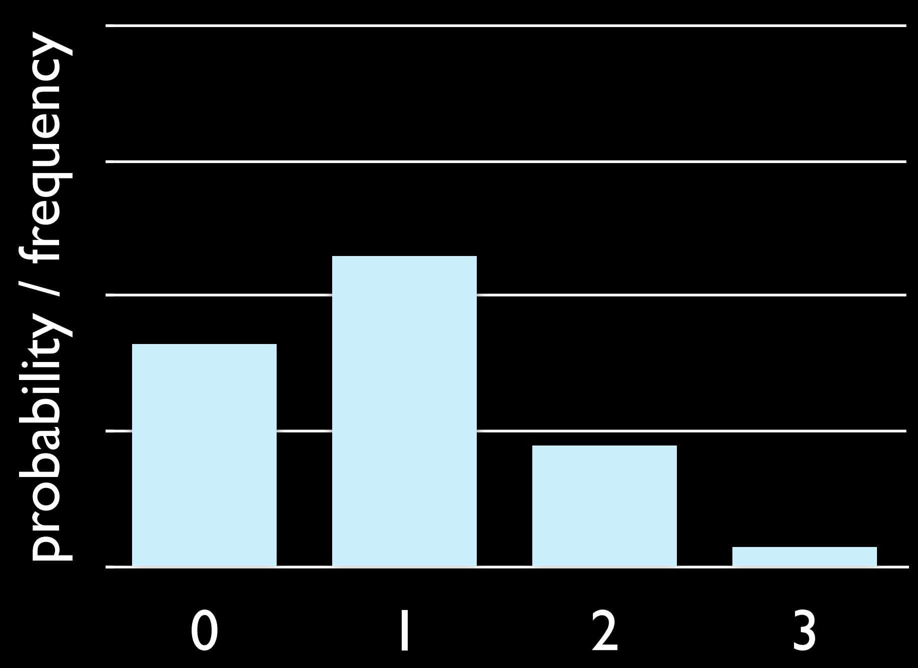

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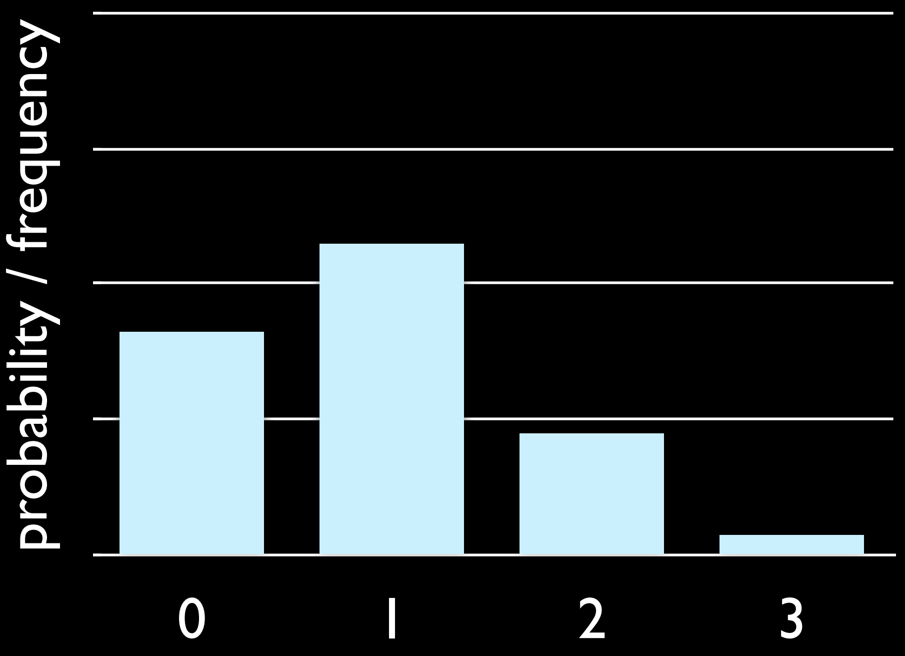

Theorem: Any computable distribution can be represented by a Church expression.

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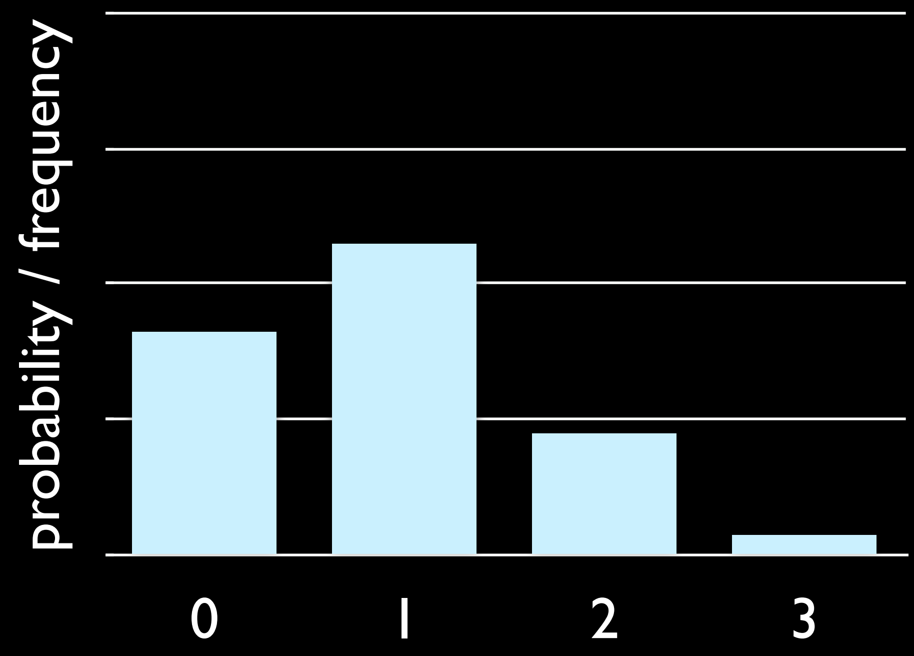

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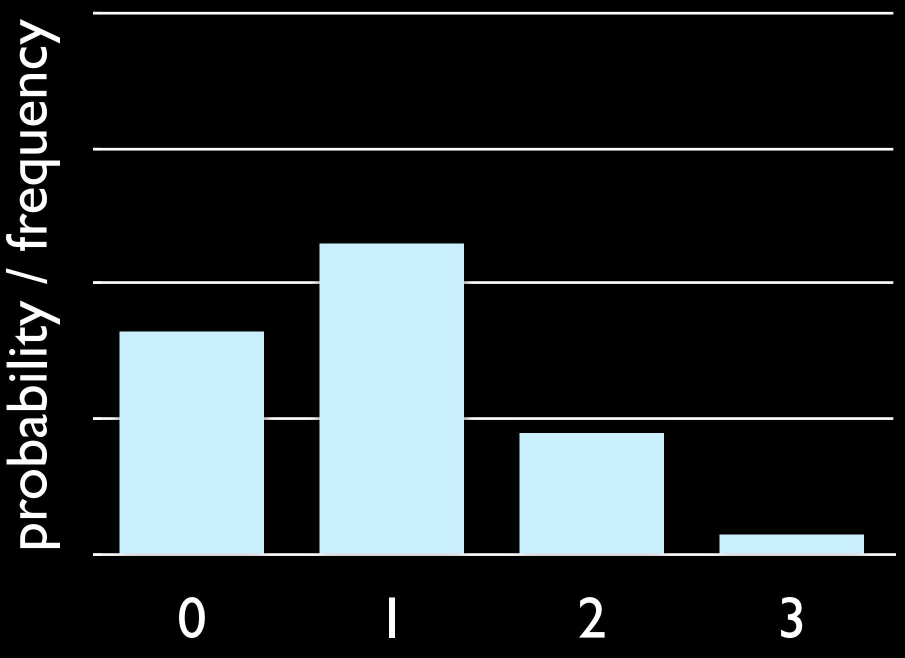

Conditioning (inference):

```
(query
  (define a (flip 0.3))
  (define b (flip 0.3))
  (define c (flip 0.3))
  (+ a b c)
  (= (+ a b) 1))
```

Goodman, Mansinghka, Roy, Bonawitz, Tenenabum (2008)

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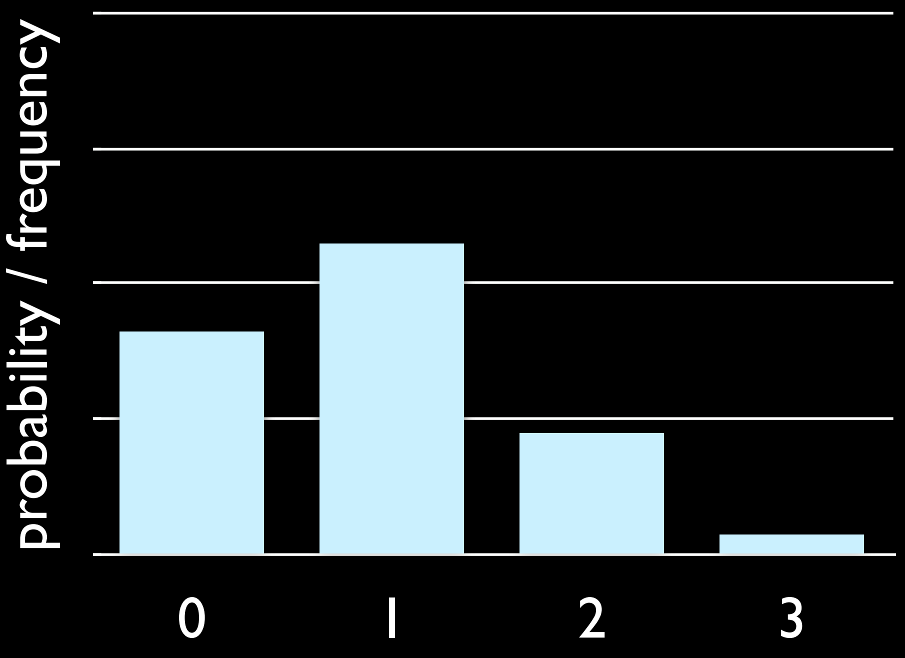

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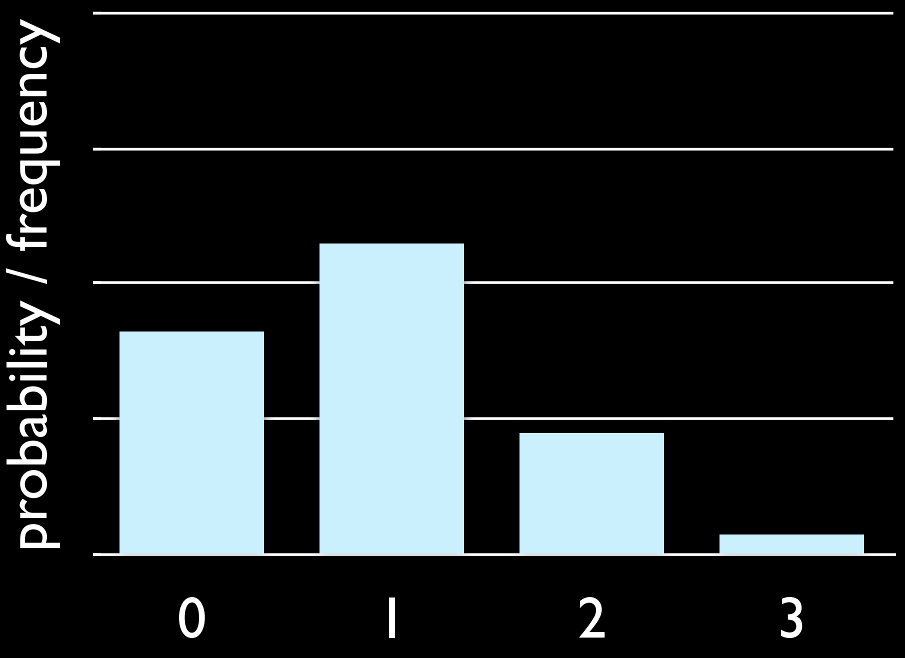

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Goodman, Mansinghka, Roy, Bonawitz, Tenenabum (2008)

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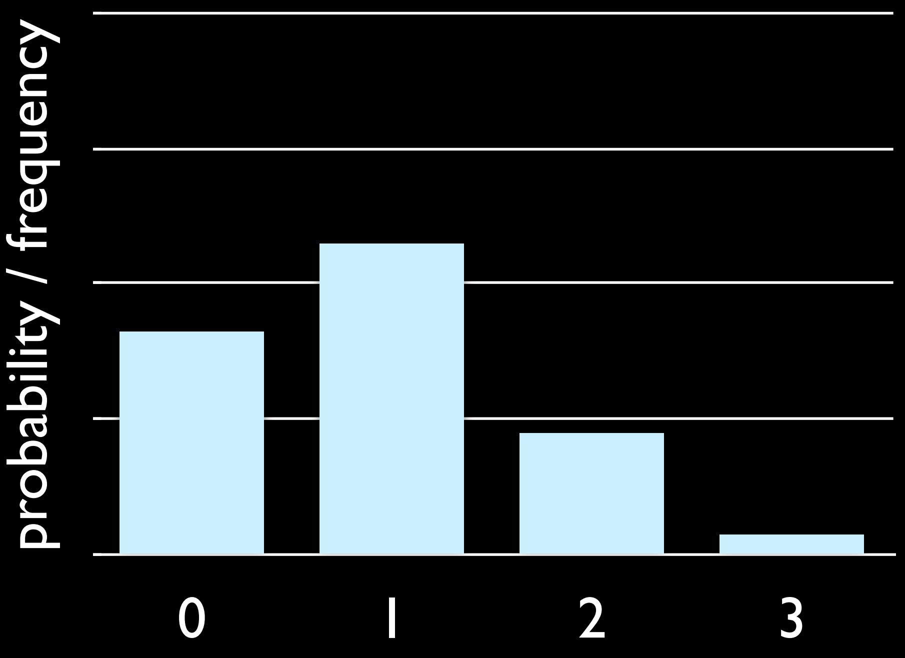

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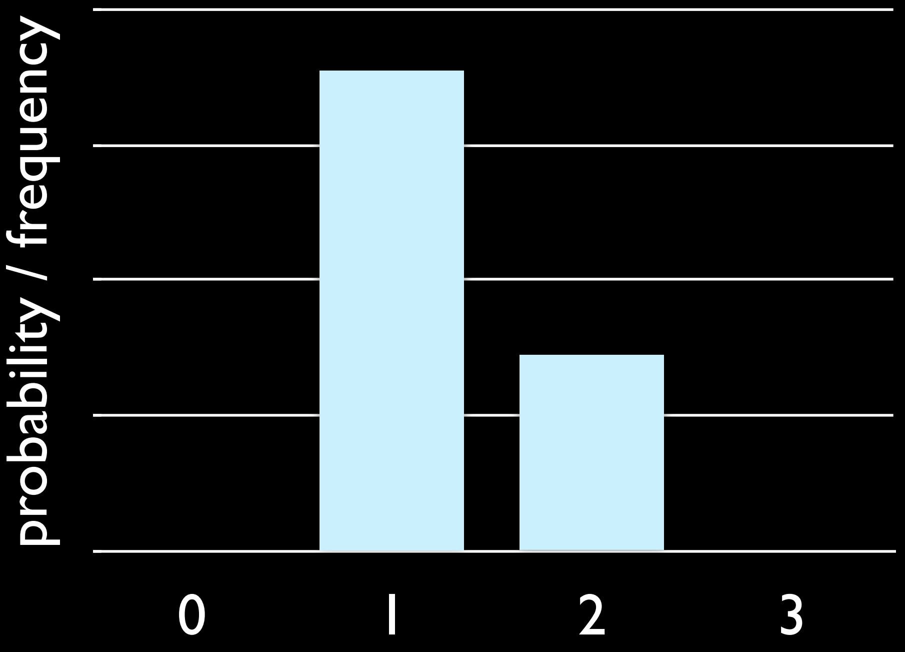
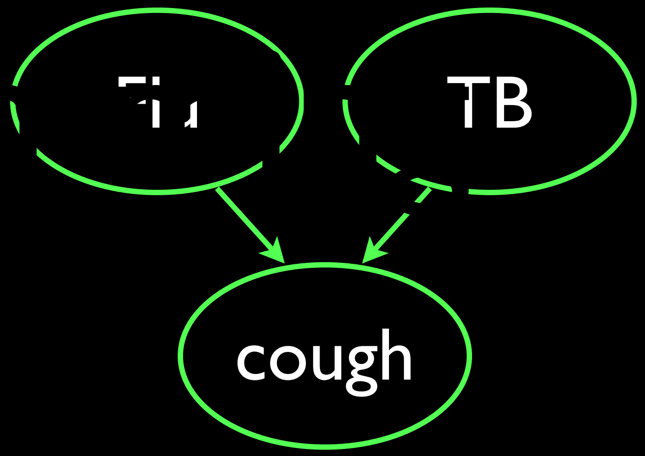


Goodman, Mansinghka, Roy, Bonawitz, Tenenabum (2008)

Inference

- Universal inference: an algorithm that does inference for any Church query. (And hopefully is efficient for a wide class.)
  - As a modeler, save implementation time: rapid prototyping.
  - For cognitive science, shows that the mind could be a universal inference engine.

Example: Bayes Net



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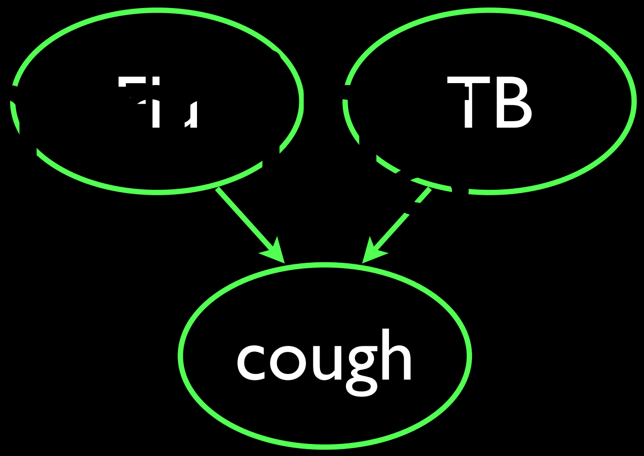

```
(define flu (flip 0.2))
(define TB  (flip 0.01))
(define cough
 (if (or flu TB)
     (flip 0.8) (flip 0.1)))
```

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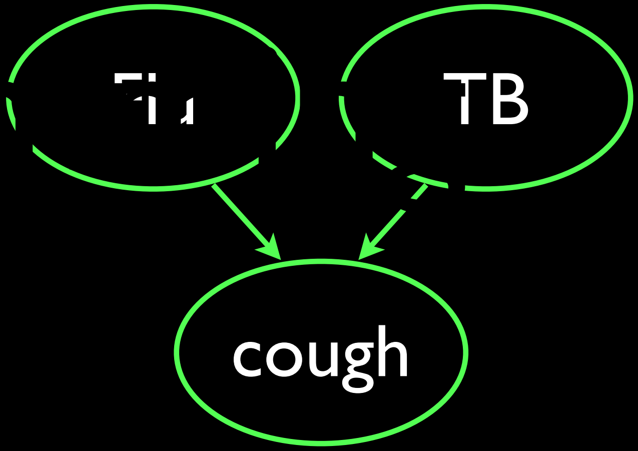

"Infer the chance of flu,
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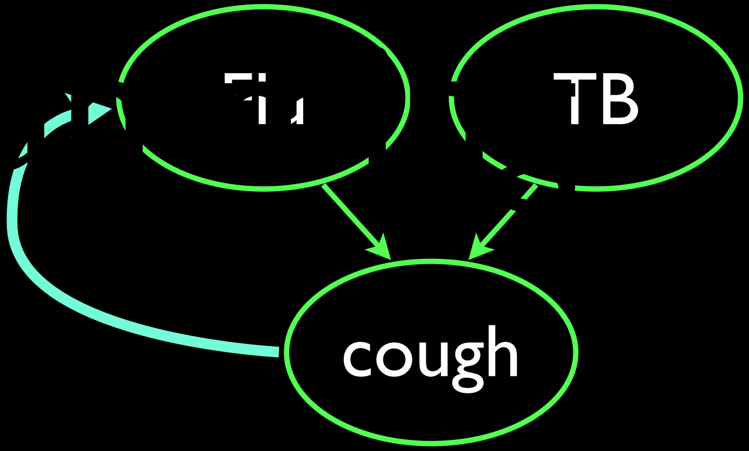

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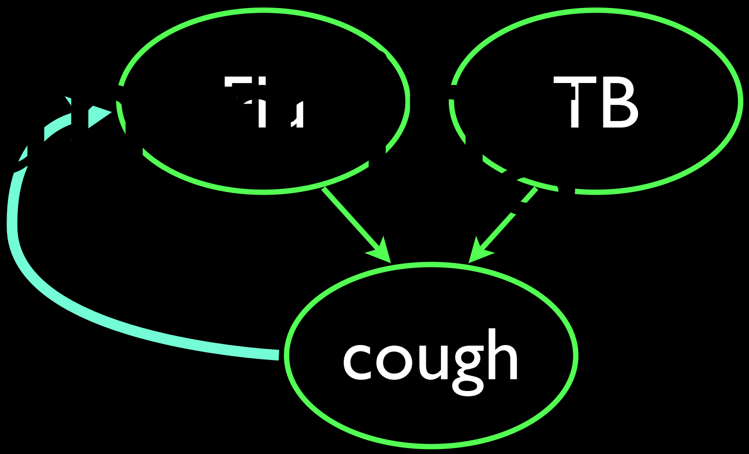

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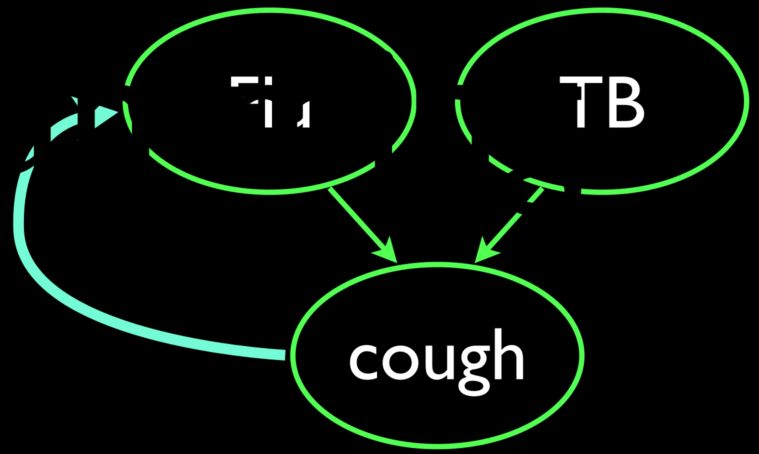
 flu
 cough)
```

```
=> true 66%
```

Example: Bayes Net

"Infer the chance of flu, given observed cough."

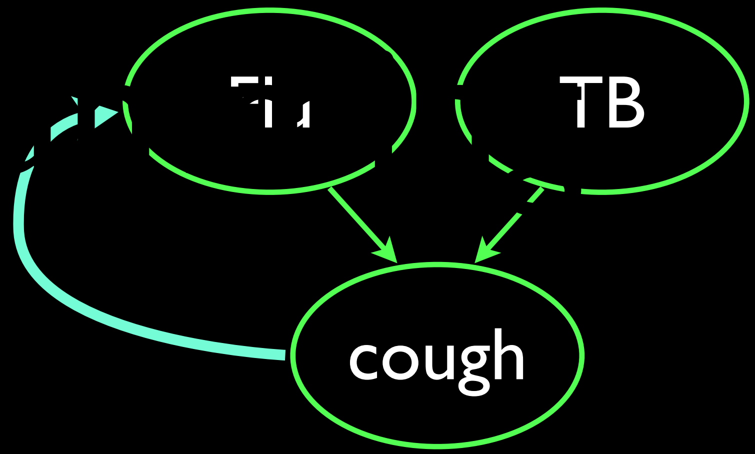


```
(query
 (define flu (flip 0.2))
 (define TB  (flip 0.01))
 (define cough
  (if (or flu TB)
      (flip 0.8) (flip 0.1)))
 flu
 (and cough TB))
```

```
=> true 66%
```

Example: Bayes Net

"Infer the chance of flu, given observed cough."

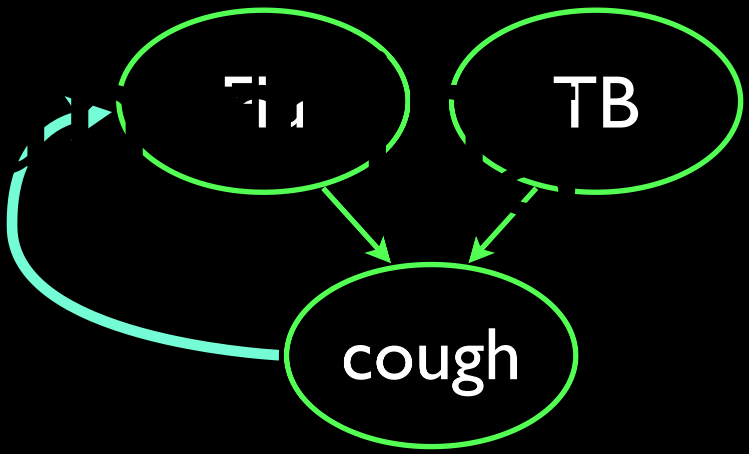


```
(query
 (define flu (flip 0.2))
 (define TB  (flip 0.01))
 (define cough
  (if (or flu TB)
      (flip 0.8) (flip 0.1)))
 flu
 (and cough TB))
```

```
=> true 20%
```

Example: Bayes Net

“Infer the chance of flu,
given observed cough.”



```
(query
 (define flu (flip 0.2))
 (define TB  (flip 0.01))
 (define cough
  (if (or flu TB)
      (flip 0.8) (flip 0.1)))
 flu
 (and cough TB))
```

```
=> true 20%
```

Example: Bayes Net

“Infer the chance of flu,
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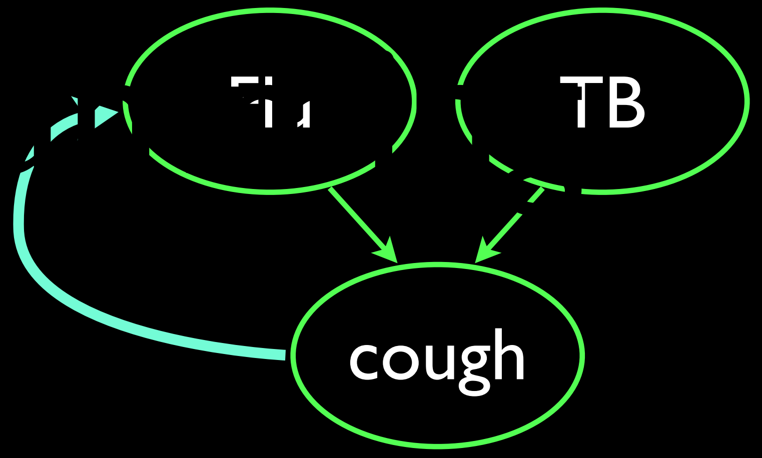
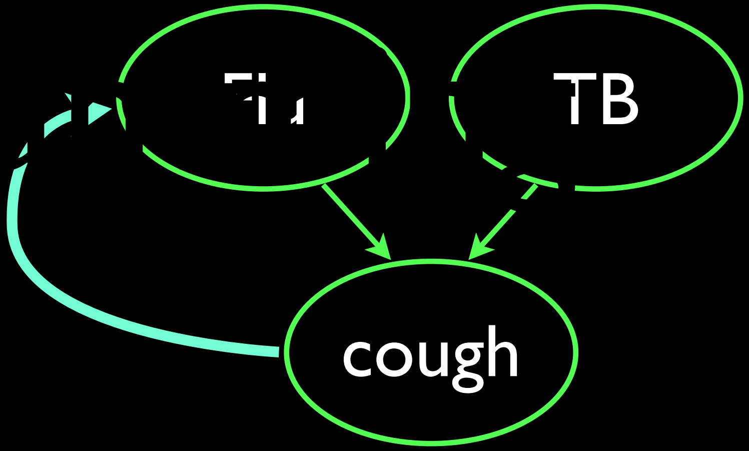


```
(query
 (define flu (flip 0.2))
 (define TB  (flip 0.01))
 (define cough
  (if (or flu TB)
      (flip 0.8) (flip 0.1)))
 flu
 (and cough TB))
```

=> true 20%

Example: Bayes Net

"Infer the chance of flu, given observed cough."



```
(query
 (define flu (flip 0.2))
 (define TB  (flip 0.01))
 (define cough
  (if (or flu TB)
      (flip 0.9) (flip 0.1)))
 flu
 (and cough TB))
```

=> true 20%

Fragment Grammars via Probabilistic Programming *(Church)*

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- Alternative to more standard mathematical formalization (see, O'Donnell, 2011).

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Fragment Grammars via Probabilistic Programming *(Church)*

- Alternative to more standard mathematical formalization (see, O'Donnell, 2011).
- Highlights relationship between formalisms (PCFGs, Adaptor Grammars, Fragment Grammars).
- Cross fertilization of ideas from the theory of programming languages.
- Caveat: Church inference algorithms do not work well for these models.

Goals

Goals

1. Get across intuitions.

Goals

1. Get across intuitions.
2. Give flavor of relationships between modeling ideas and programming ideas.

```
(define unfold
 (lambda (symbol)
  (if (terminal? symbol)
   symbol
   (map unfold (sample-rhs symbol)))))
```

```
(define adapted-unfold
 (PYMem a b
  (lambda (symbol)
    (if (terminal? symbol)
     symbol
     (map unfold (sample-rhs symbol))))))
```

```
(define stochastic-lazy-unfold
  (lambda (symbol)
   (if (terminal? symbol)
     symbol
     (map delay-or-unfold (sample-rhs symbol)))))
```

```
(define delay-or-unfold
  (PYMem a b (lambda (symbol)
   (if (flip)
     (delay (stochastic-lazy-unfold symbol))
     (stochastic-lazy-unfold symbol)))))
```

$$G^{\mathtt{a}}_{\mathtt{pcfg}}(d) = \begin{cases} \displaystyle\sum_{r \in R_{\mathcal{G}} : \mathtt{a} \rightarrow \mathtt{root}(\hat{d}_i), \cdots, \mathtt{root}(\hat{d}_k)} \theta_r \prod_{i=1}^{k} G^{\mathtt{root}(\hat{d}_i)}_{\mathtt{pcfg}}(\hat{d}_i) & \mathtt{root}(d) = \mathtt{a} \in V_{\mathcal{G}} \\ 1 & \mathtt{root}(d) = \mathtt{a} \in T_{\mathcal{G}} \end{cases}$$

$$G^{\mathtt{a}}_{\mathtt{AG}}(d) = \begin{cases} \displaystyle\sum_{r \in R_{\mathcal{G}} : \mathtt{a} \to \mathtt{root}(\hat{d}_i), \cdots, \mathtt{root}(\hat{d}_k)} \theta_r \prod_{i=1}^{k} \mathrm{mem}\{G^{\mathtt{root}(\hat{d}_i)}_{\mathtt{AG}}\}(\hat{d}_i) & \mathtt{root}(d) = \mathtt{a} \in V_{\mathcal{G}} \\ 1 & \mathtt{root}(d) = \mathtt{a} \in T_{\mathcal{G}} \end{cases}$$

$$\mathrm{mem}\{G^{\mathtt{A}}_{\mathtt{AG}}\} \sim \mathrm{PYP}(a^{\mathtt{A}}, b^{\mathtt{A}}, G^{\mathtt{A}}_{\mathtt{AG}})$$

$$L^{\mathtt{A}}(d) = \sum_{r \in R_{\mathcal{G}}:\mathtt{A} \rightarrow \mathtt{root}(\hat{d}_i), \cdots, \mathtt{root}(\hat{d}_k)} \theta_r \prod_{i=1}^{k} \left[\nu_{\hat{d}_i} G_{\mathtt{FG}}^{\mathtt{root}(\hat{d}_i)}(\hat{d}_i) + (1 - \nu_{\hat{d}_i}) 1 \right]$$

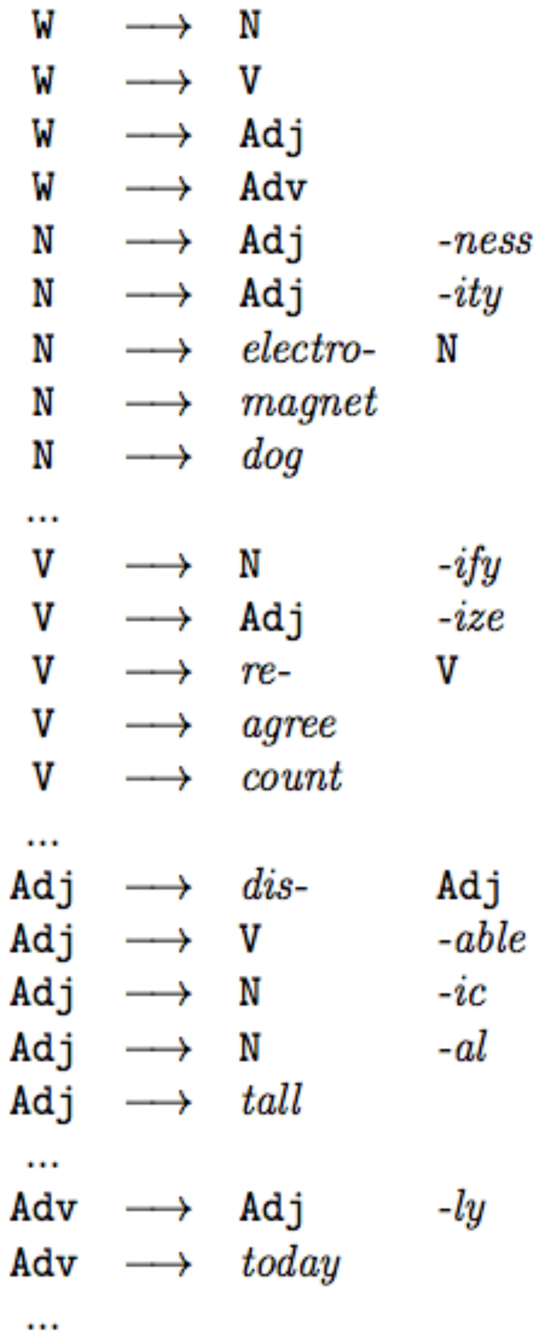
$$G_{\mathtt{FG}}^{\mathtt{a}}(d) = \begin{cases} \displaystyle\sum_{s \in \mathtt{prefix}(d)} \mathrm{mem}\{L^{\mathtt{a}}\}(s) \prod_{i=1}^{n} G_{\mathtt{FG}}^{\mathtt{root}(s_i^{'})}(s_i^{'}) & \mathtt{root}(d) = \mathtt{a} \in V_{\mathcal{G}} \\ 1 & \mathtt{root}(d) = \mathtt{a} \in T_{\mathcal{G}} \end{cases}$$

$$\mathrm{mem}\{L^{\mathtt{A}}\} \sim \mathrm{PYP}(a^{\mathtt{A}}, b^{\mathtt{A}}, L^{\mathtt{A}})$$

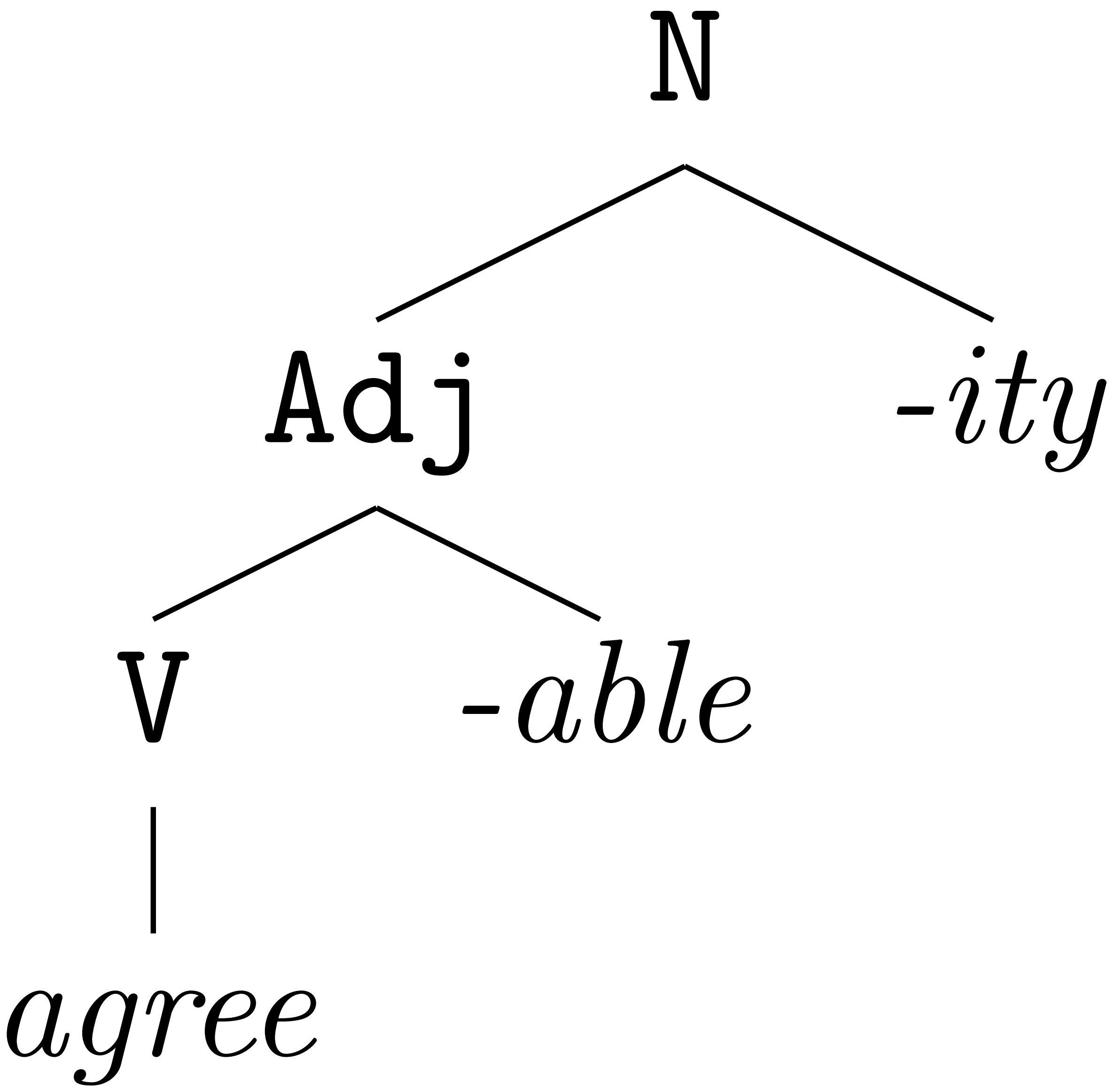
Fragment Grammars via Probabilistic Programming

1. Stochastic computation via `unfold`
2. Stochastic reuse via memoization
3. Partial computations via stochastic laziness

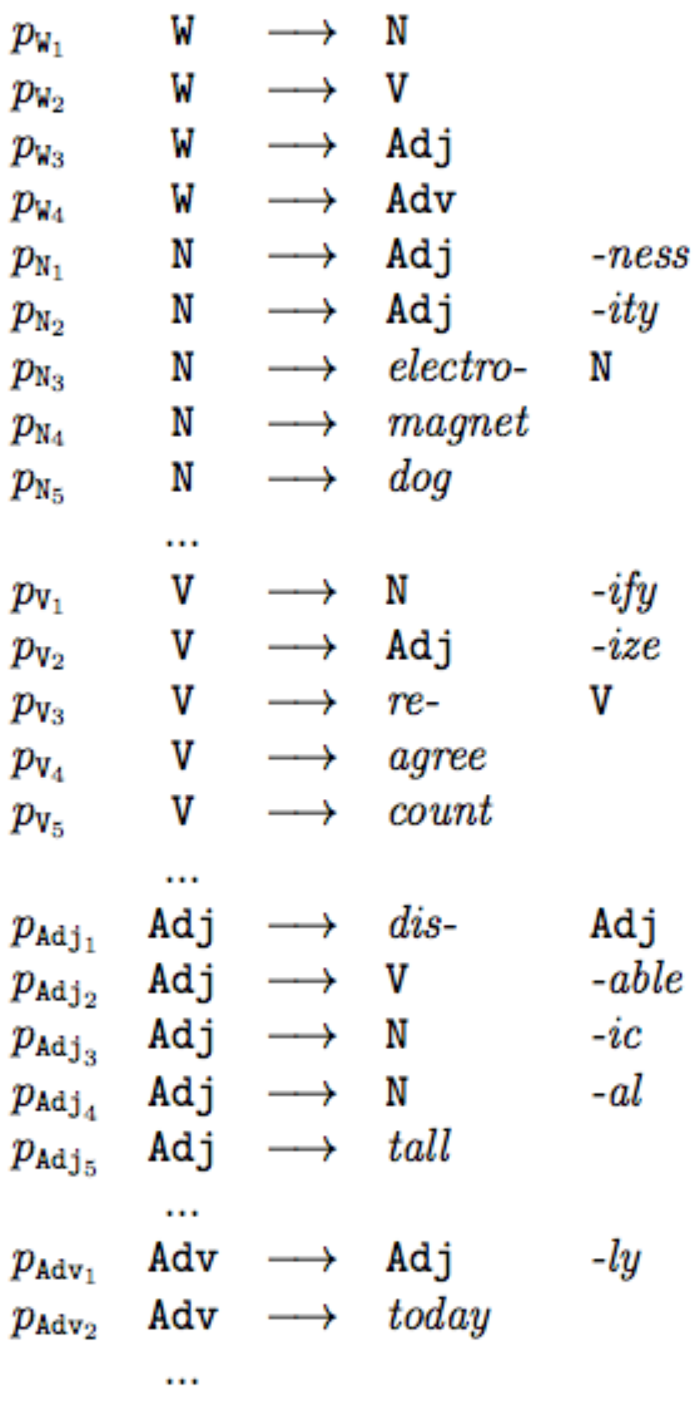
Context Free Grammars



W	⟶	N	
W	⟶	V	
W	⟶	Adj	
W	⟶	Adv	
N	⟶	Adj	*-ness*
N	⟶	Adj	*-ity*
N	⟶	*electro-*	N
N	⟶	*magnet*	
N	⟶	*dog*	
...			
V	⟶	N	*-ify*
V	⟶	Adj	*-ize*
V	⟶	*re-*	V
V	⟶	*agree*	
V	⟶	*count*	
...			
Adj	⟶	*dis-*	Adj
Adj	⟶	V	*-able*
Adj	⟶	N	*-ic*
Adj	⟶	N	*-al*
Adj	⟶	*tall*	
...			
Adv	⟶	Adj	*-ly*
Adv	⟶	*today*	
...			

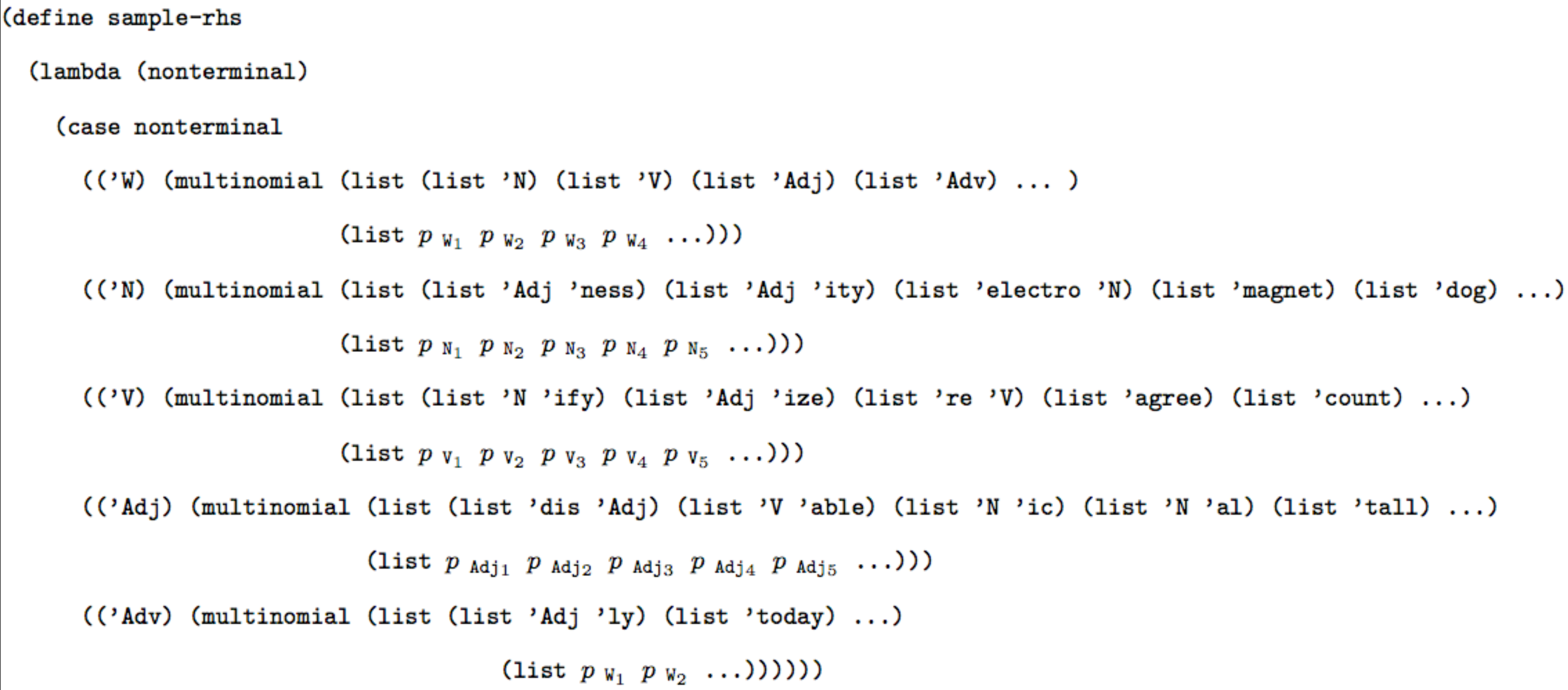


Declarative Knowledge of Constituent Structure



$p_{\mathtt{W}_1}$	W	$\longrightarrow$	N	
$p_{\mathtt{W}_2}$	W	$\longrightarrow$	V	
$p_{\mathtt{W}_3}$	W	$\longrightarrow$	Adj	
$p_{\mathtt{W}_4}$	W	$\longrightarrow$	Adv	
$p_{\mathtt{N}_1}$	N	$\longrightarrow$	Adj	*-ness*
$p_{\mathtt{N}_2}$	N	$\longrightarrow$	Adj	*-ity*
$p_{\mathtt{N}_3}$	N	$\longrightarrow$	*electro-*	N
$p_{\mathtt{N}_4}$	N	$\longrightarrow$	*magnet*	
$p_{\mathtt{N}_5}$	N	$\longrightarrow$	*dog*	
	…			
$p_{\mathtt{V}_1}$	V	$\longrightarrow$	N	*-ify*
$p_{\mathtt{V}_2}$	V	$\longrightarrow$	Adj	*-ize*
$p_{\mathtt{V}_3}$	V	$\longrightarrow$	*re-*	V
$p_{\mathtt{V}_4}$	V	$\longrightarrow$	*agree*	
$p_{\mathtt{V}_5}$	V	$\longrightarrow$	*count*	
	…			
$p_{\mathtt{Adj}_1}$	Adj	$\longrightarrow$	*dis-*	Adj
$p_{\mathtt{Adj}_2}$	Adj	$\longrightarrow$	V	*-able*
$p_{\mathtt{Adj}_3}$	Adj	$\longrightarrow$	N	*-ic*
$p_{\mathtt{Adj}_4}$	Adj	$\longrightarrow$	N	*-al*
$p_{\mathtt{Adj}_5}$	Adj	$\longrightarrow$	*tall*	
	…			
$p_{\mathtt{Adv}_1}$	Adv	$\longrightarrow$	Adj	*-ly*
$p_{\mathtt{Adv}_2}$	Adv	$\longrightarrow$	*today*	
	…			

Declarative Knowledge of Constituent Structure



```
(define sample-rhs
  (lambda (nonterminal)
    (case nonterminal
      (('W) (multinomial (list (list 'N) (list 'V) (list 'Adj) (list 'Adv) ... )
                         (list $p_{W_1}$ $p_{W_2}$ $p_{W_3}$ $p_{W_4}$ ...)))
      (('N) (multinomial (list (list 'Adj 'ness) (list 'Adj 'ity) (list 'electro 'N) (list 'magnet) (list 'dog) ...)
                         (list $p_{N_1}$ $p_{N_2}$ $p_{N_3}$ $p_{N_4}$ $p_{N_5}$ ...)))
      (('V) (multinomial (list (list 'N 'ify) (list 'Adj 'ize) (list 're 'V) (list 'agree) (list 'count) ...)
                         (list $p_{V_1}$ $p_{V_2}$ $p_{V_3}$ $p_{V_4}$ $p_{V_5}$ ...)))
      (('Adj) (multinomial (list (list 'dis 'Adj) (list 'V 'able) (list 'N 'ic) (list 'N 'al) (list 'tall) ...)
                           (list $p_{Adj_1}$ $p_{Adj_2}$ $p_{Adj_3}$ $p_{Adj_4}$ $p_{Adj_5}$ ...)))
      (('Adv) (multinomial (list (list 'Adj 'ly) (list 'today) ...)
                                     (list $p_{W_1}$ $p_{W_2}$ ...))))))
```

Fundamental Recursive Computation: `unfold`

```
(define unfold
 (lambda (symbol)
  (if (terminal? symbol)
   symbol
   (map unfold (sample-rhs symbol)))))
```

Fundamental Recursive Computation: `unfold`

```
(define unfold
 (lambda (symbol)
  (if (terminal? symbol)
   symbol
   (map unfold (sample-rhs symbol)))))
```

Choose a right-hand side for symbol:

N → Adj *-ity*

Fundamental Recursive Computation: `unfold`

```
(define unfold
  (lambda (symbol)
    (if (terminal? symbol)
      symbol
      (map unfold (sample-rhs symbol)))))
```

Fundamental Recursive Computation: `unfold`

```
(define unfold
 (lambda (symbol)
  (if (terminal? symbol)
   symbol
   (map unfold (sample-rhs symbol)))))
```

Recursively apply `unfold` to each symbol on right-hand side

Computation Trace

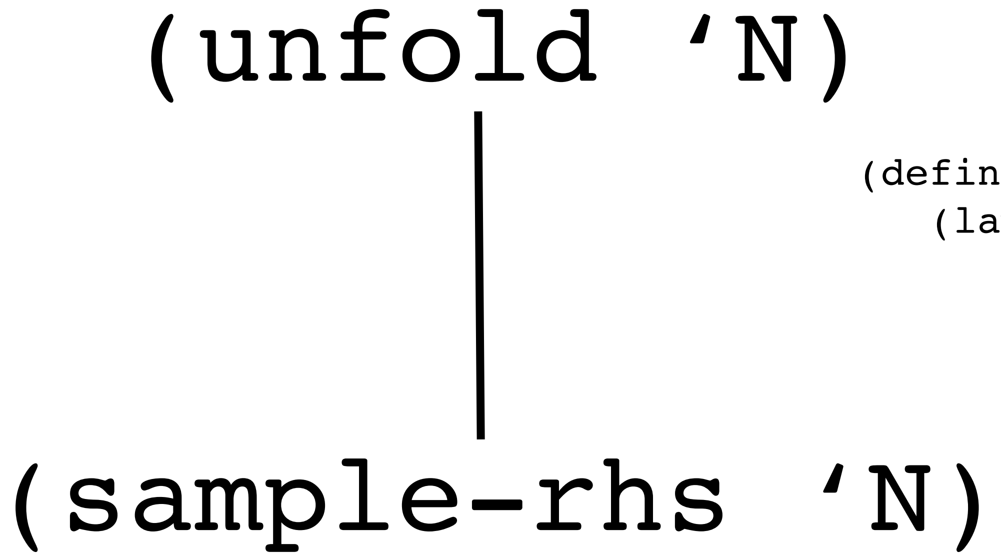
```
(unfold ‘N)
```

Computation Trace

```
(unfold ‘N)
```

```
(define unfold
   (lambda (symbol)
      (if (terminal? symbol)
         symbol
         (map unfold (sample-rhs symbol)))))
```

Computation Trace

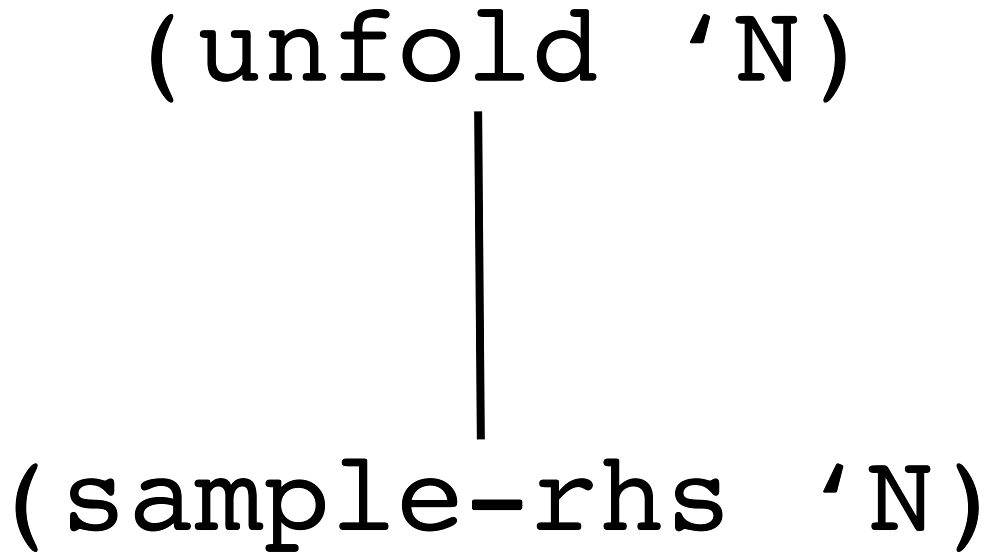


(unfold ‘N)

(sample-rhs ‘N)

```
(define unfold
   (lambda (symbol)
      (if (terminal? symbol)
         symbol
         (map unfold (sample-rhs symbol)))))
```

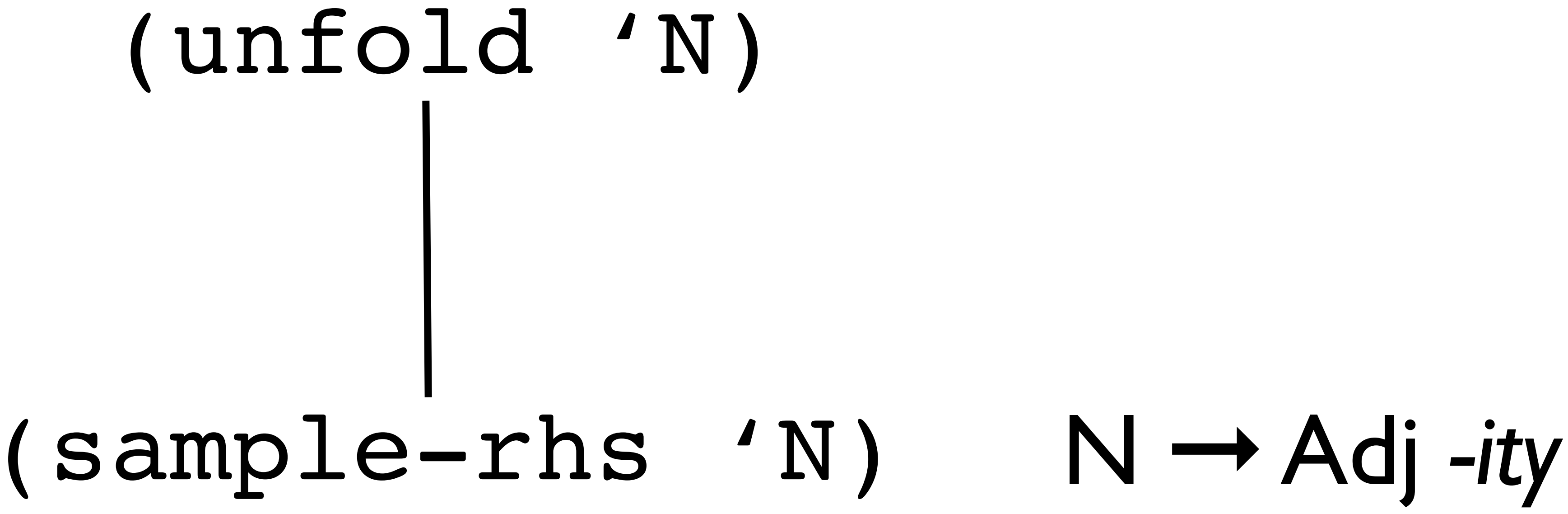
Computation Trace



(unfold ‘N)

(sample-rhs ‘N)

Computation Trace

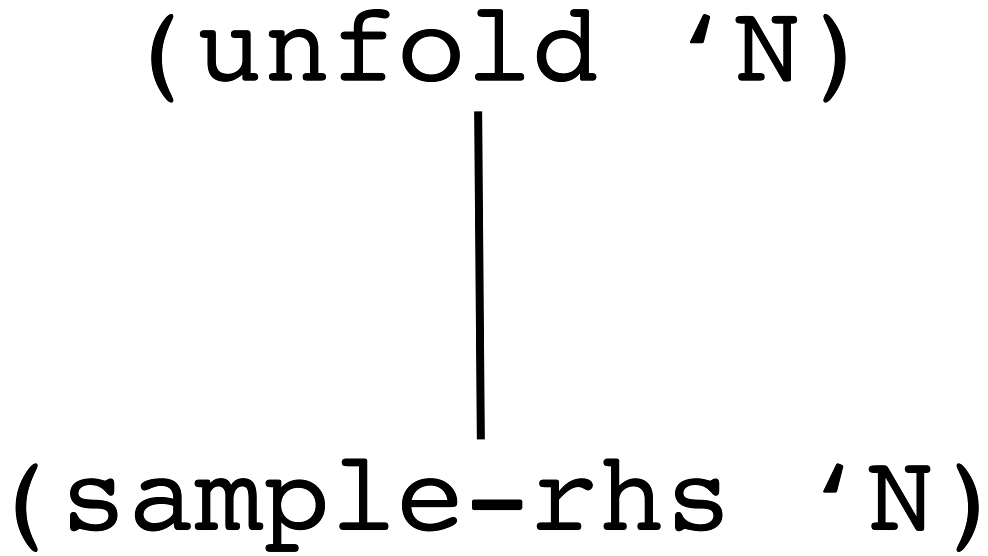


`(unfold ‘N)`

`(sample-rhs ‘N)`

N → Adj *-ity*

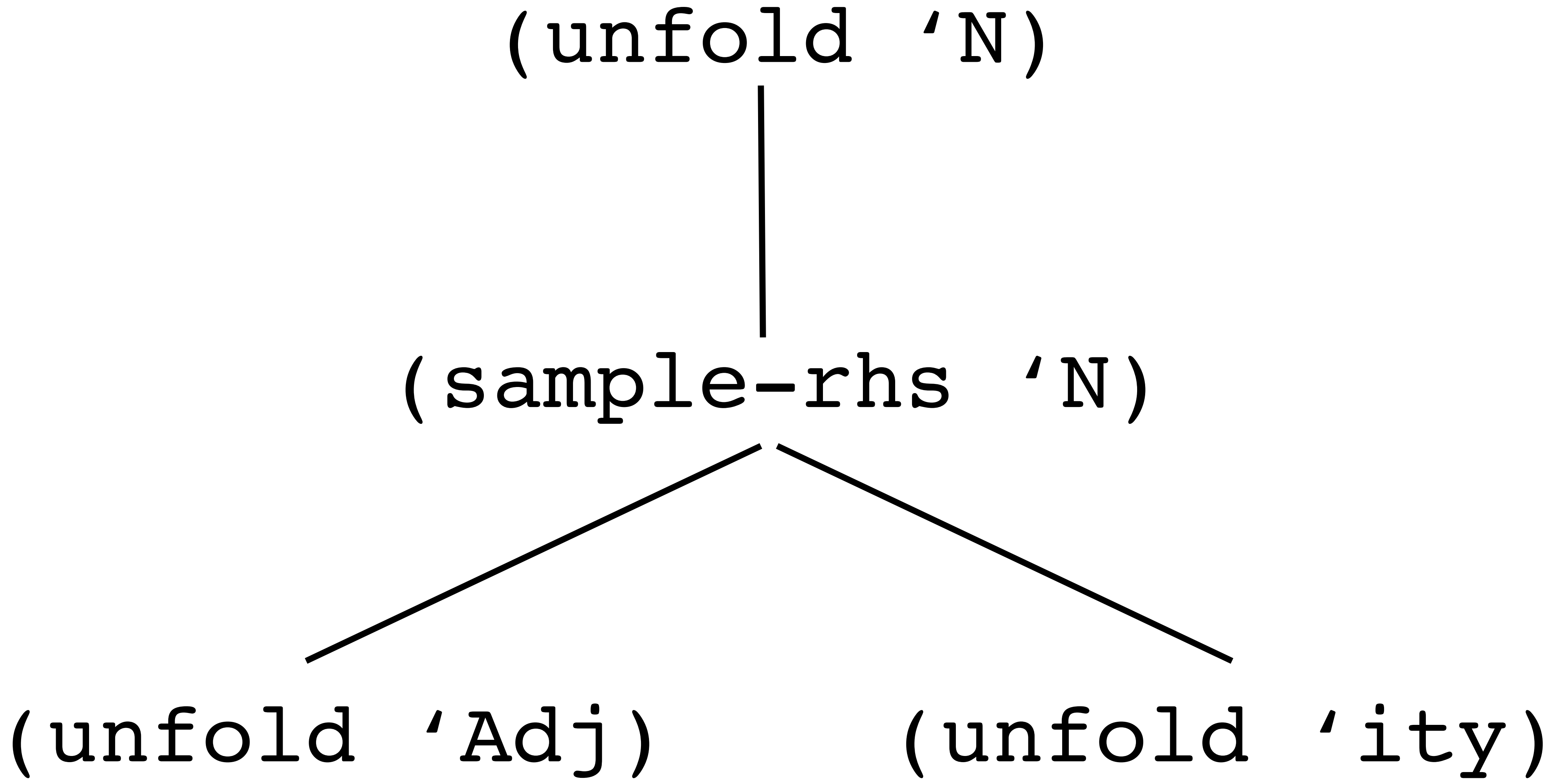
Computation Trace



```
(unfold ‘N)

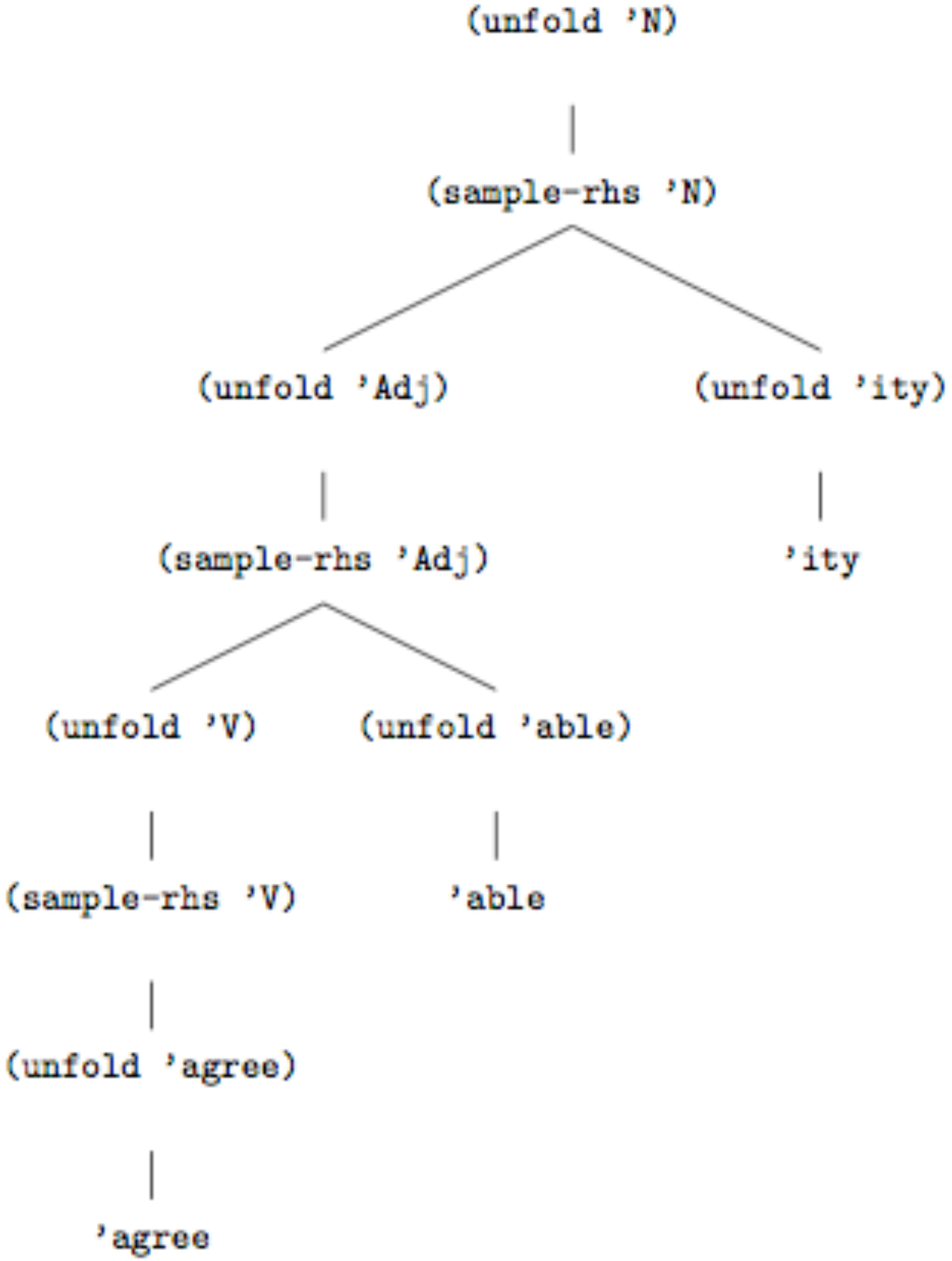
(sample-rhs ‘N)
```

Computation Trace



```
(unfold ‘N)
(sample-rhs ‘N)
(unfold ‘Adj)
(unfold ‘ity)
```

Computation Trace



(unfold 'N)

(sample-rhs 'N)

(unfold 'Adj)

(unfold 'ity)

(sample-rhs 'Adj)

'ity

(unfold 'V)

(unfold 'able)

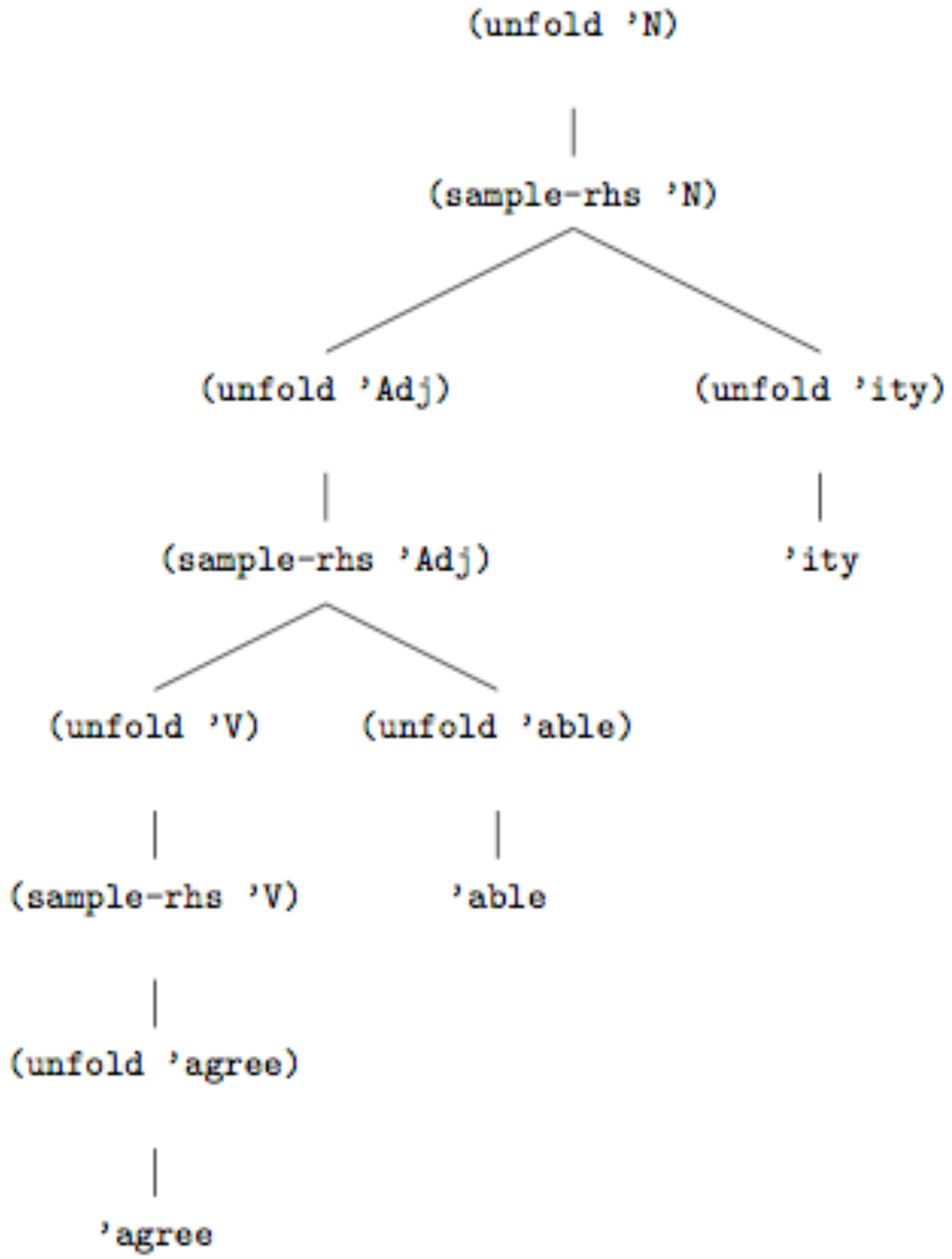
(sample-rhs 'V)

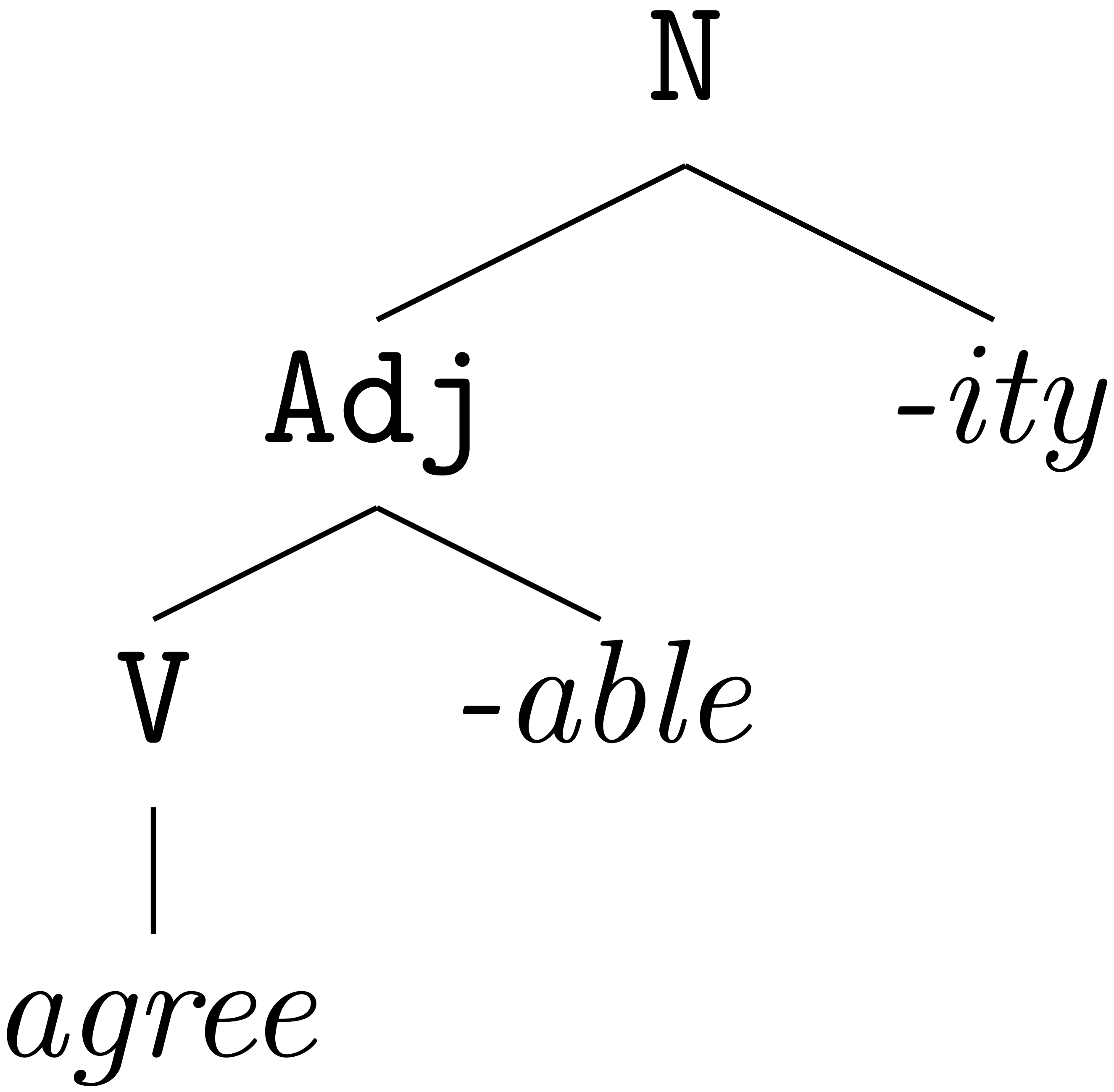
'able

(unfold 'agree)

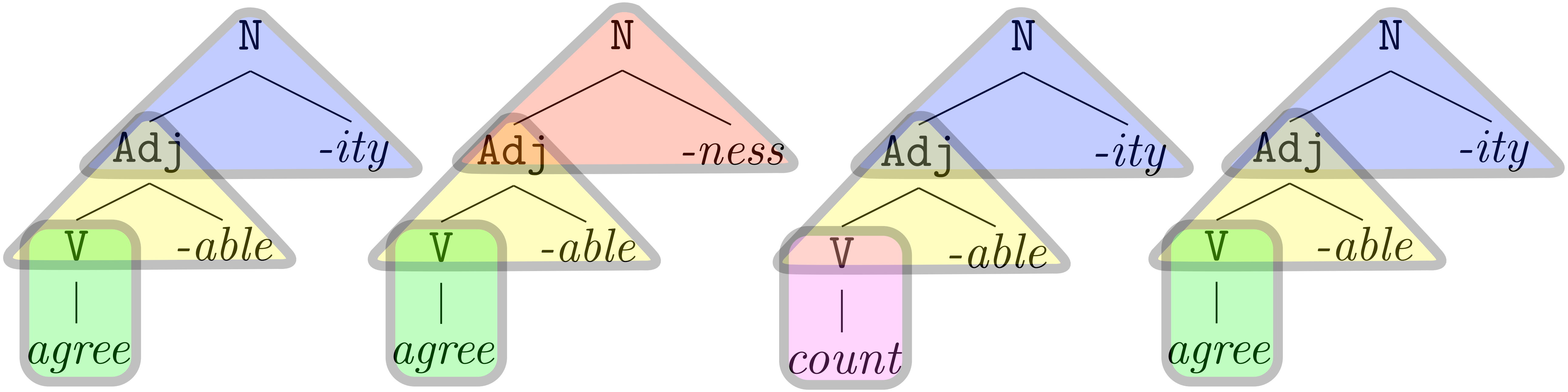
'agree

Trace as Tree





Reusability for PCFGs



Fragment Grammars via Probabilistic Programming

1. Stochastic computation via `unfold`

2. Stochastic reuse via memoization

3. Partial computations via stochastic laziness

Memoization

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- Store outputs of earlier computations in a table

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- When function is called with new arguments, then compute and store in table

Memoization

- Store outputs of earlier computations in a table
- When function is called with particular arguments then grab from table if stored
- When function is called with new arguments, then compute and store in table
- Higher-order function: `mem`

Reuse through Memoization

```
(define eye-color
  (lambda (person)
    (if (flip 0.5) ‘blue ‘brown)))
```

Reuse through Memoization

```
(define eye-color
  (lambda (person)
    (if (flip 0.5) ‘blue ‘brown)))
```

```
    (eye-color ‘bob)  => ‘blue
```

Reuse through Memoization

```
(define eye-color
  (lambda (person)
    (if (flip 0.5) ‘blue ‘brown)))

   (eye-color ‘bob)  => ‘blue
   (eye-color ‘bob)  => ‘brown
```

Reuse through Memoization

```
(define eye-color
  (lambda (person)
    (if (flip 0.5) ‘blue ‘brown)))

    (eye-color ‘bob)  => ‘blue
    (eye-color ‘bob)  => ‘brown
    (eye-color ‘bob)  => ‘blue
```

Reuse through Memoization

```
(define eye-color
  (lambda (person)
    (if (flip 0.5) ‘blue ‘brown)))

    (eye-color ‘bob)  => ‘blue
    (eye-color ‘bob)  => ‘brown
    (eye-color ‘bob)  => ‘blue
    (eye-color ‘bob)  => ‘brown
```

Reuse through Memoization

```
(define eye-color
  (lambda (person)
    (if (flip 0.5) ‘blue ‘brown)))

    (eye-color ‘bob)  => ‘blue
    (eye-color ‘bob)  => ‘brown
    (eye-color ‘bob)  => ‘blue
    (eye-color ‘bob)  => ‘brown
                    ...
```

Reuse through Memoization

```
(define eye-color
  (mem (lambda (person)
   (if (flip 0.5) ‘blue brown))))
```

Reuse through Memoization

```
(define eye-color
  (mem (lambda (pers
    (if (flip 0.5) ‘blue brown))))
```

Anywhere in the program where (**eye-color** ‘bob) is used, we will *reuse* same value.

Reuse through Memoization

```
(define eye-color
  (mem (lambda (pers
   (if (flip 0.5) ‘blue brown))))
```

Anywhere in the program where (**eye-color** ‘bob) is used, we will *reuse* same value.

```
(eye-color ‘bob)  => ‘blue
```

Reuse through Memoization

```
(define eye-color
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```

Anywhere in the program where (**eye-color** ‘bob) is used, we will *reuse* same value.

```
(eye-color ‘bob)  => ‘blue
(eye-color ‘bob)  => ‘blue
```

Reuse through Memoization

```
(define eye-color
  (mem (lambda (pers
   (if (flip 0.5) ‘blue brown))))
```

Anywhere in the program where (**eye-color** ‘bob) is used, we will *reuse* same value.

```
   (eye-color ‘bob)  => ‘blue
   (eye-color ‘bob)  => ‘blue
   (eye-color ‘bob)  => ‘blue
```

Reuse through Memoization

```
(define eye-color
  (mem (lambda (pers
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```
(eye-color 'bob)  => 'blue
(eye-color 'bob)  => 'blue
(eye-color 'bob)  => 'blue
(eye-color 'bob)  => 'blue
```

Reuse through Memoization

```
(define eye-color
  (mem (lambda (pers
   (if (flip 0.5) ‘blue brown))))
```

Anywhere in the program where (**eye-color** ‘bob) is used, we will *reuse* same value.

```
   (eye-color ‘bob)  => ‘blue
   (eye-color ‘bob)  => ‘blue
   (eye-color ‘bob)  => ‘blue
   (eye-color ‘bob)  => ‘blue
                 ...
```

Stochastic Reusability

- Deterministic memoization always returns same value after first call, but sometimes we want to **probabilistically** favor reuse.

Stochastic Reusability

```
(define location
  (lambda (person)
   (sample-location-in-world)))
```

Stochastic Reusability

```
(define location
  (lambda (person)
   (sample-location-in-world)))


(location ‘bob)  => ‘UCLA
```

Stochastic Reusability

```
(define location
  (lambda (person)
    (sample-location-in-world)))


(location ‘bob)  => ‘UCLA
(location ‘bob)  => ‘Antarctica
```

Stochastic Reusability

```
(define location
  (lambda (person)
   (sample-location-in-world)))
```

```
(location ‘bob)  => ‘UCLA
(location ‘bob)  => ‘Antarctica
(location ‘bob)  => ‘London
```

Stochastic Reusability

```
(define location
  (lambda (person)
   (sample-location-in-world)))
```

```
(location ‘bob)  => ‘UCLA
(location ‘bob)  => ‘Antarctica
(location ‘bob)  => ‘London
(location ‘bob)  => ‘Thailand
```

Stochastic Reusability

```
(define location
  (lambda (person)
   (sample-location-in-world)))



(location ‘bob)  => ‘UCLA
(location ‘bob)  => ‘Antarctica
(location ‘bob)  => ‘London
(location ‘bob)  => ‘Thailand
                ...
```

Stochastic Reusability

```
(define location
  (stochastic-mem (lambda (person)
   (sample-location-in-world))))
```

Stochastic Reusability

```
(define location
  (stochastic-mem (lambda (person)
   (sample-location-in-world))))


   (location ‘bob)  => ‘home
```

Stochastic Reusability

```
(define location
  (stochastic-mem (lambda (person)
   (sample-location-in-world))))
```

```
(location ‘bob)  => ‘home
(location ‘bob)  => ‘office
```

Stochastic Reusability

```
(define location
  (stochastic-mem (lambda (person)
   (sample-location-in-world))))


   (location ‘bob)   => ‘home
   (location ‘bob)   => ‘office
   (location ‘bob)   => ‘home
```

Stochastic Reusability

```
(define location
  (stochastic-mem (lambda (person)
   (sample-location-in-world))))
```

```
   (location ‘bob)  => ‘home
   (location ‘bob)  => ‘office
   (location ‘bob)  => ‘home
   (location ‘bob)  => ‘home
```

Stochastic Reusability

```
(define location
  (stochastic-mem (lambda (person)
   (sample-location-in-world))))
```

```
(location ‘bob)  => ‘home
(location ‘bob)  => ‘office
(location ‘bob)  => ‘home
(location ‘bob)  => ‘home
                 ...
```

Stochastic Memoization

(Goodman et al., 2008; Johnson et al., 2007)

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- Adaptor Grammars: Anything that can be computed can be stored and reused *probabilistically*.

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(Goodman et al., 2008; Johnson et al., 2007)

- Adaptor Grammars: Anything that can be computed can be stored and reused *probabilistically*.
- Memoization distribution: *Pitman-Yor Processes* (Pitman & Yor, 1995).

Stochastic Memoization

(Goodman et al., 2008; Johnson et al., 2007)

- Adaptor Grammars: Anything that can be computed can be stored and reused *probabilistically*.
- Memoization distribution: *Pitman-Yor Processes* (Pitman & Yor, 1995).
- Stochastic memoization + PCFGs = Adaptor Grammars.

Pitman-Yor Process

Pitman-Yor Process

- Generalization of the Chinese Restaurant Process

Pitman-Yor Process

- Generalization of the Chinese Restaurant Process
- Two parameters:

Pitman-Yor Process

- Generalization of the Chinese Restaurant Process
- Two parameters:
  - $a \in [0,1]$

Pitman-Yor Process

- Generalization of the Chinese Restaurant Process
- Two parameters:
  - $a \in [0,1]$
  - $b > -a$

Pitman-Yor Process

- Generalization of the Chinese Restaurant Process
- Two parameters:
  - a ∈ [0,1]
  - b > -a

Probability of Reuse

$$\frac{y_i - a}{N + b}$$

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  - a ∈ [0,1]
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y_i: Total number of observations of value i

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$$\frac{a \cdot K + b}{N + b}$$

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- Two parameters:
    - a ∈ [0,1]
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y_i: Total number of observations of value i

N: Total number of observations

K: Total number of values

Probability of Reuse

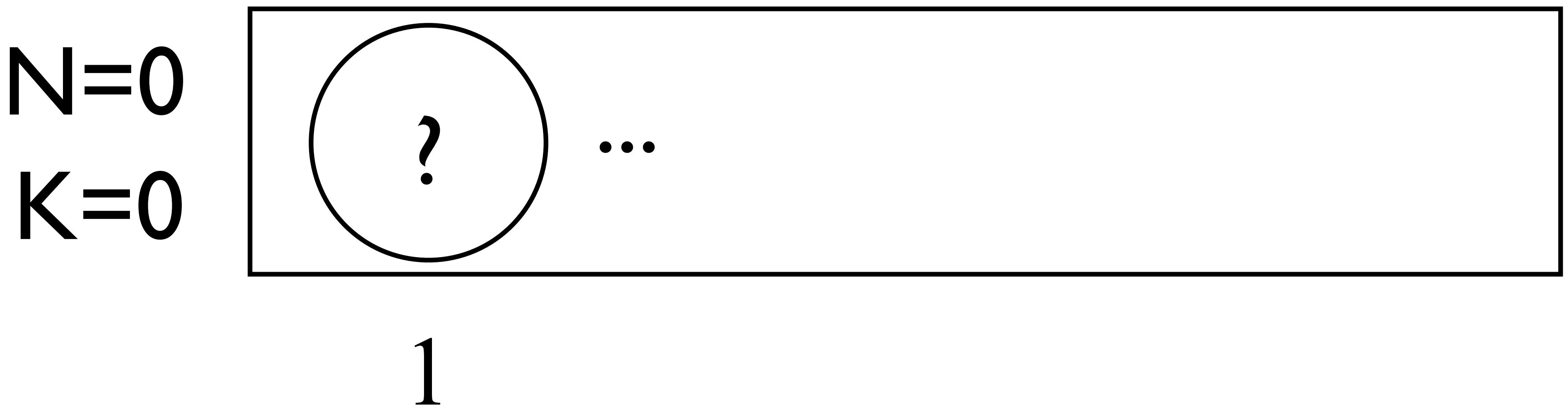
$$\frac{y_i - a}{N + b}$$

Probability of Novelty

$$\frac{a \cdot K + b}{N + b}$$

```
(func arg1 ... argN)
```

(PYMem a b func)



N=0

K=0

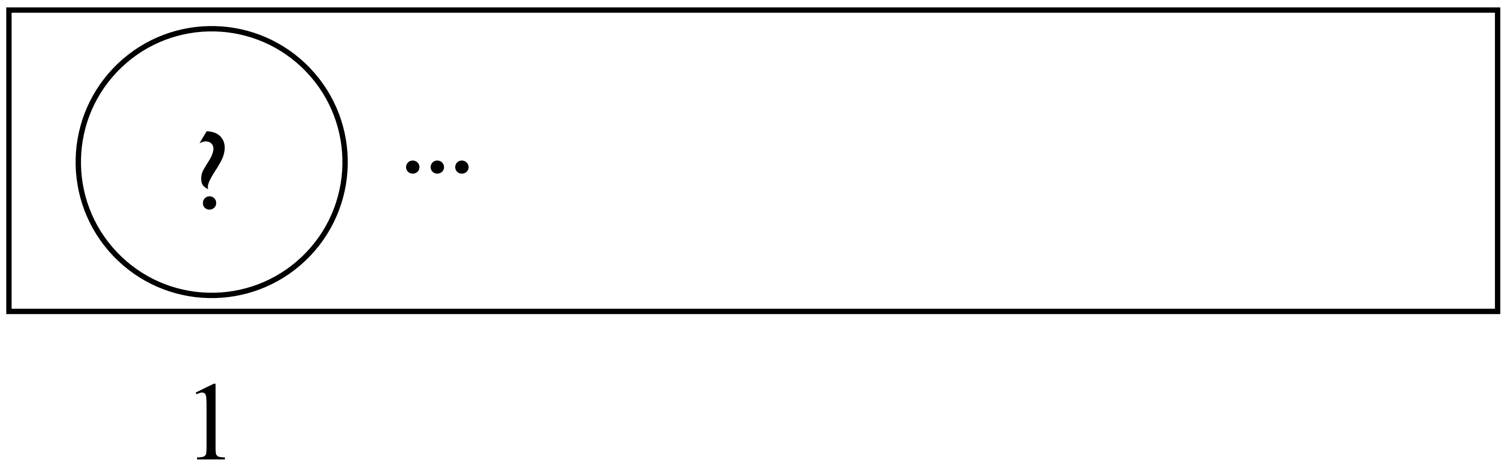
?

...

1

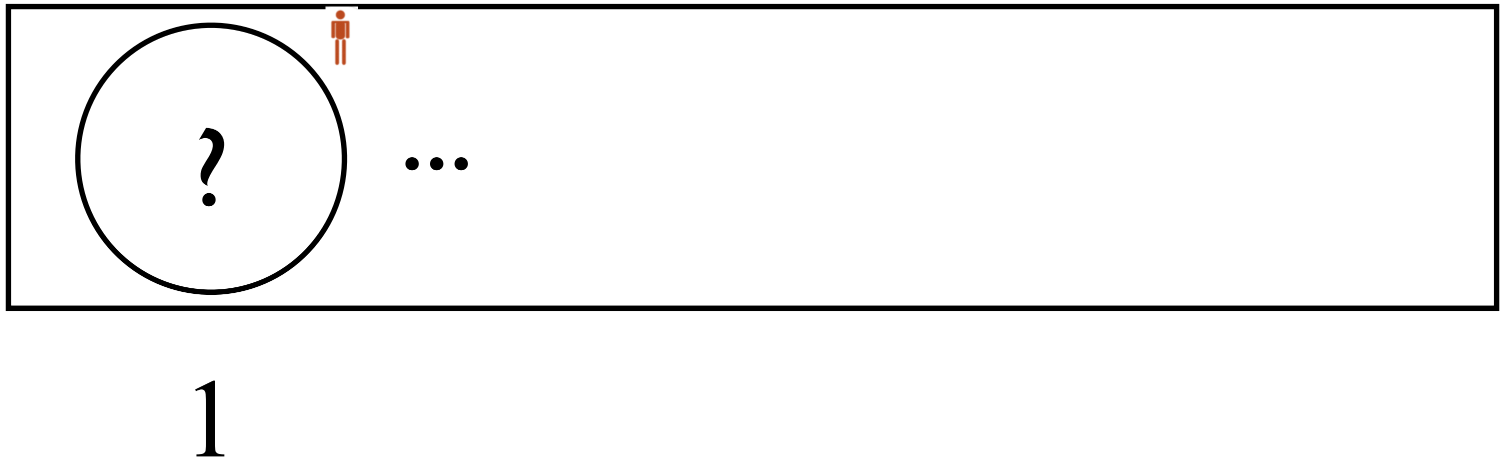
N=0

K=0



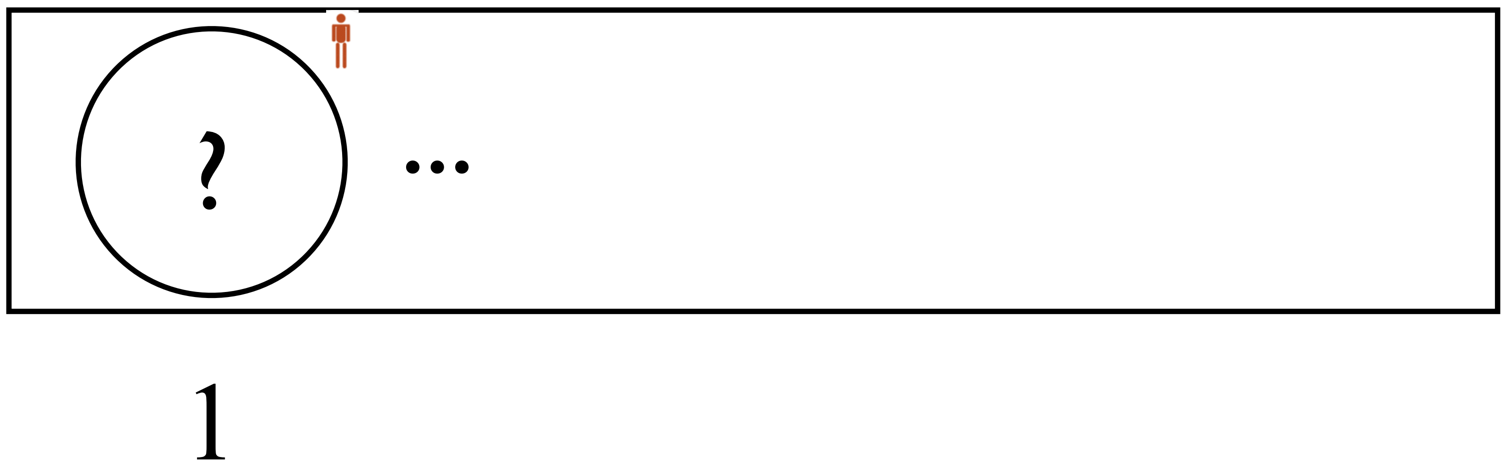
N=0

K=0



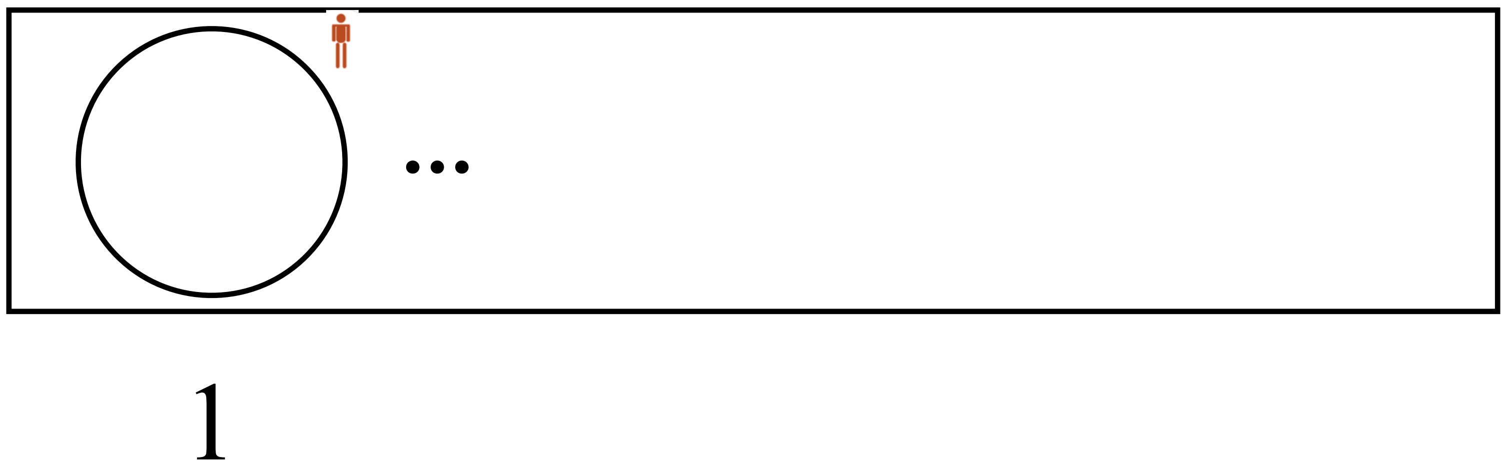
N=1

K=0

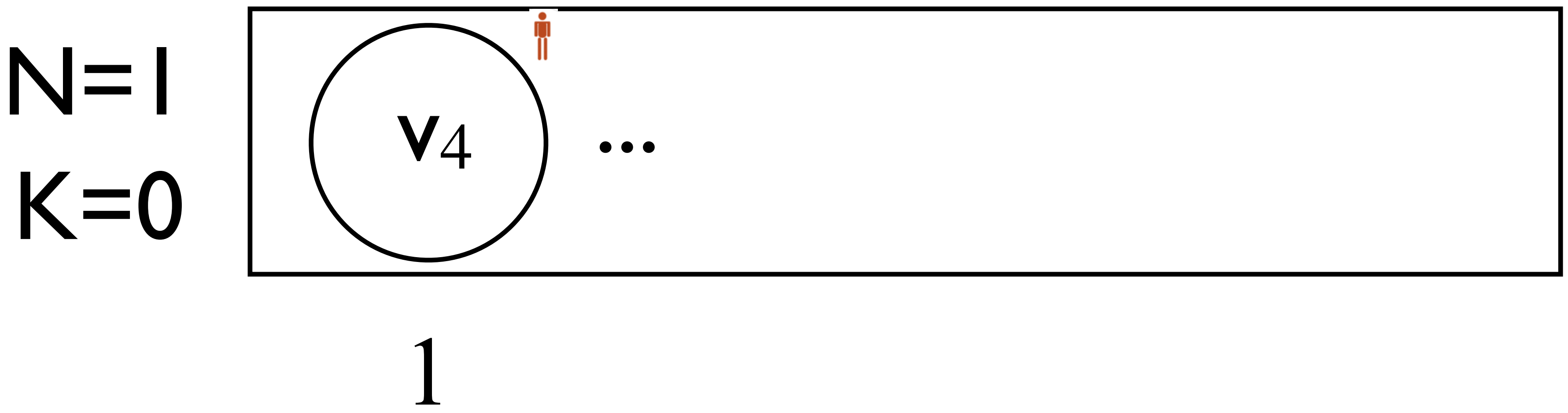


N=1

K=0



v_4 ~ `(func arg1 ...)`



N=1

K=0

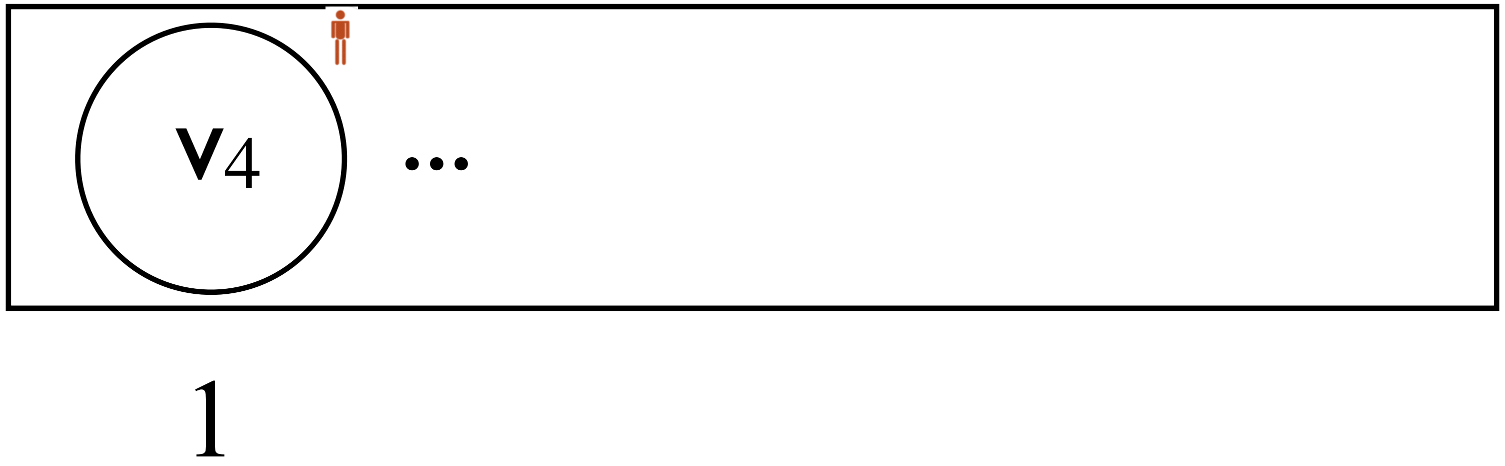
v_4

...

1

N=1

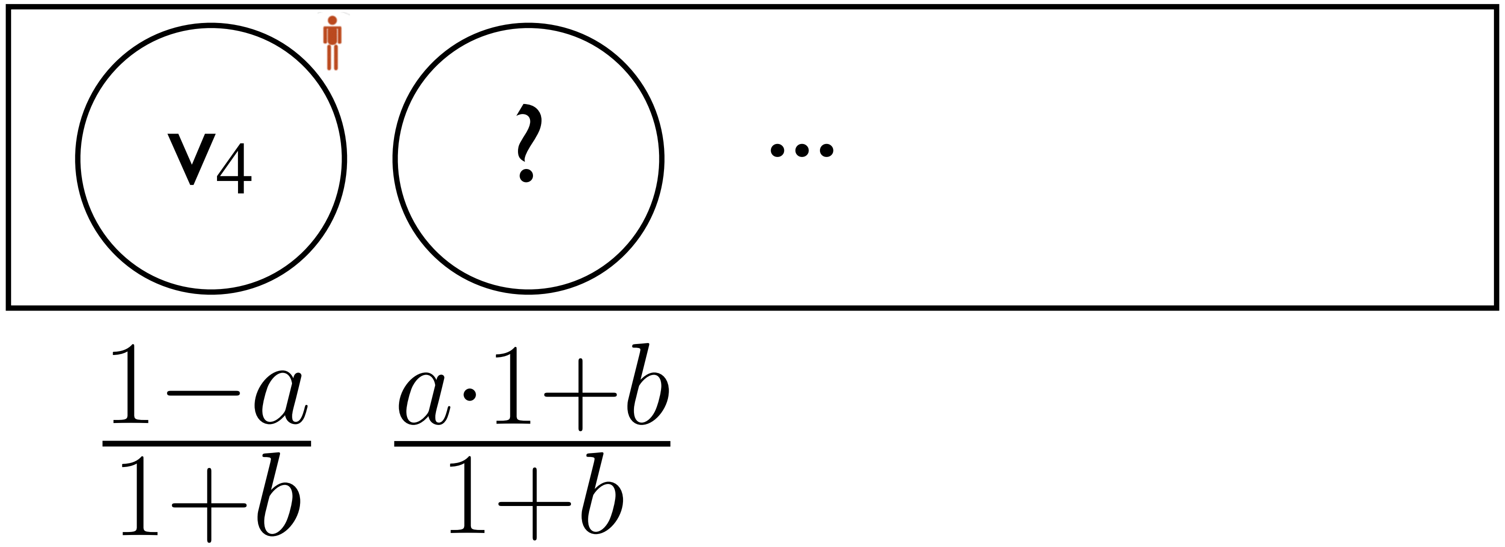
K=1



Samples: v_4

N=1

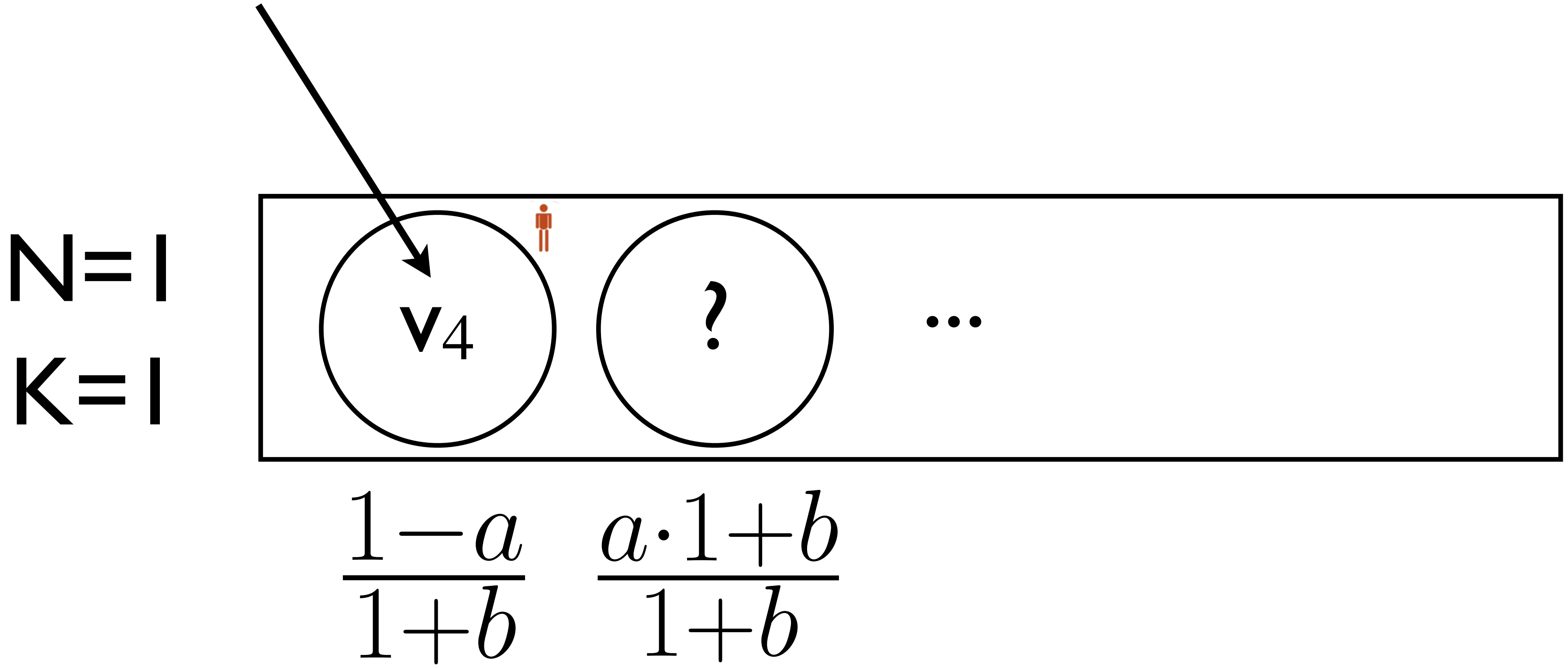
K=1



$$\frac{1-a}{1+b} \quad \frac{a \cdot 1+b}{1+b}$$

Samples: v_4

$$\frac{y_i - a}{N + b}$$



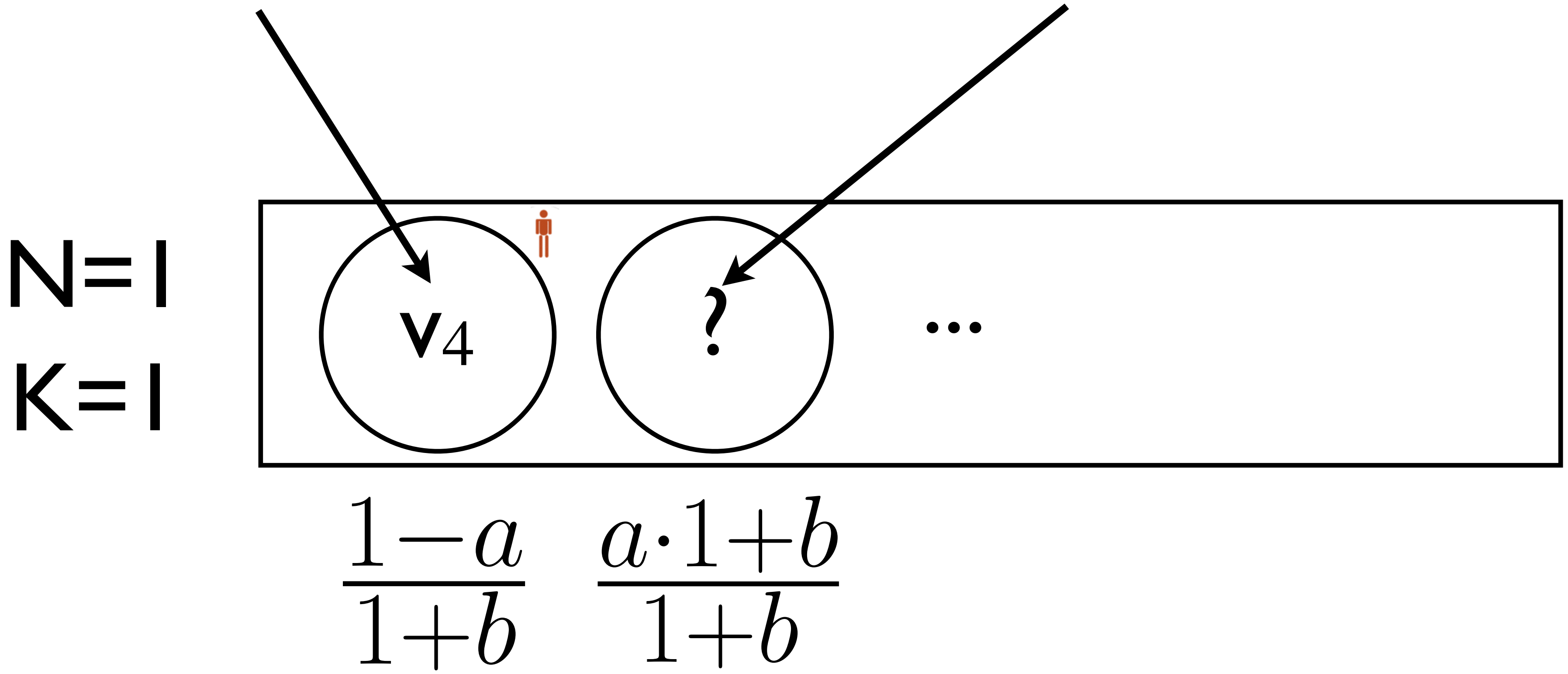
N=1

K=1

$\frac{1-a}{1+b}$ $\frac{a \cdot 1+b}{1+b}$

Samples: v_4

$$\frac{y_i - a}{N + b} \qquad \frac{a \cdot K + b}{N + b}$$

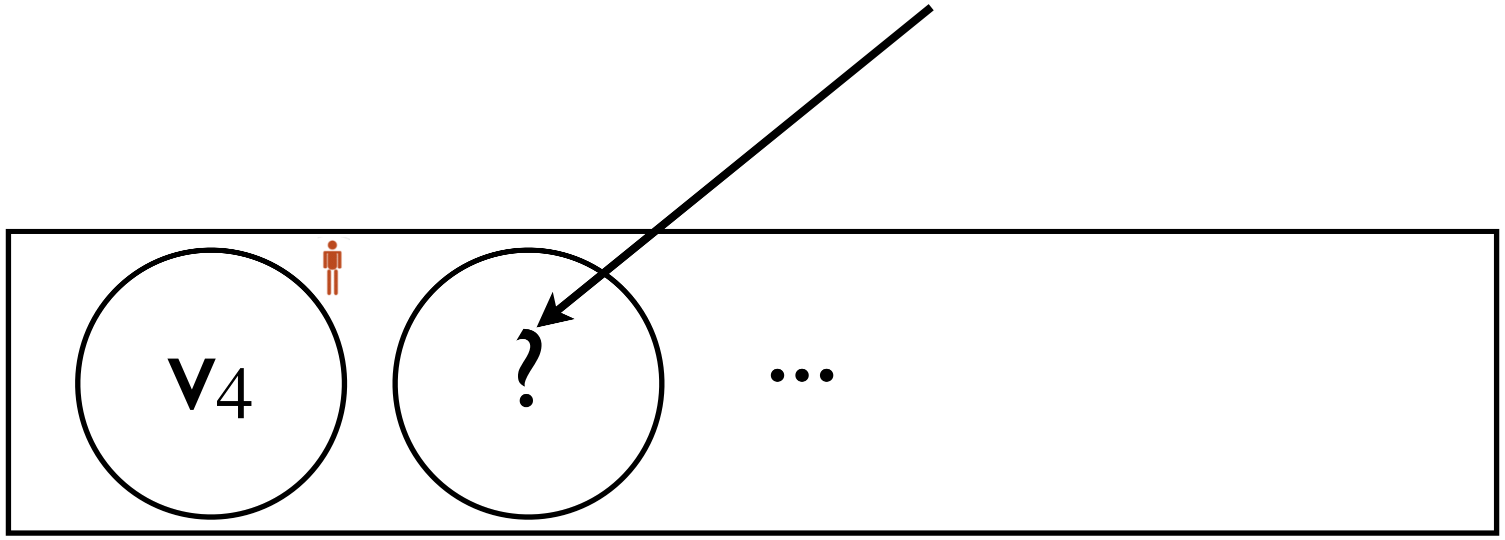


Samples: v_4

$$\frac{a \cdot K + b}{N + b}$$

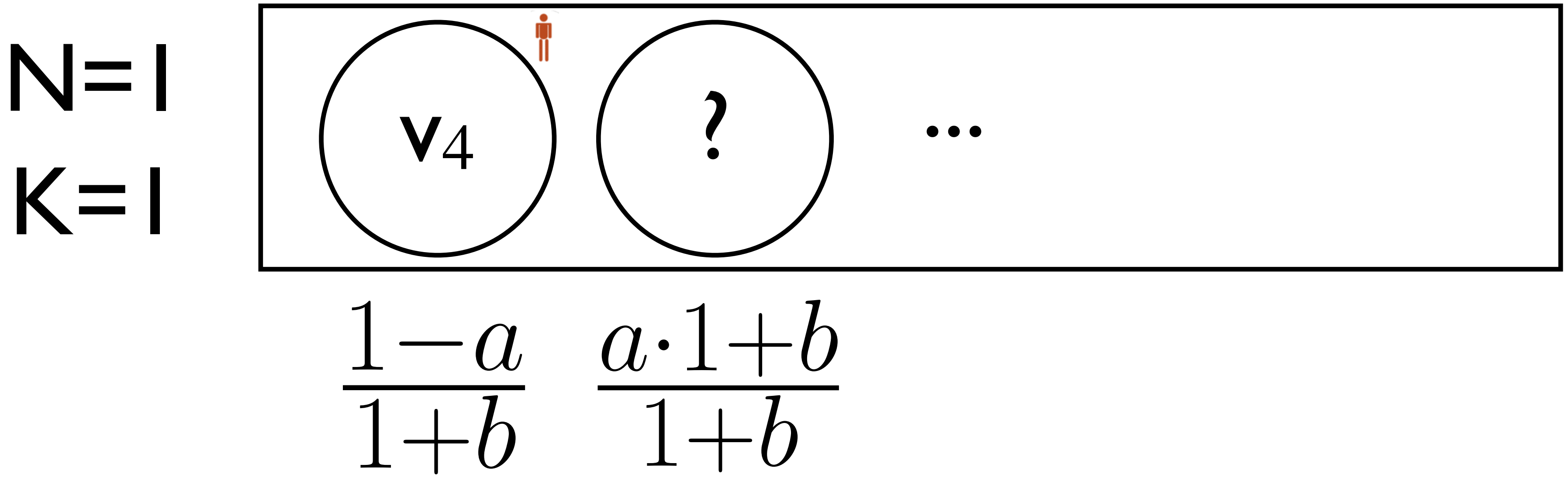
N=1

K=1



$\frac{1-a}{1+b}$ $\frac{a \cdot 1+b}{1+b}$

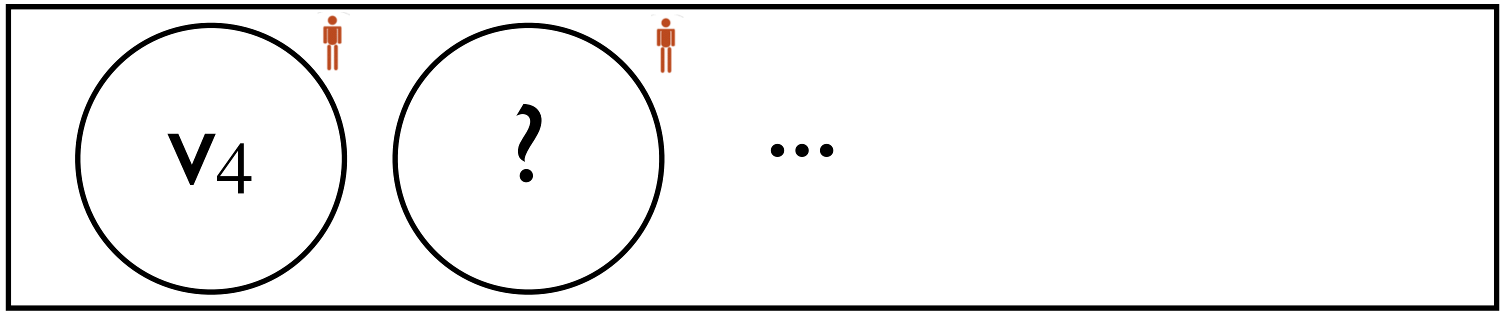
Samples: v_4



Samples: v_4

N=1

K=1

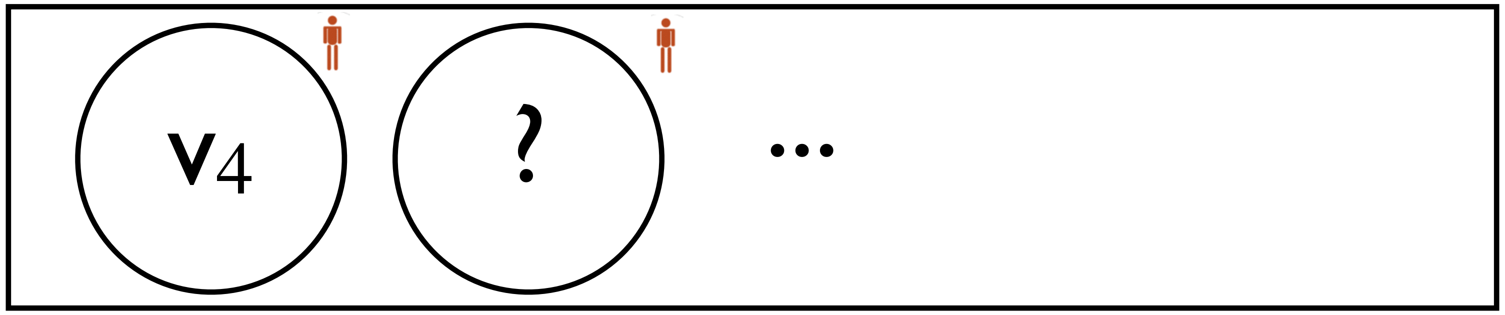


$$\frac{1-a}{1+b} \qquad \frac{a\cdot 1+b}{1+b}$$

Samples: v_4

N=2

K=1



v_4

?

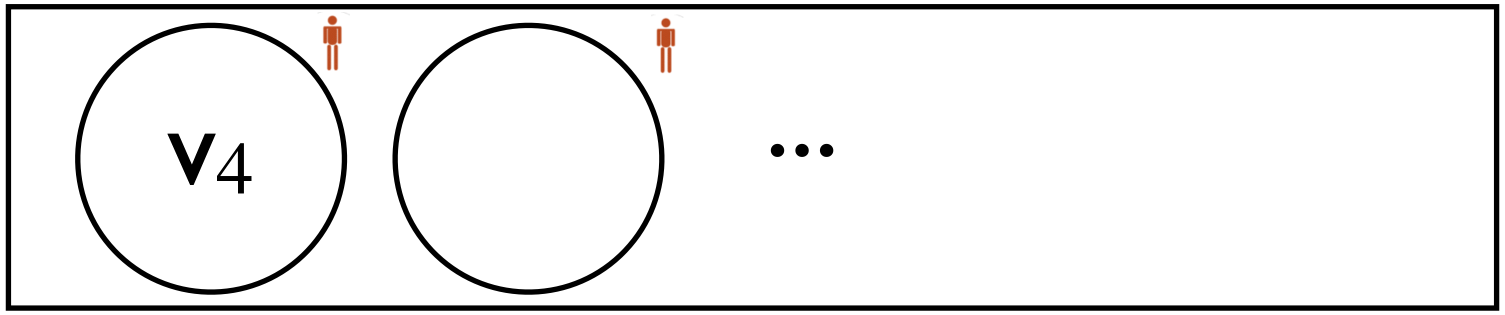
...

$$\frac{1-a}{1+b} \quad \frac{a \cdot 1+b}{1+b}$$

Samples: v_4

N=2

K=1



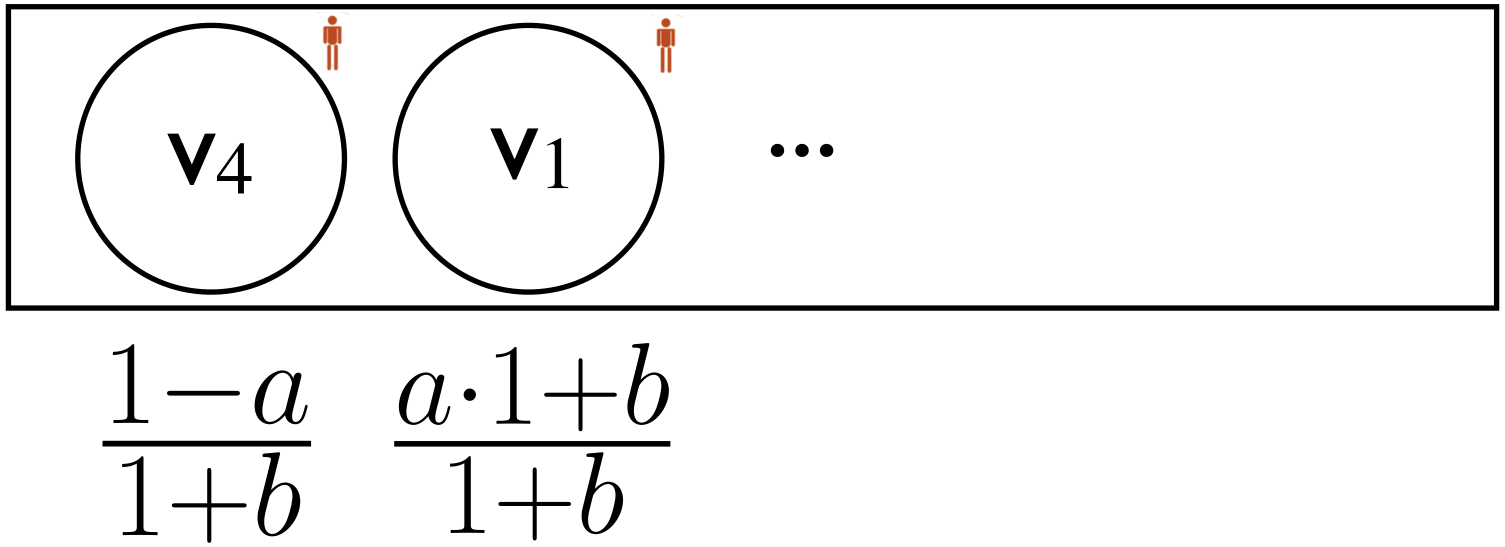
$\frac{1-a}{1+b}$ $\frac{a\cdot 1+b}{1+b}$

Samples: v_4

$v_1 \sim$ `(func arg1 ...)`

N=2

K=1

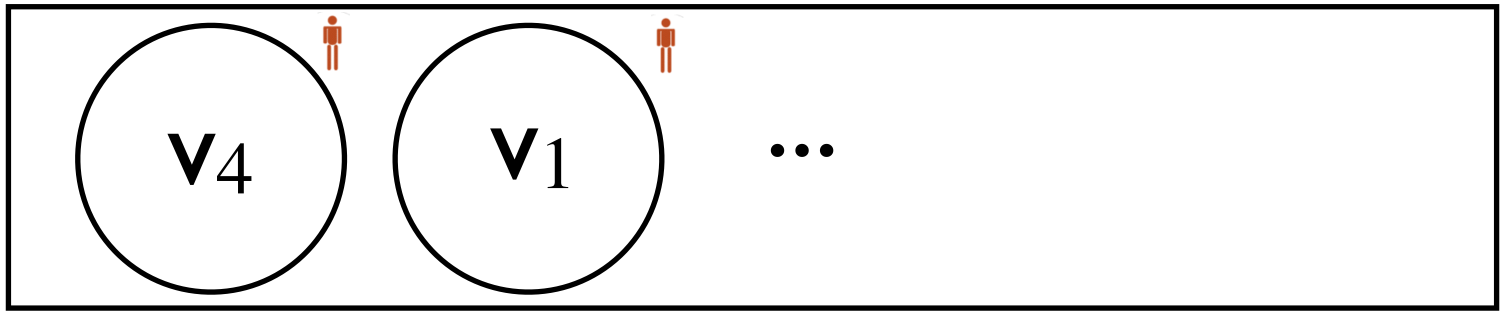


$$\frac{1-a}{1+b} \quad \frac{a\cdot 1+b}{1+b}$$

Samples: v_4

N=2

K=2



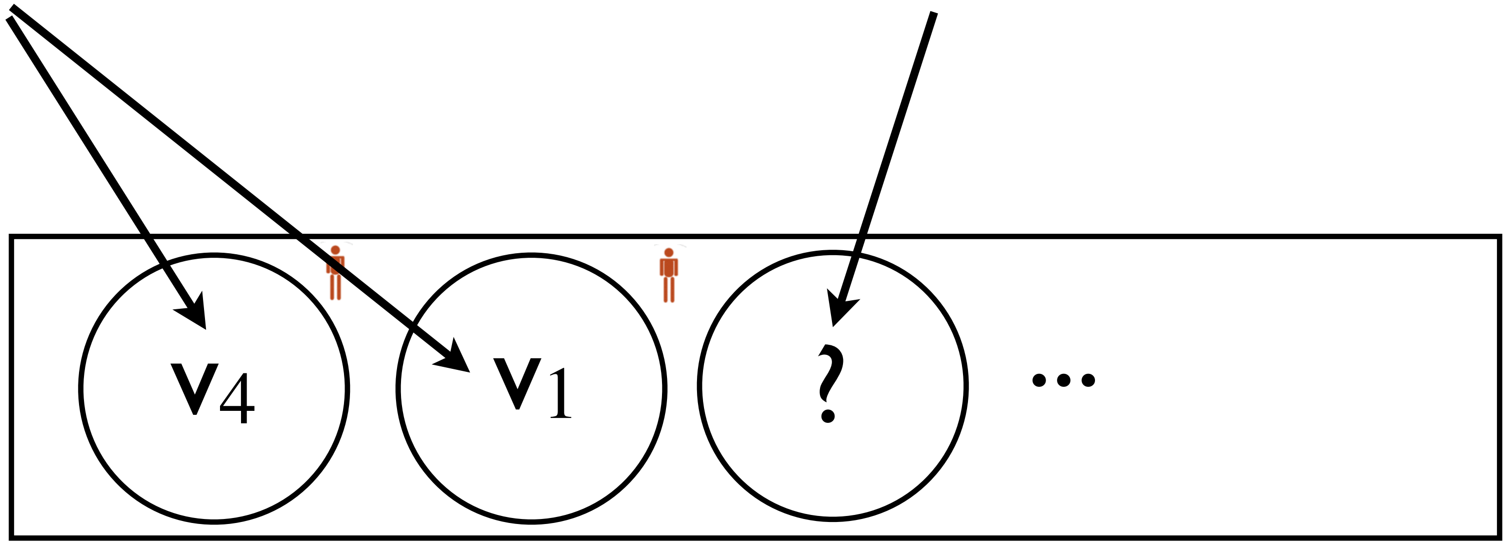
$$\frac{1-a}{1+b} \quad \frac{a\cdot 1+b}{1+b}$$

Samples: v_4, v_1

$$\frac{y_i - a}{N + b} \qquad \frac{a \cdot K + b}{N + b}$$

N=2

K=2



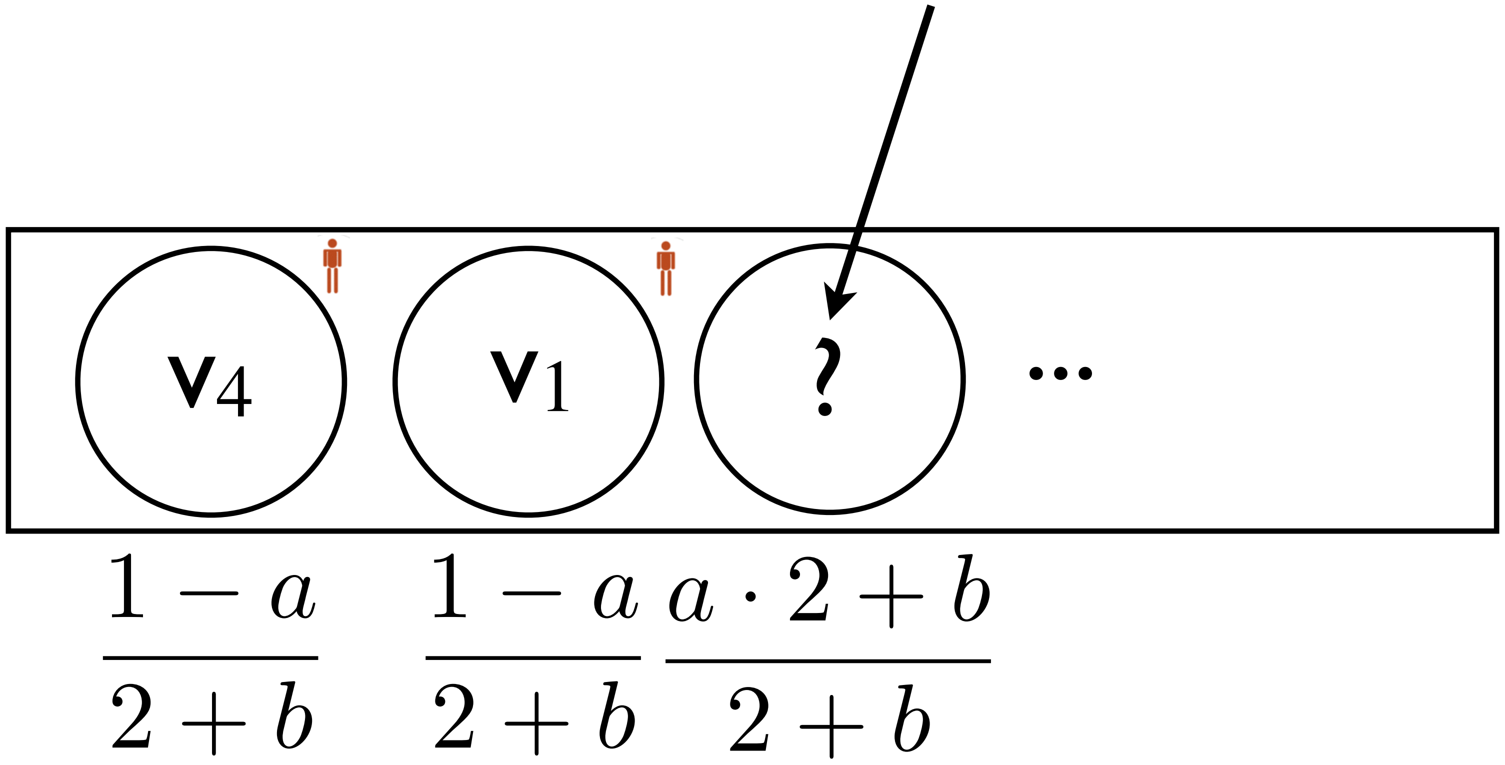
$$\frac{1-a}{2+b} \quad \frac{1-a}{2+b} \quad \frac{a \cdot 2 + b}{2+b}$$

Samples: v_4, v_1

$$\frac{a \cdot K + b}{N + b}$$

N=2

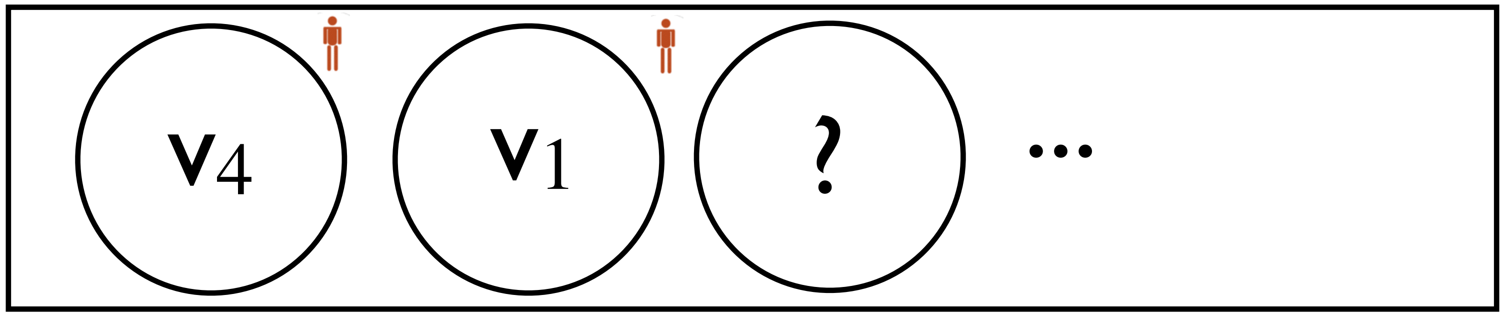
K=2



Samples: v_4, v_1

N=2

K=2

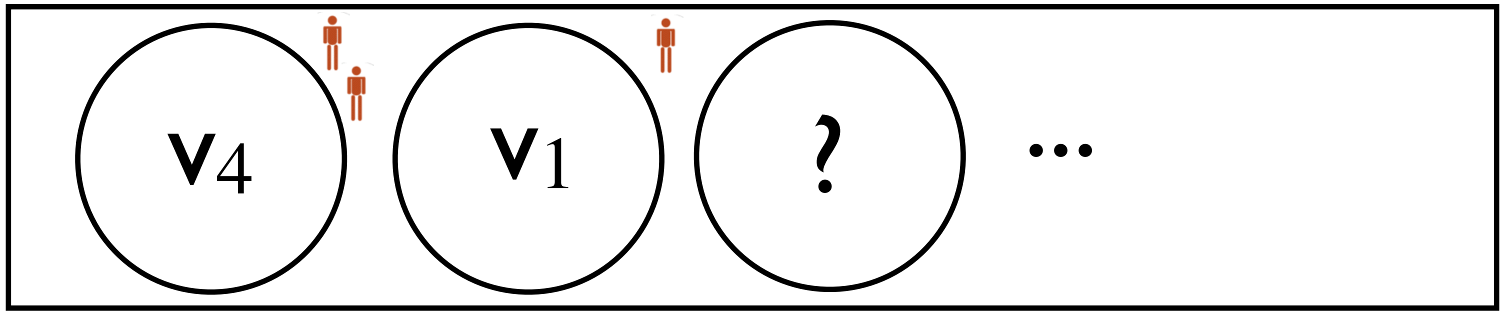


$$\frac{1-a}{2+b} \quad \frac{1-a}{2+b} \quad \frac{a \cdot 2+b}{2+b}$$

Samples: v_{4}, v_{1}

N=2

K=2

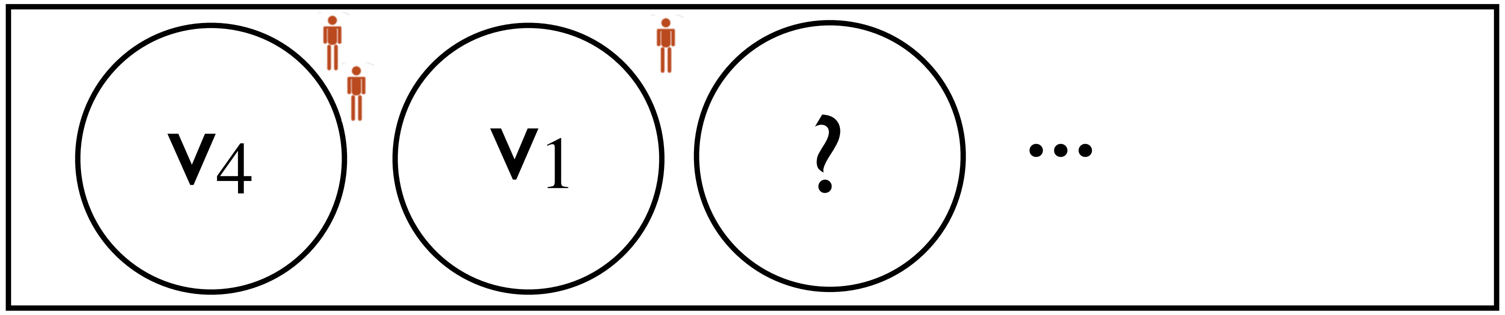


$\frac{1-a}{2+b}$ $\frac{1-a}{2+b}$ $\frac{a \cdot 2+b}{2+b}$

Samples: v_4, v_1

N=3

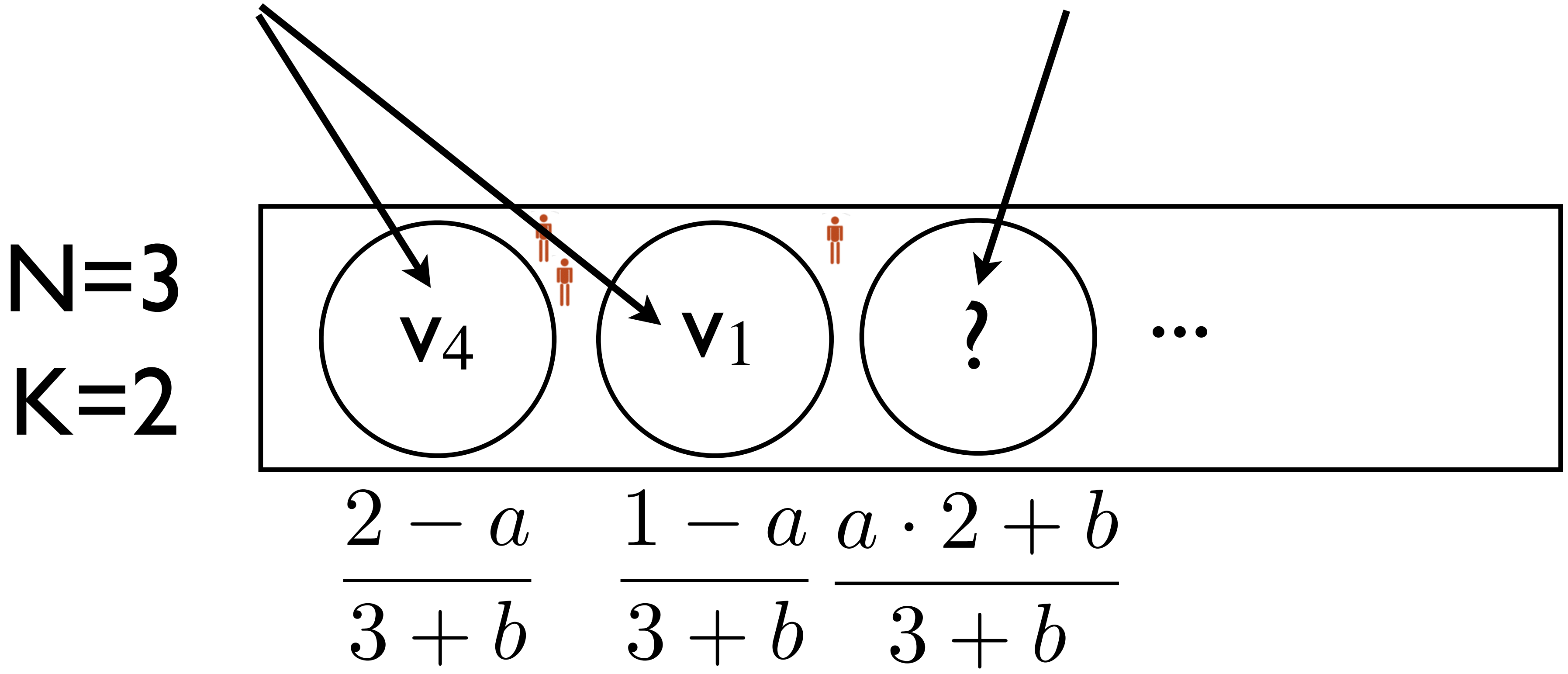
K=2



$$\frac{1-a}{2+b} \quad \frac{1-a}{2+b} \quad \frac{a\cdot 2+b}{2+b}$$

Samples: v$_4$, v$_1$, v$_4$

$$\frac{y_i - a}{N + b} \qquad \frac{a \cdot K + b}{N + b}$$



$$\frac{2 - a}{3 + b} \quad \frac{1 - a}{3 + b} \quad \frac{a \cdot 2 + b}{3 + b}$$

Samples: v_4, v_1, v_4

Properties of PYPs

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- Rich get richer, concentrates distribution on a few values.

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- Prefers fewer customers/tables/tables-per-customer.

Properties of PYPs

- Rich get richer, concentrates distribution on a few values.
- Prefers fewer customers/tables/tables-per-customer.
- Prefers to generate novel values proportional to how often novelty has been generated in the past.

Adaptor Grammars

(Johnson et al., 2007)

```
(define adapted-unfold
 (PYMem a b
  (lambda (symbol)
    (if (terminal? symbol)
     symbol
     (map unfold (sample-rhs symbol))))))
```

Properties of Adaptor Grammars

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- Reuse previous computations (subtrees).

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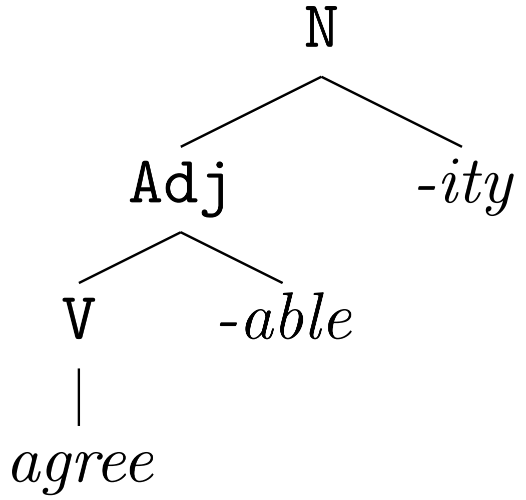

Properties of Adaptor Grammars

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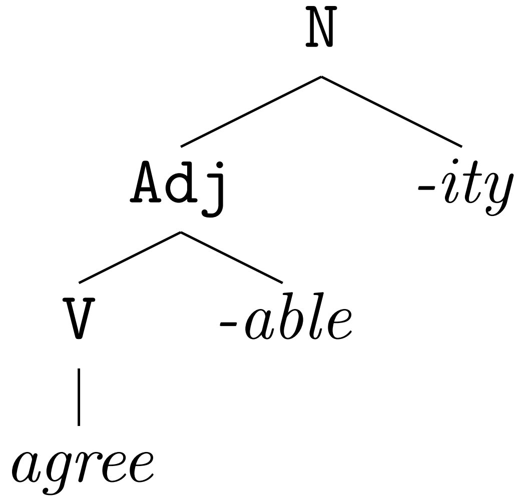
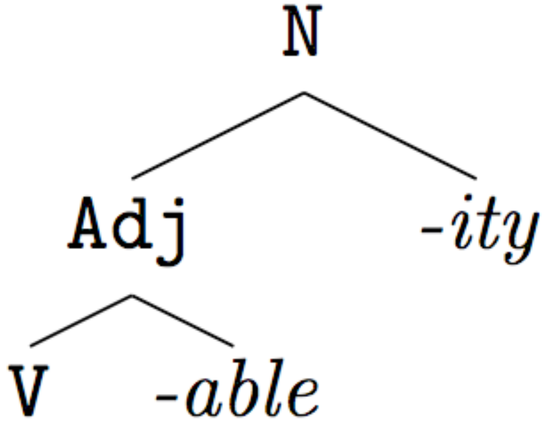
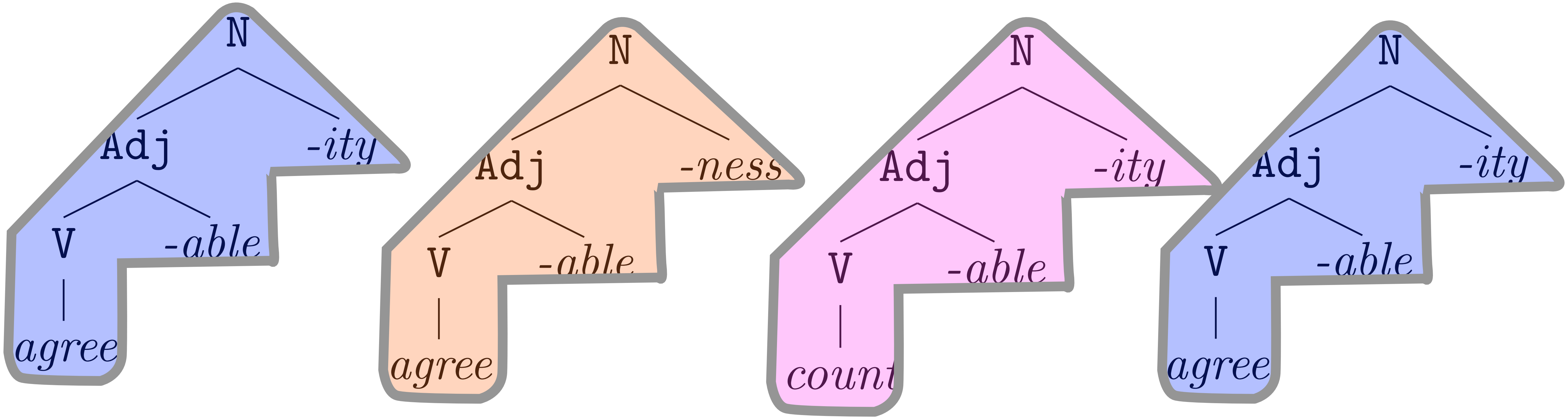


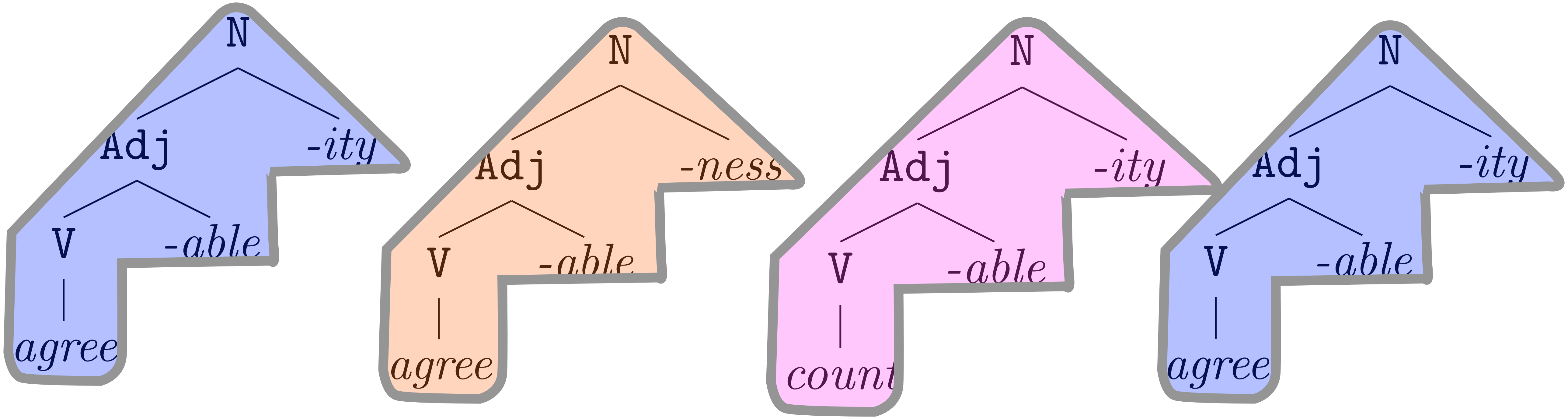


Reusability for Adaptor Grammars



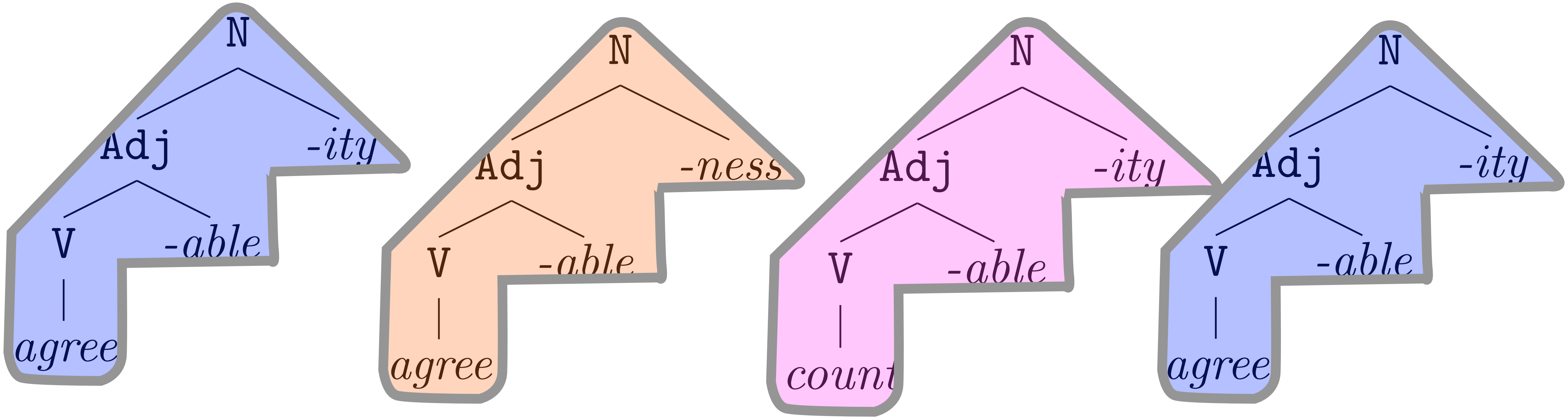
Reusability for Adaptor Grammars

1. Always possible to use base grammar.



Reusability for Adaptor Grammars

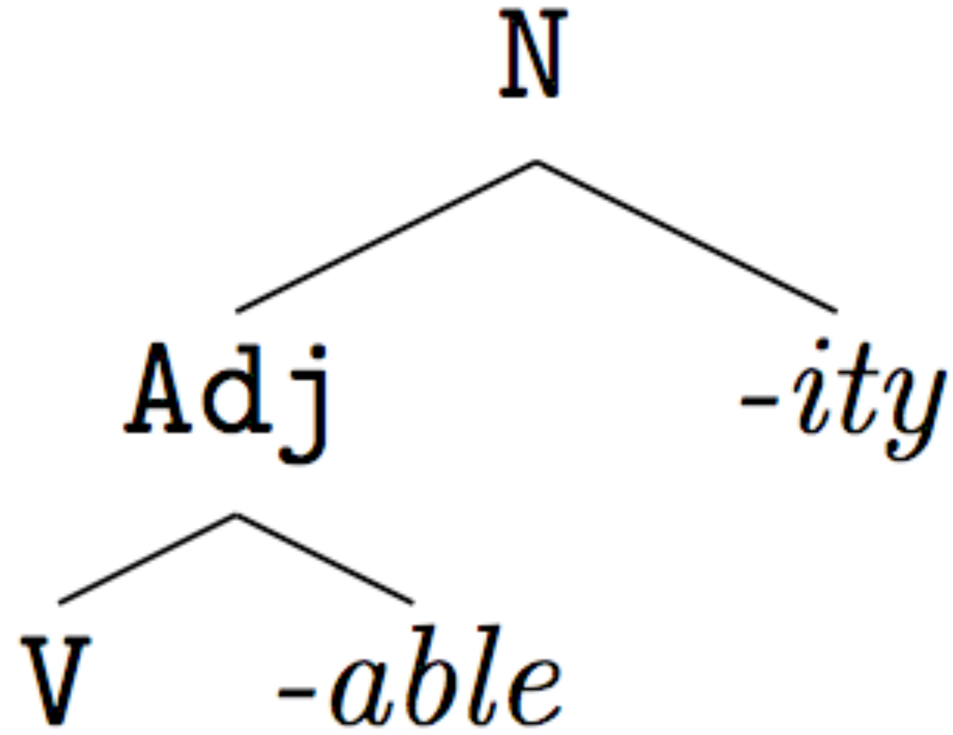
1. Always possible to use base grammar.
2. Fully recursive.



Fragment Grammars via Probabilistic Programming

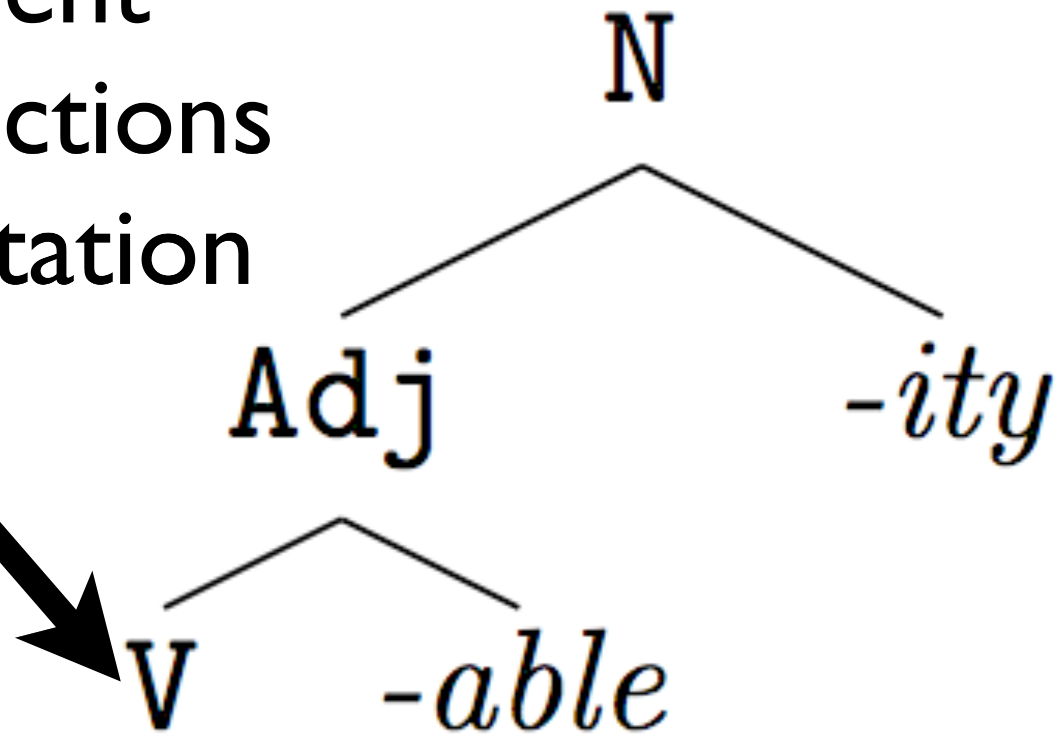
1. Stochastic computation via `unfold`
2. Stochastic reuse via memoization
3. Partial computations via stochastic laziness

Goal: Represent Partial Computations



Goal: Represent Partial Computations

Variables represent “delayed” instructions for later computation



Lazy and Eager Evaluation

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- Eager Evaluation: Do as much work as early as possible.

Lazy and Eager Evaluation

- Eager Evaluation: Do as much work as early as possible.
- Lazy Evaluation: Delay work until it is absolutely necessary to continue computation.

Example

```
(define add3
 (lambda (x y z)
   (+ x y z)))
```

Eager Evaluation

```
(add3 (+ 1 2 3) (* 2 4) (- 3 1))
```

Eager Evaluation

```
(add3 (+ 1 2 3) (* 2 4) (- 3 1))
```

Eager Evaluation

```
(add3 6 (* 2 4) (- 3 1))
```

Eager Evaluation

```
(add3 6 (* 2 4) (- 3 1))
```

Eager Evaluation

```
(add3 6 8 (- 3 1))
```

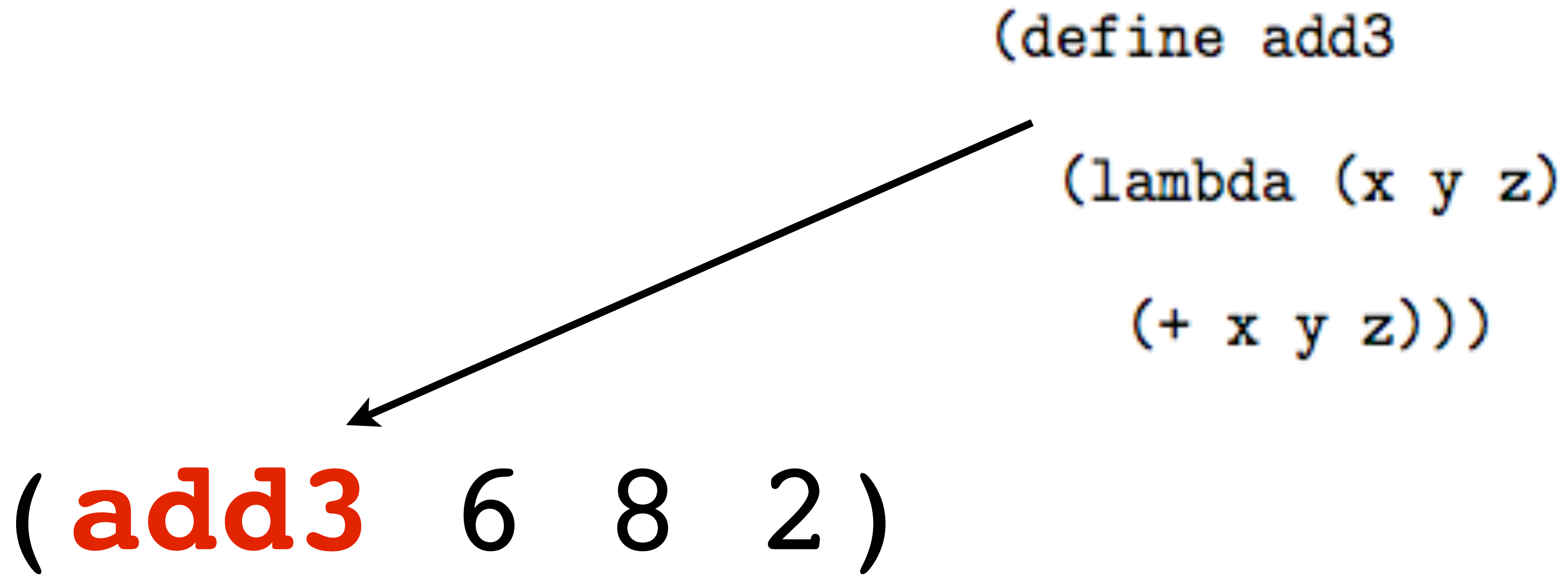
Eager Evaluation

```
(add3 6 8 (- 3 1))
```

Eager Evaluation

```
(add3 6 8 2)
```

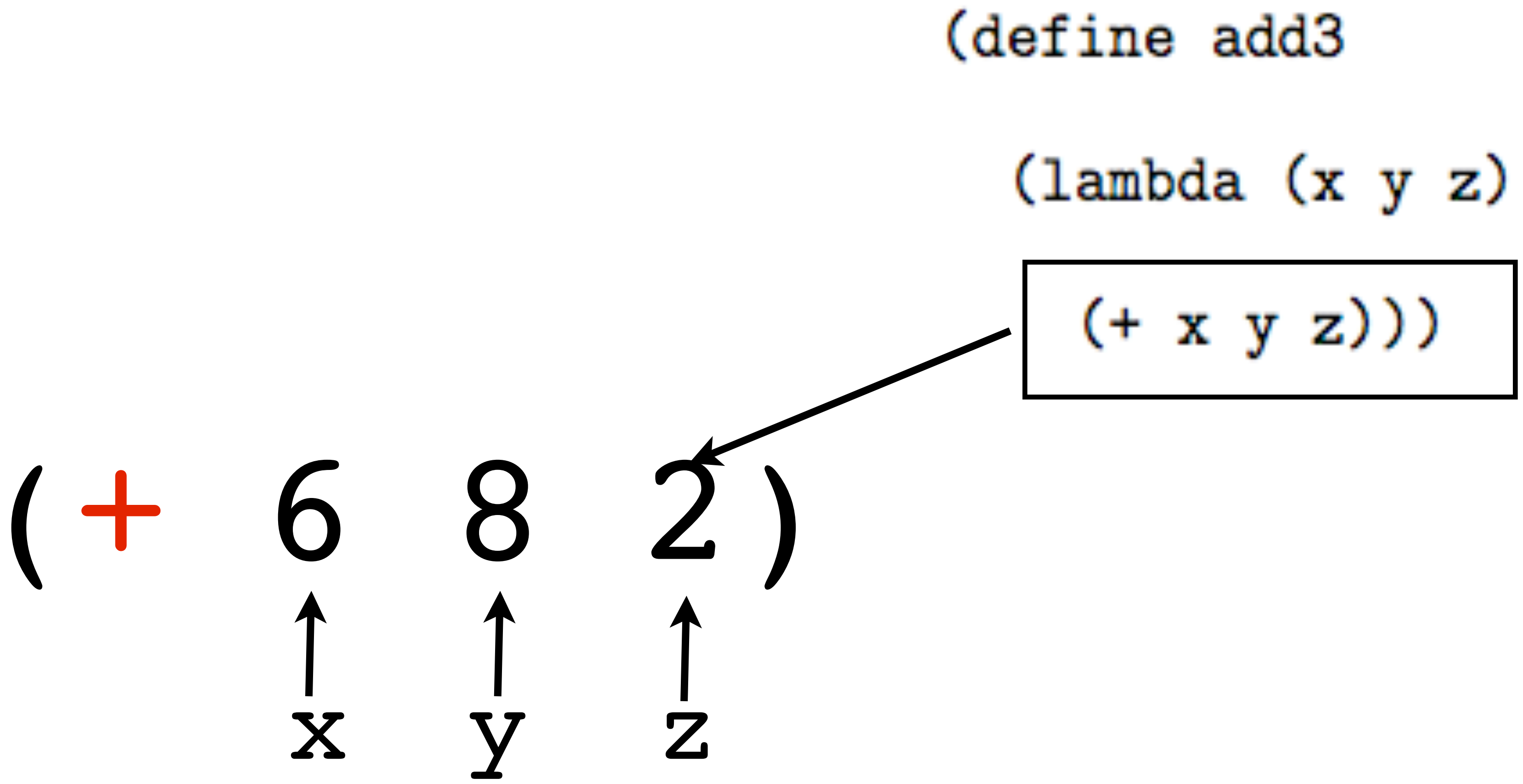
Eager Evaluation



```
(define add3
  (lambda (x y z)
    (+ x y z)))
```

```
(add3 6 8 2)
```

Eager Evaluation



```
(define add3

  (lambda (x y z)

    (+ x y z)))
```

(+ 6 8 2)

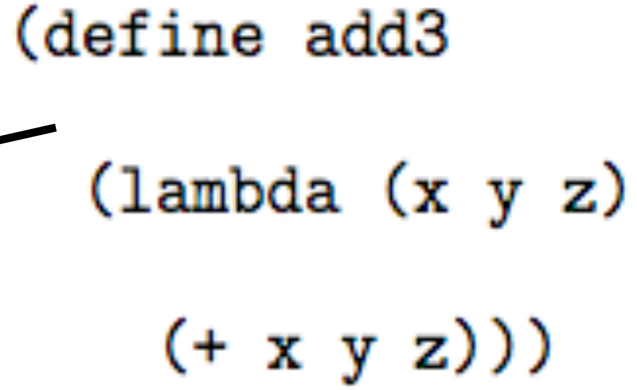
Eager Evaluation

```
16
```

Lazy Evaluation

```
(add3 (+ 1 2 3) (* 2 4) (- 3 1))
```

Lazy Evaluation



```
(define add3
  (lambda (x y z)
    (+ x y z)))
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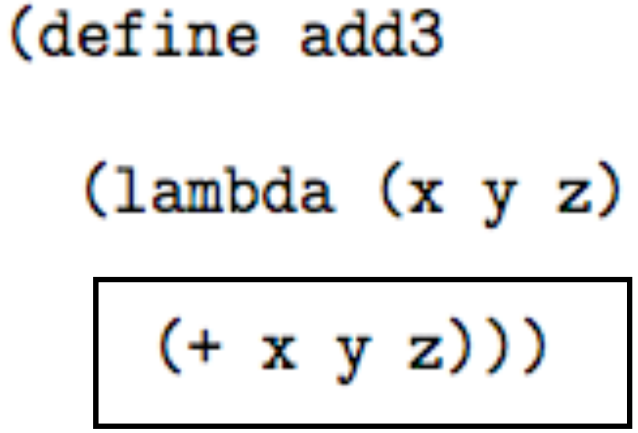
```
(add3 (+ 1 2 3) (* 2 4) (- 3 1))
```

Lazy Evaluation



```
(define add3
  (lambda (x y z)
    (+ x y z)))
```

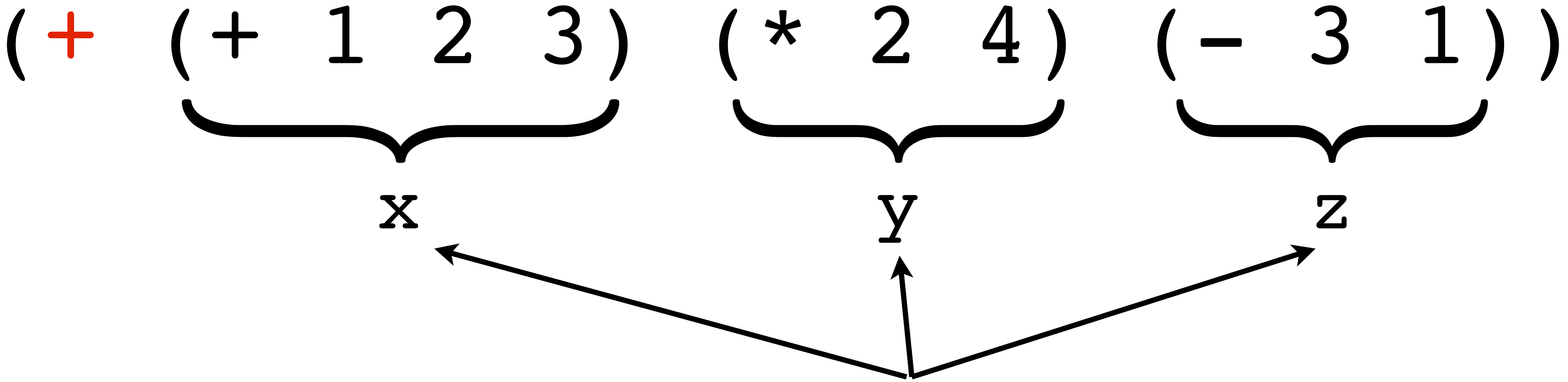
$$(\textcolor{red}{+}\ \underbrace{(+\ 1\ 2\ 3)}_{x}\ \underbrace{(*\ 2\ 4)}_{y}\ \underbrace{(-\ 3\ 1)}_{z})$$

Lazy Evaluation

```
(define add3

  (lambda (x y z)

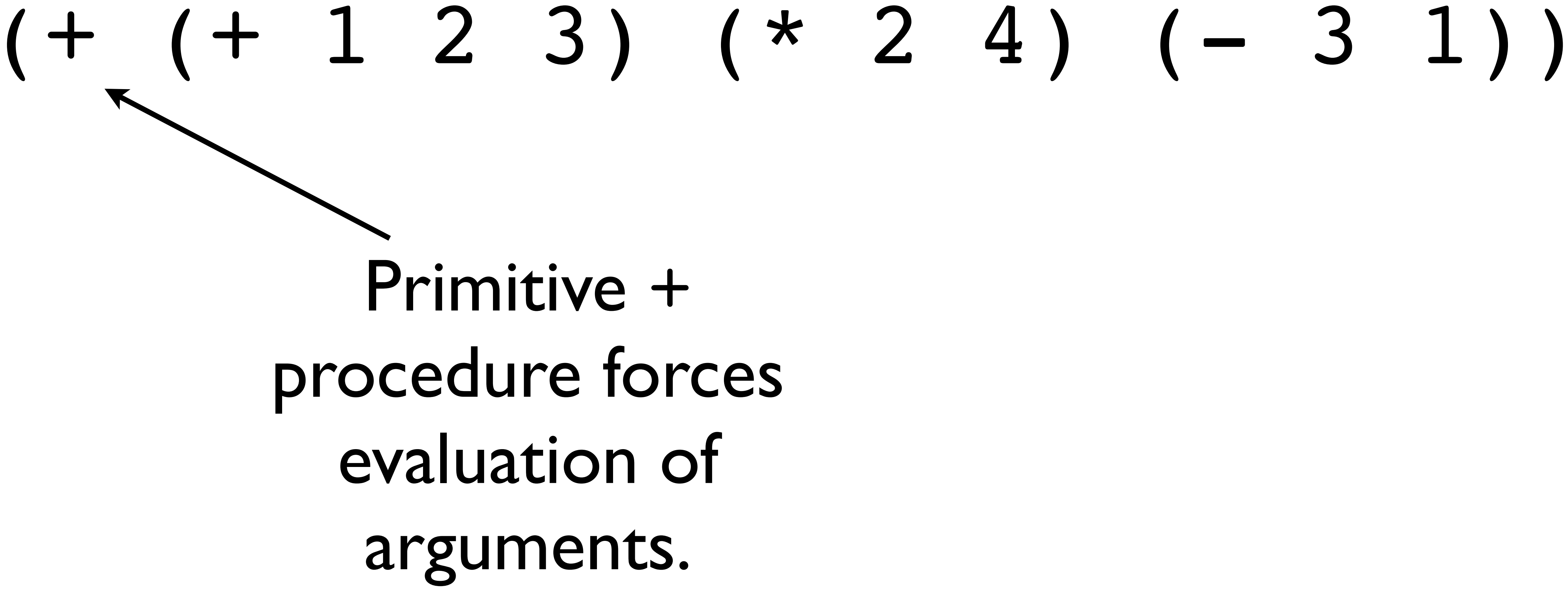
    (+ x y z)))
```



```
(+ (+ 1 2 3) (* 2 4) (- 3 1))
```

Argument expressions are delayed until their values are needed by another computation.

Lazy Evaluation



```
(+ (+ 1 2 3) (* 2 4) (- 3 1))
```

Primitive + procedure forces evaluation of arguments.

Lazy Evaluation

```
(+ (+ 1 2 3) (* 2 4) (- 3 1))
```

Lazy Evaluation

```
(+ 16 (* 2 4) (- 3 1))
```

Lazy Evaluation

```
(+ 16 (* 2 4) (- 3 1))
```

Lazy Evaluation

```
(+ 16 8 (- 3 1))
```

Lazy Evaluation

```
(+ 16 8 (- 3 1))
```

Lazy Evaluation

```
(+ 16 8 2)
```

Lazy Evaluation

```
16
```

λ-calculus: Order of Evaluation

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- *Applicative order* (eager evaluation): evaluate arguments first, then apply function.

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- *Church-Rosser theorem*: Order doesn't matter for deterministic λ-calculus.

λ-calculus: Order of Evaluation

- *Applicative order* (eager evaluation): evaluate arguments first, then apply function.
- *Normal order* (lazy evaluation): copy arguments into procedure, only evaluate when needed.
- *Church-Rosser theorem*: Order doesn't matter for deterministic λ-calculus.
- Does matter for Ψλ-calculus!

Ψλ-calculus: Order of Evaluation

```
(define **same?**
 (lambda (x)
   (equal? x x)))
```

Ψλ-calculus: Order of Evaluation

```
(define same?
 (lambda (x)
   (equal? x x)))
```

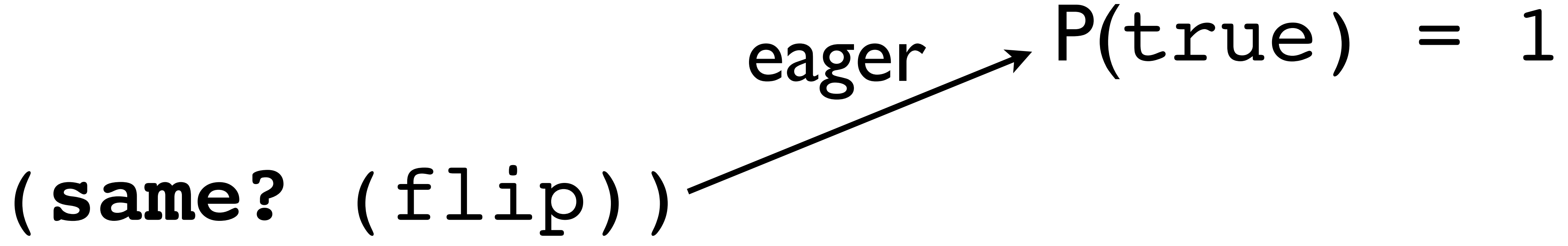
Ψλ-calculus: Order of Evaluation

```
(define same?
  (lambda (x)
    (equal? x x)))
```

```
(same? (flip))
```

Ψλ-calculus: Order of Evaluation

```
(define **same?**
  (lambda (x)
    (equal? x x)))
```



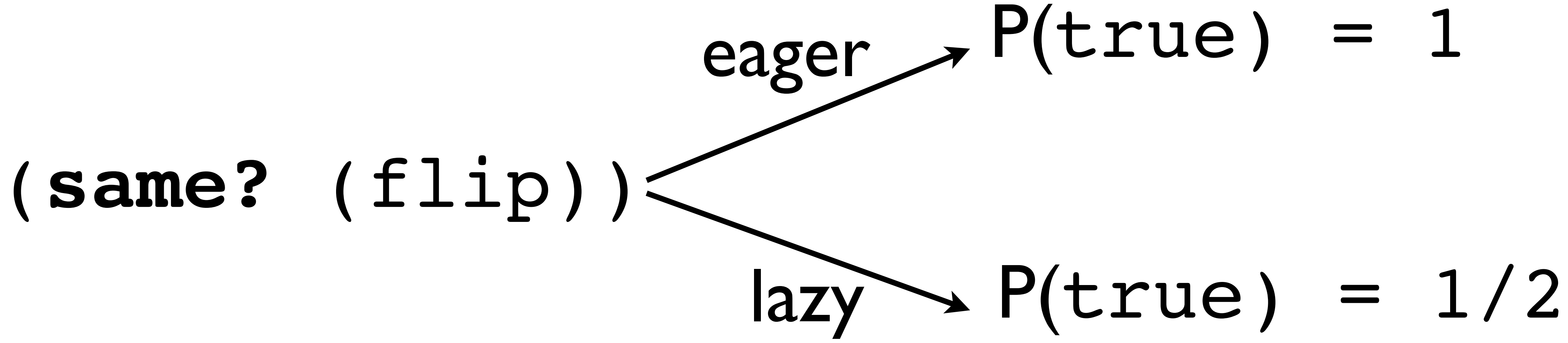
```
(same? (flip))
```

eager

P(true) = 1

Ψλ-calculus: Order of Evaluation

```
(define same?
 (lambda (x)
   (equal? x x)))
```



(same? (flip))

eager → P(true) = 1

lazy → P(true) = 1/2

Tradeoff

- Laziness allows you to delay computation and, thus, **preserve randomness** and variability until the last possible moment.
- Eagerness allows you to determine random choices early in computation and, thus, **share** choices across different parts of a program.

Random Evaluation Order

Random Evaluation Order

- Idea: Stochastically mix lazy and eager evaluation in Ψλ-calculus.

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- Ultimately allow **learning** of which computations should be performed in advance and which should be delayed.

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- Assume eager evaluation strategy and add `delay` primitive.

Random Evaluation Order

- Idea: Stochastically mix lazy and eager evaluation in Ψλ-calculus.
- Ultimately allow **learning** of which computations should be performed in advance and which should be delayed.
- Assume eager evaluation strategy and add `delay` primitive.
- Apply to `unfold` (can be applied fully generally).

Stochastic Lazy `unfold`

```
(define stochastic-lazy-unfold
 (lambda (symbol)
   (if (terminal? symbol)
     symbol
     (map delay-or-unfold (sample-rhs symbol)))))
```

Stochastic Lazy `unfold`

```
(define stochastic-lazy-unfold
 (lambda (symbol)
   (if (terminal? symbol)
    symbol
    (map delay-or-unfold (sample-rhs symbol)))))
```

Stochastic Lazy `unfold`

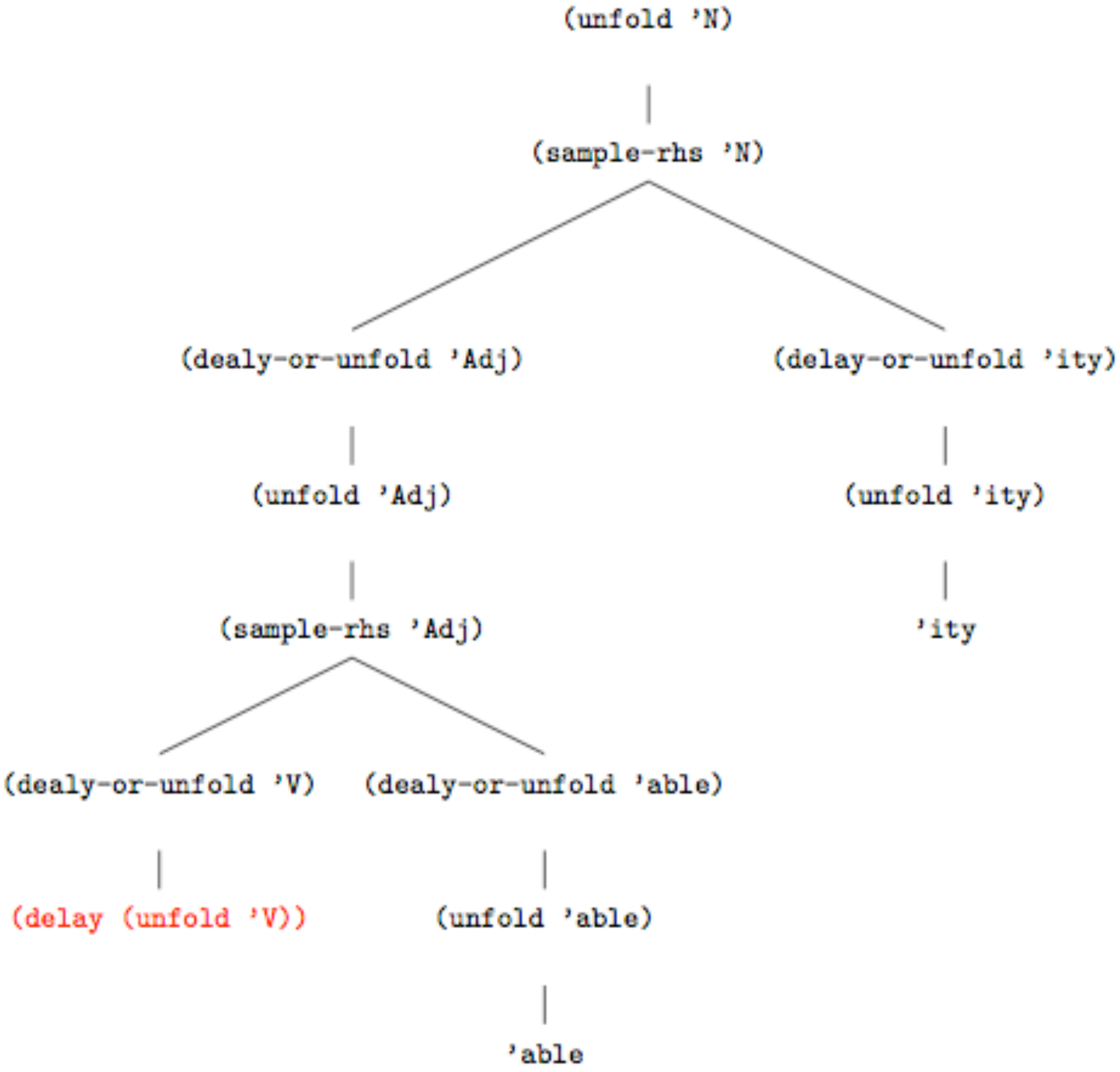
```
(define delay-or-unfold
 (lambda (symbol)
  (if (flip)
   (delay (stochastic-lazy-unfold symbol))
   (stochastic-lazy-unfold symbol))))
```

Stochastic Lazy `unfold`

```
(define stochastic-lazy-unfold
  (lambda (symbol)
    (if (terminal? symbol)
      symbol
      (map delay-or-unfold (sample-rhs symbol)))))

(define delay-or-unfold
  (lambda (symbol)
    (if (flip)
      (delay (stochastic-lazy-unfold symbol))
      (stochastic-lazy-unfold symbol))))
```

Computation Trace with Delay



(unfold 'N)

(sample-rhs 'N)

(dealy-or-unfold 'Adj)

(delay-or-unfold 'ity)

(unfold 'Adj)

(unfold 'ity)

(sample-rhs 'Adj)

'ity

(dealy-or-unfold 'V)

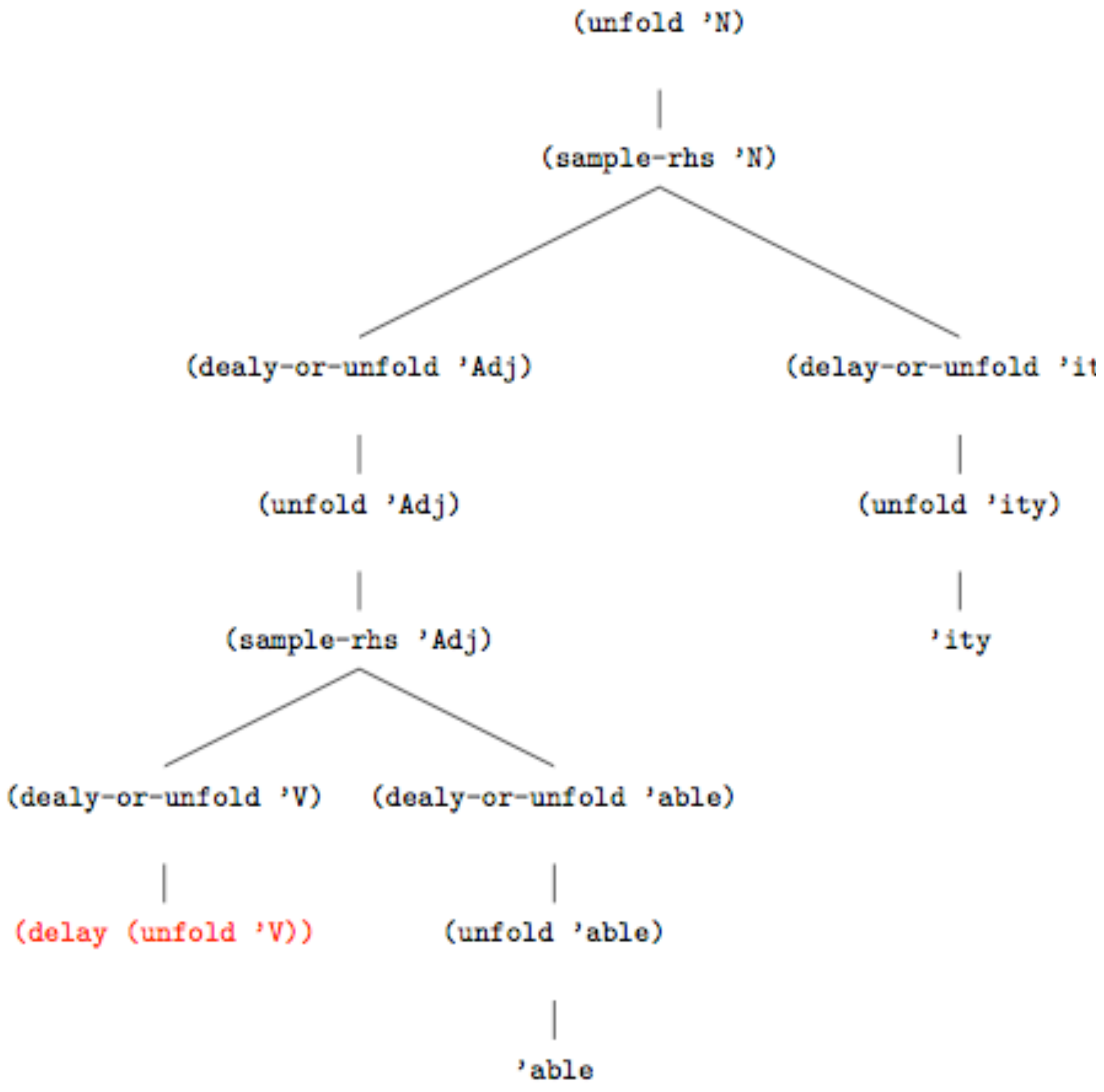
(dealy-or-unfold 'able)

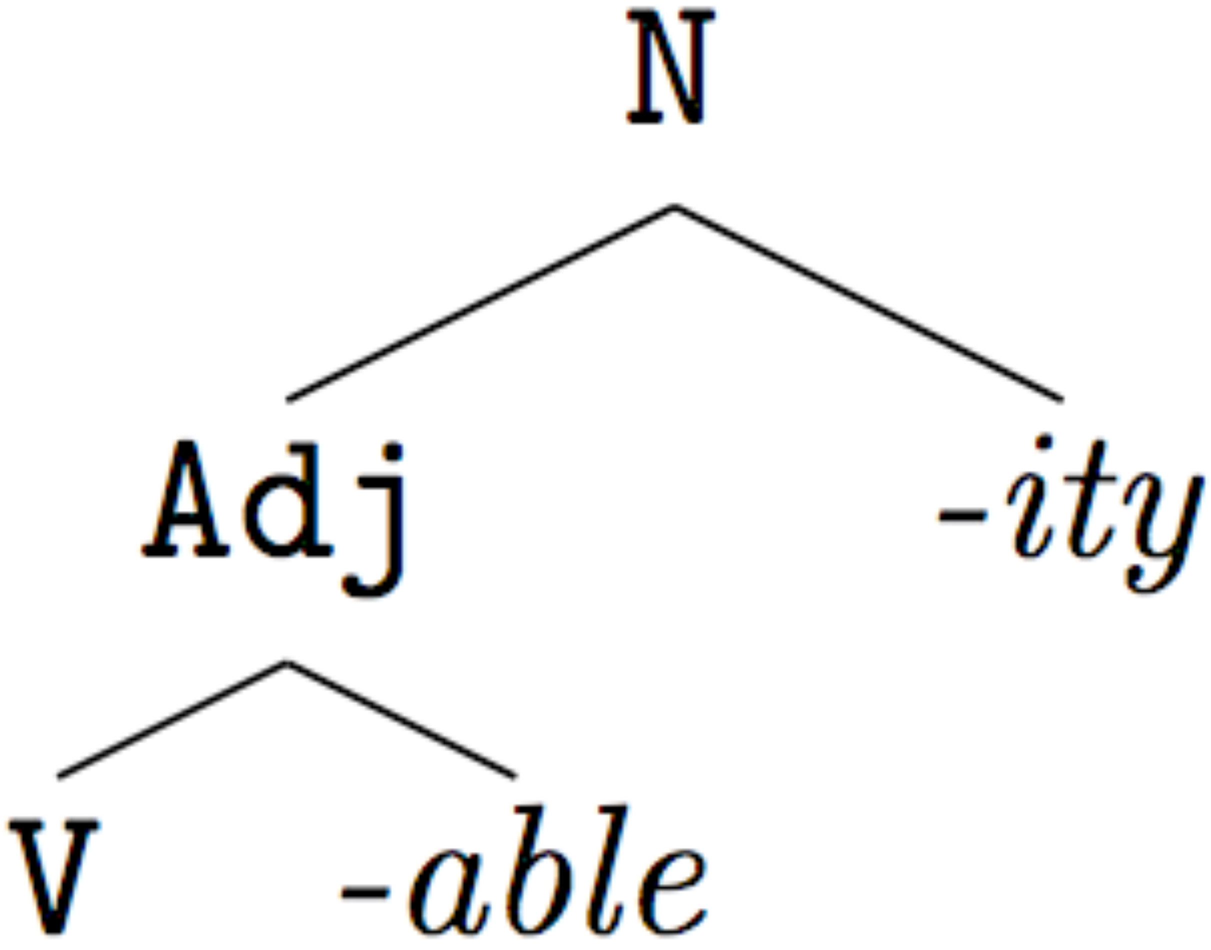
(delay (unfold 'V))

(unfold 'able)

'able

Computation Trace with Delay





Reusing Delayed Computations

Reusing Delayed Computations

- Need to be able to reuse partial evaluations.

Reusing Delayed Computations

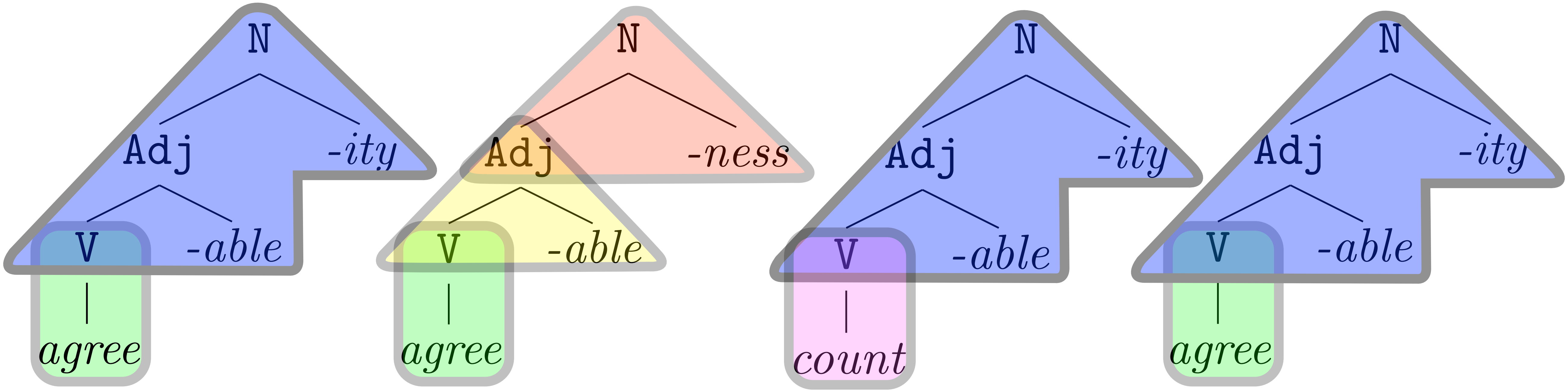
- Need to be able to reuse partial evaluations.
- Memoize stochastically lazy unfold.

Fragment Grammars

```
(define stochastic-lazy-unfold
  (lambda (symbol)
   (if (terminal? symbol)
    symbol
    (map delay-or-unfold (sample-rhs symbol)))))
```

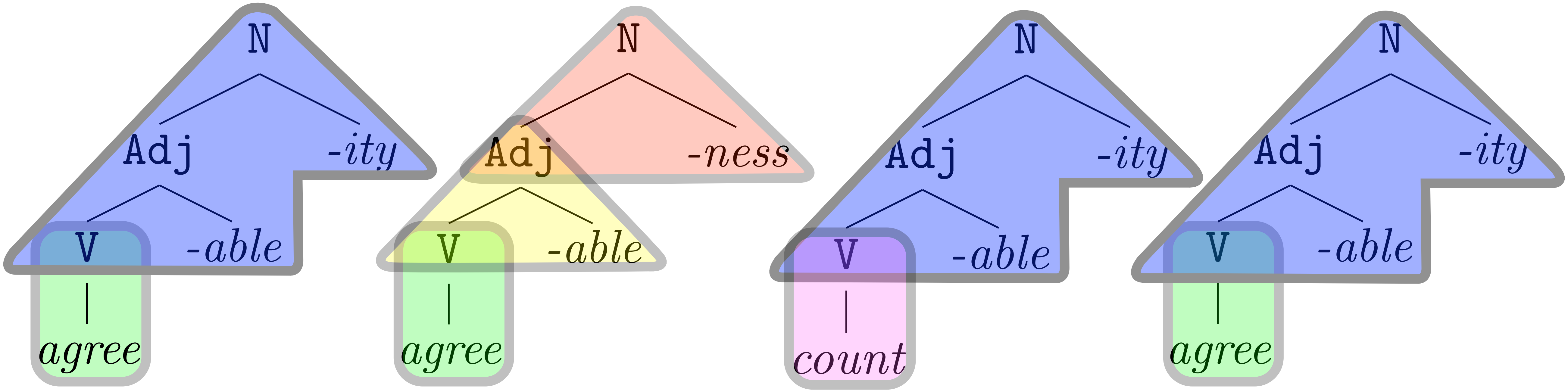
```
(define delay-or-unfold
  (PYMem a b (lambda (symbol)
   (if (flip)
    (delay (stochastic-lazy-unfold symbol))
    (stochastic-lazy-unfold symbol)))))
```

Fragment Grammar
Reusable Computations



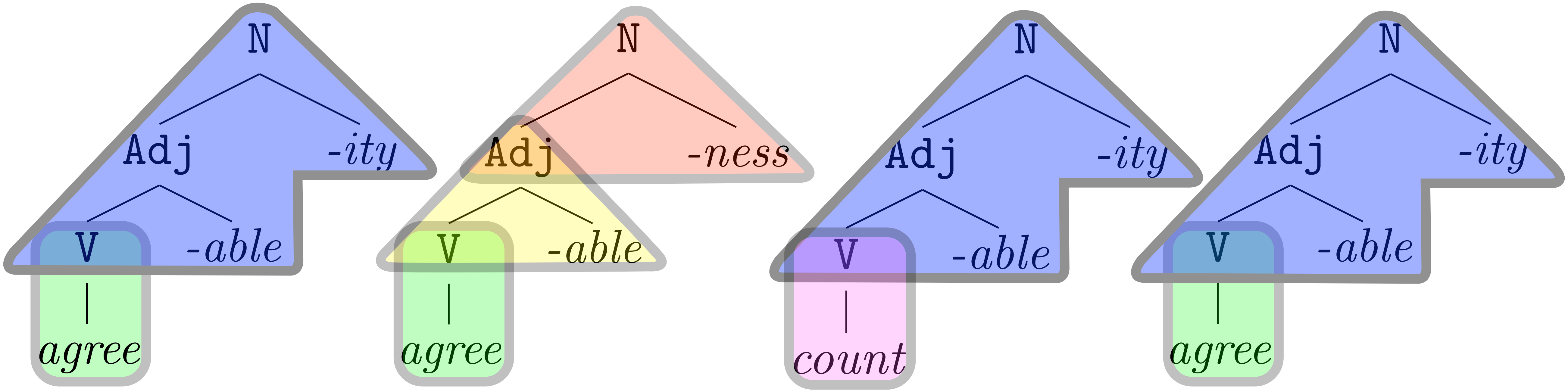
Fragment Grammar
Reusable Computations

1. Always possible to use base grammar.



Fragment Grammar
Reusable Computations

1. Always possible to use base grammar.
2. Fully recursive.



Outline

1. The Proposal.
2. Five Models of Productivity and Reuse.
3. English Derivational Morphology
4. Conclusion

Five Models

Five Models

- 4 approaches to productivity and reuse.

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- State-of-the-art probabilistic models.

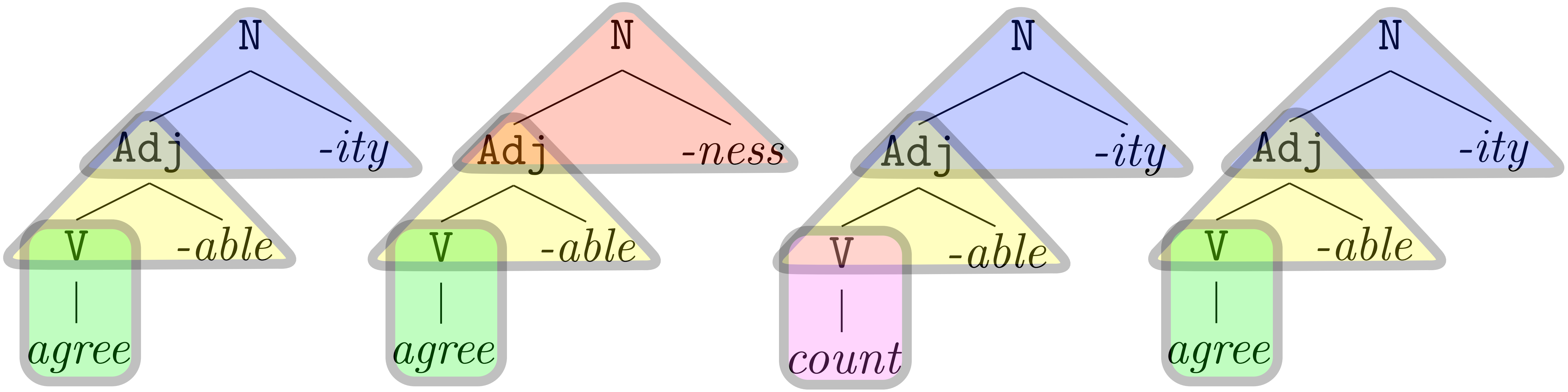
Five Models

- 4 approaches to productivity and reuse.
- Capture historical proposals from the literature.
- State-of-the-art probabilistic models.
    - Allow for variability and learning.

MDPCFG

Multinomial-Dirichlet Context-Free Grammars
(*Full-Parsing*)

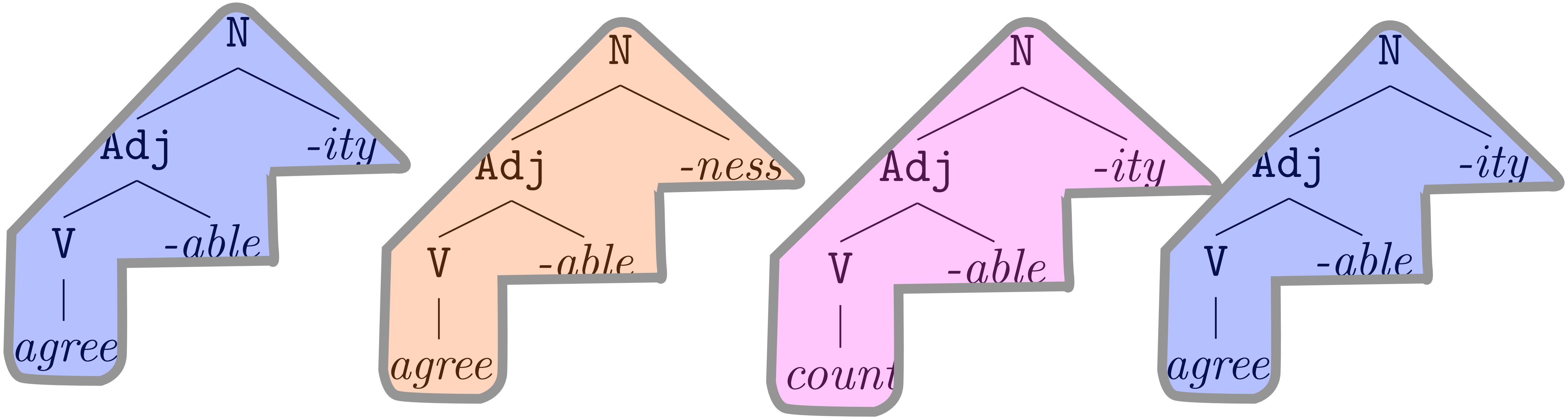
- All generalizations are productive
- Formalization: *Multinomial-Dirichlet Probabilistic Context-free Grammar* (MDPCFG; Johnson, et al. 2007a)



MAG

MAP Adaptor Grammars
(Full-entry)

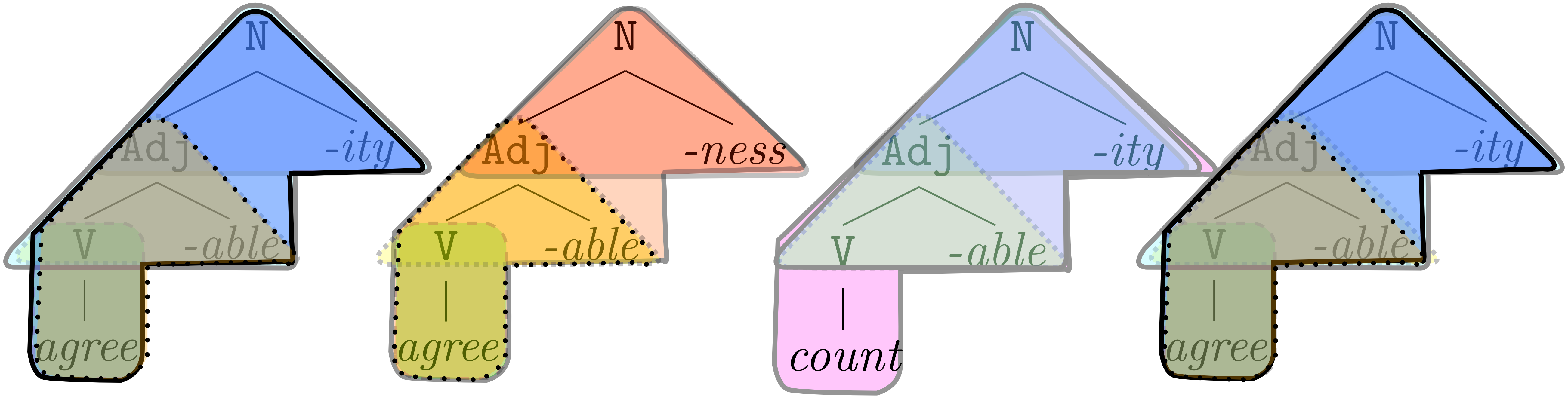
- Store whole form after *first* use.
- Formalization: *Adaptor Grammars* (AG; Johnson, et al. 2007).
- Always possible to compute productively with small probability; Fully recursive.
- Formalizes classic lexicalist theories (e.g., Jackendoff, 1975).



DOP1/GDMN

Data-Oriented Parsing
(Exemplar-based)

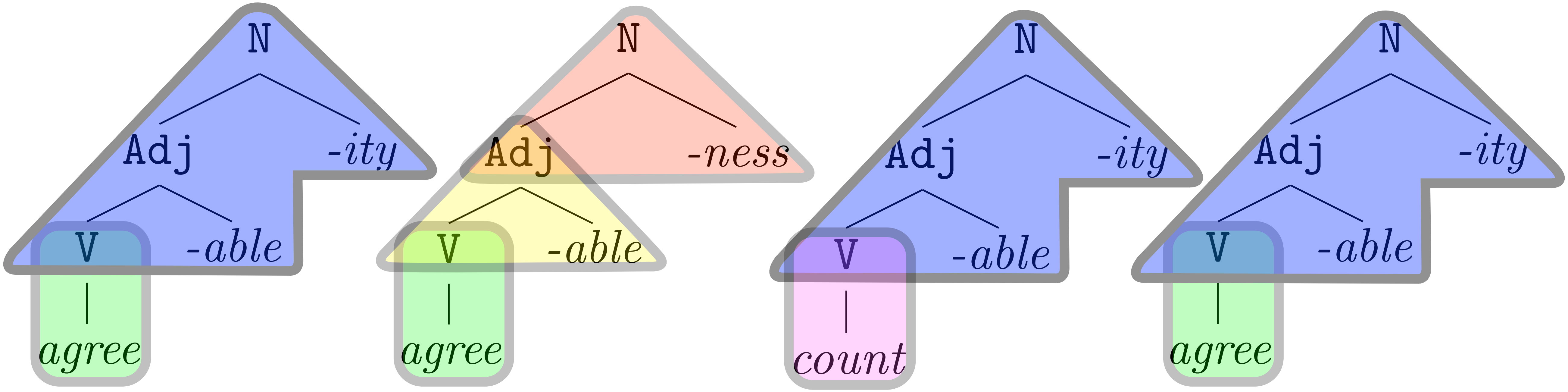
- Store *all* generalizations consistent with input
- Formalization: *Data-Oriented Parsing 1* (DOP1; Bod, 1998), *Data-Oriented Parsing: Goodman Estimator* (GDMN; Goodman, 2003)
- Recently proposed as models of syntax (e.g., Snider, 2009; Bod, 2009)



FG

Fragment Grammars
(Inference-based)

- Store *best* set of subcomputations for explaining the data.
- Formalization: *Fragment Grammars* (FG; O'Donnell, et al. 2009)
- Generalization of *Adaptor Grammars*



Outline

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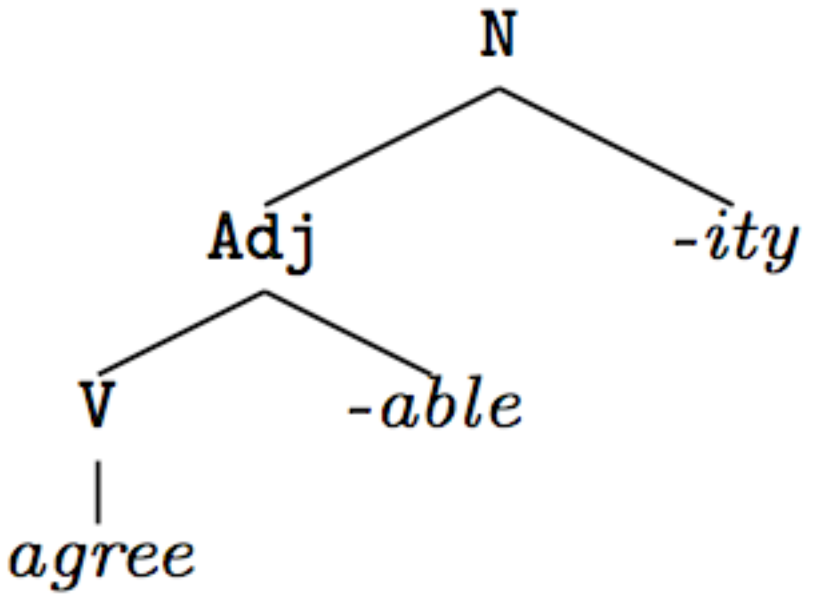
English Derivational Morphology

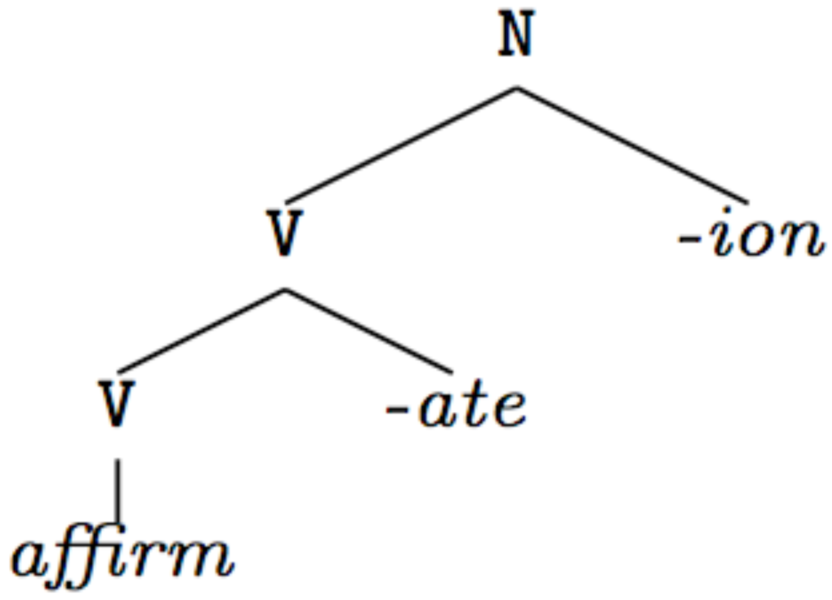
Productive	+ness *(goodness)*, +ly *(quickly)*
Semi-productive	+ity *(ability)*, +or *(operator)*
Unproductive	+th *(width)*

Simulations

- Words from CELEX.
- Extensive heuristic parsing/hand correction.
- Input format.
  - No phonology or semantics.

Derivational Inputs





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1. Individual suffix productivity differences (-ness/-ity/-th).
2. Suffix sequences.

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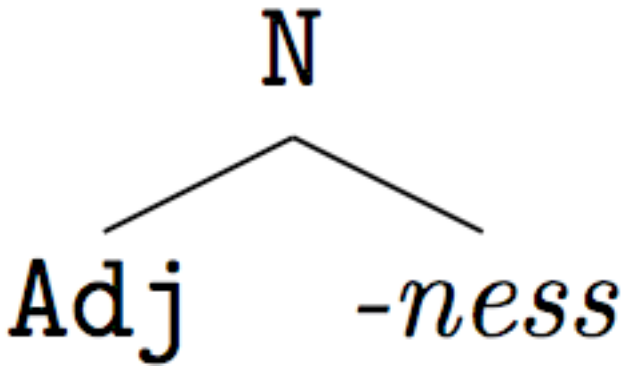
1. Individual suffix productivity differences (-ness/-ity/-th).
2. Suffix combinations.

Productivity

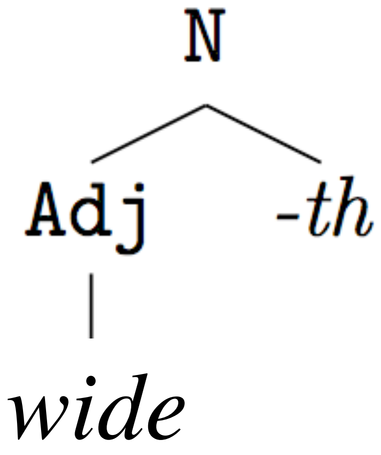
- No gold-standard dataset or measure.
  - E.g., Large databases of *wug*-tests or naturalness judgments.
- Analyses.
  1. Examples of highly productive affixes.
  2. Convergence with other theoretical measures.

How is Productivity Represented?

- Relative probability of fragments with or without variables.



V.



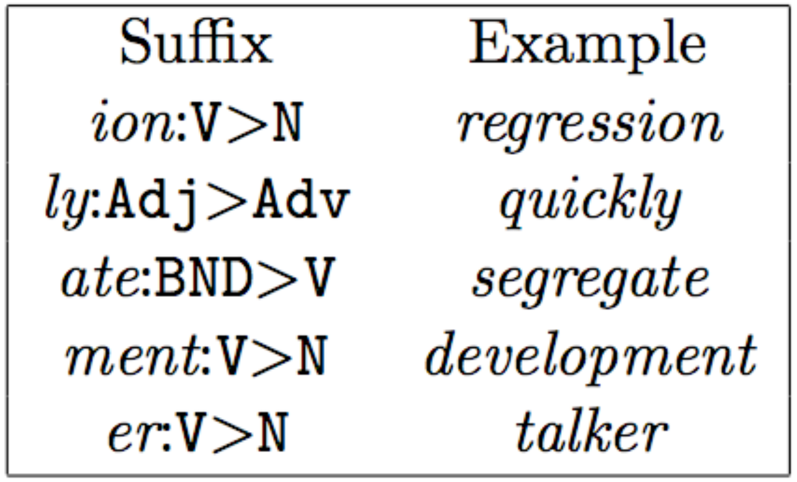
Productivity Analyses

➡ 1. Examples of highly productive suffixes.

2. Convergence with other theoretical measures.

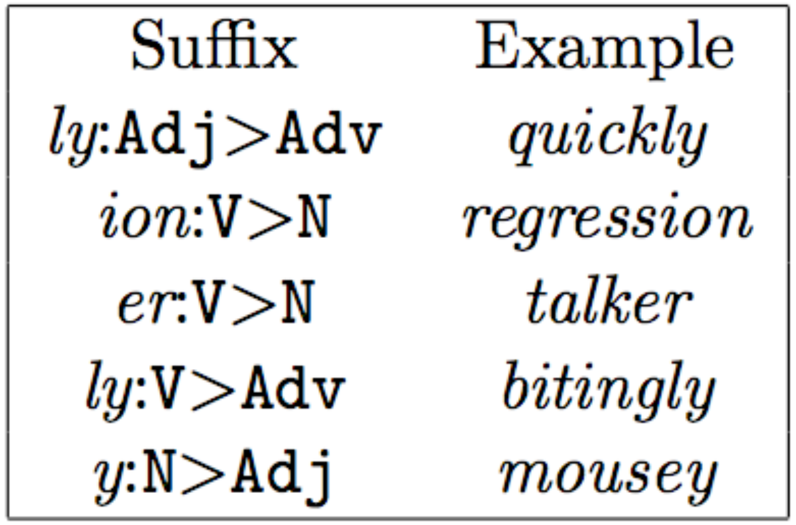
Top 5 Most Productive Suffixes

MDPCFG *(Full-Parsing)*



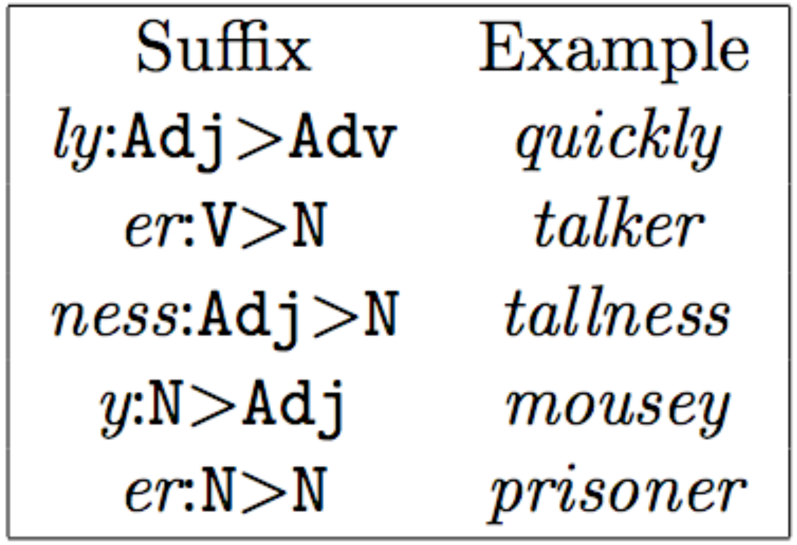
Suffix	Example
ion:V>N	*regression*
ly:Adj>Adv	*quickly*
ate:BND>V	*segregate*
ment:V>N	*development*
er:V>N	*talker*

MAG *(Full-listing)*



Suffix	Example
ly:Adj>Adv	*quickly*
ion:V>N	*regression*
er:V>N	*talker*
ly:V>Adv	*bitingly*
y:N>Adj	*mousey*

FG *(Inference-based)*



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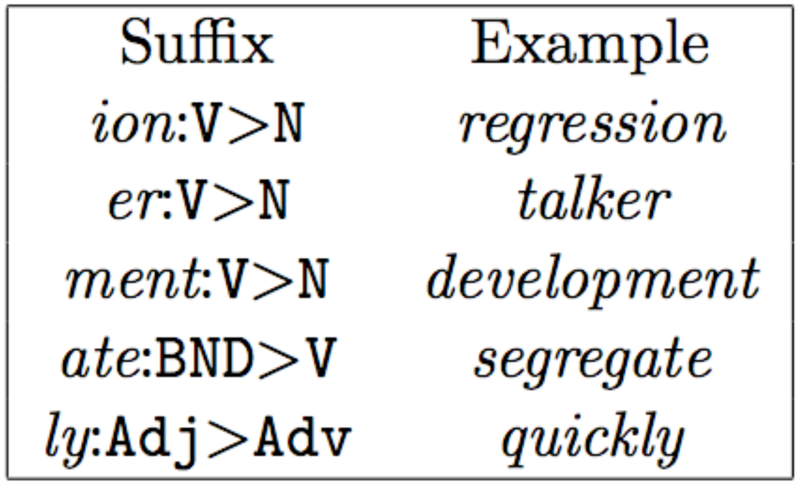
er:N>N	*prisoner*

DOP1 *(Exemplar)*



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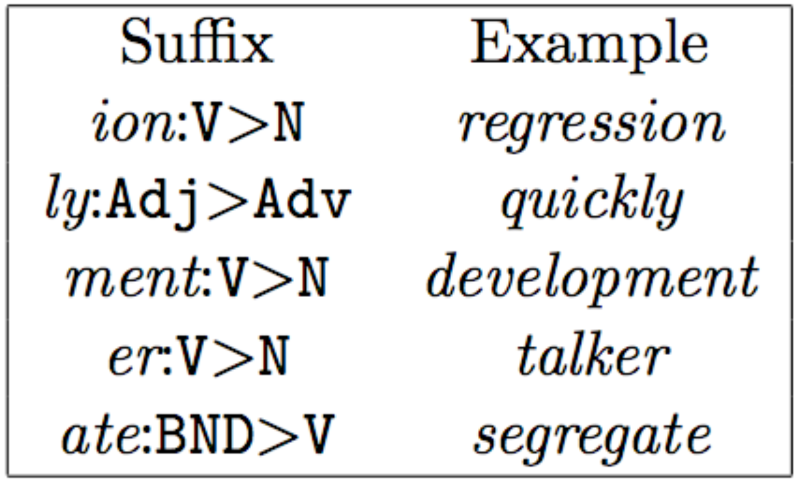
GDMN *(Exemplar)*



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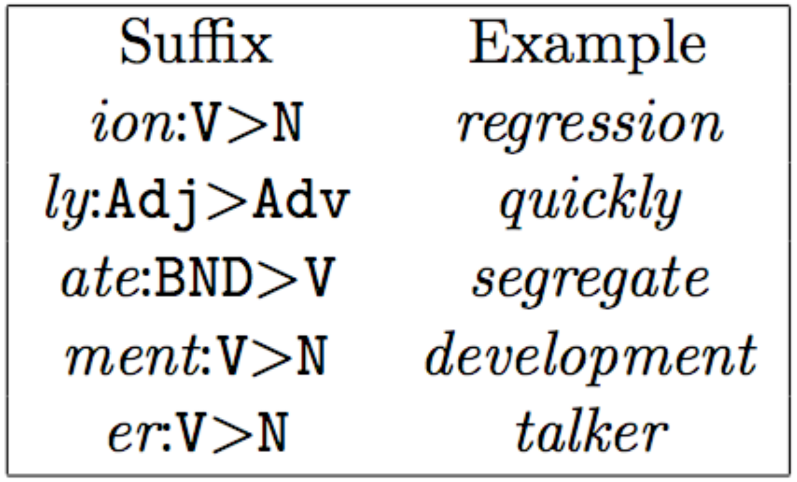

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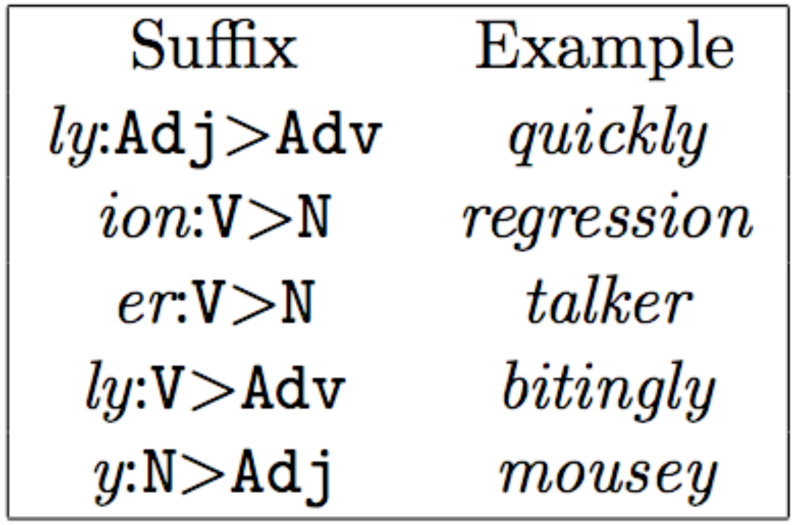
MAG *(Full-listing)*



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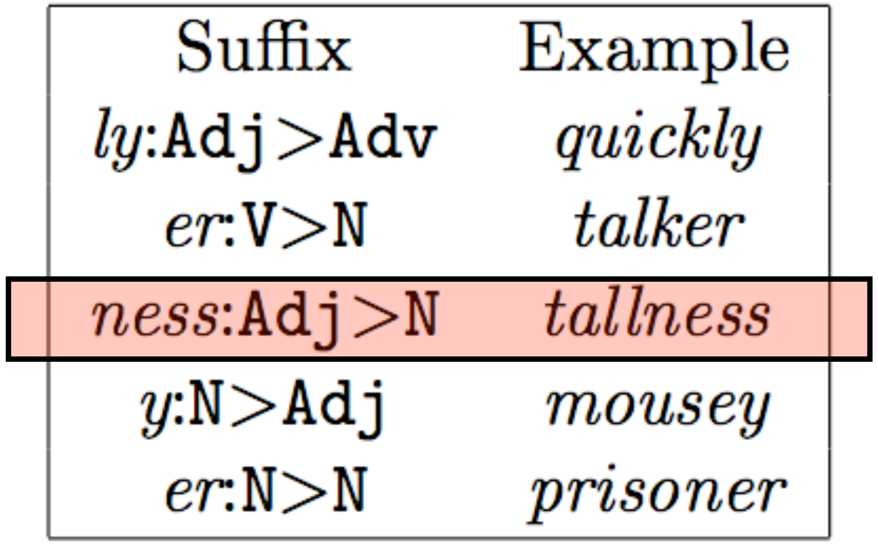

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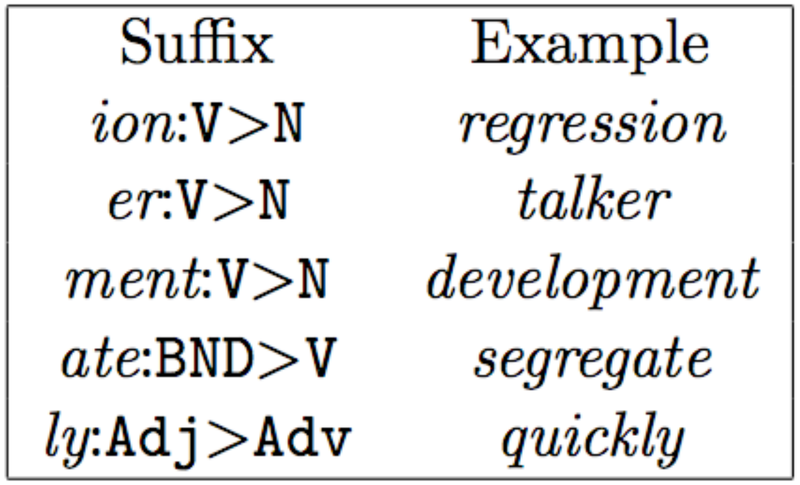
DOP1 *(Exemplar)*



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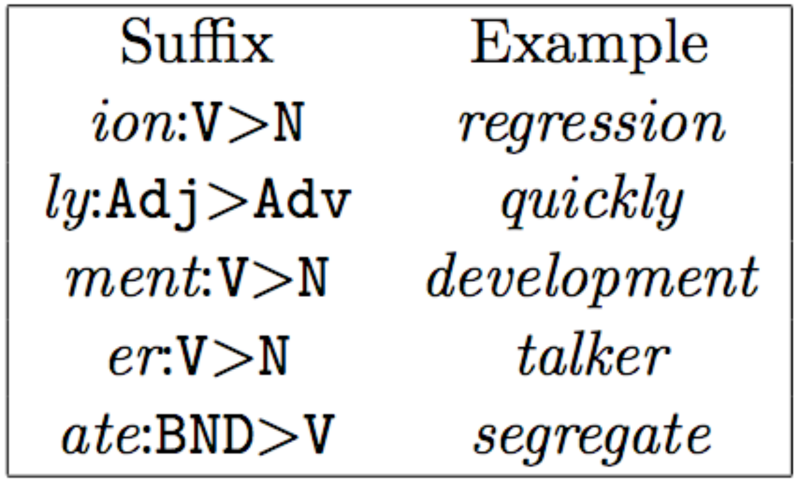
GDMN *(Exemplar)*



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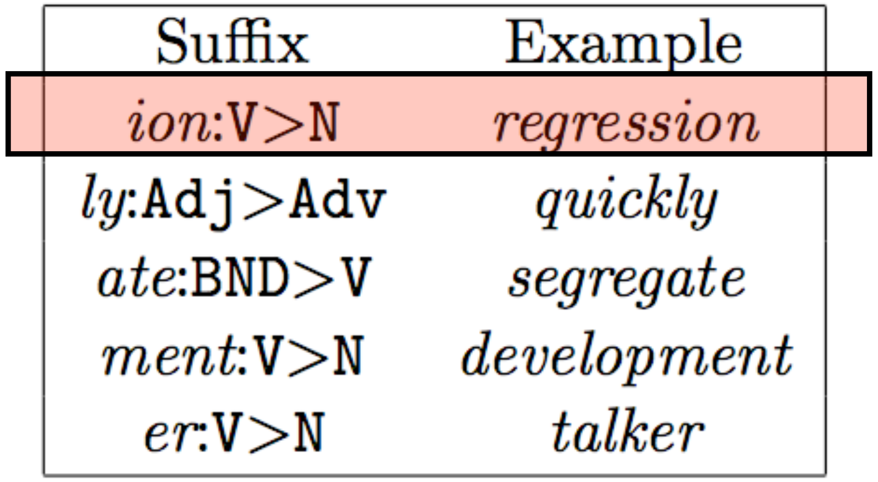

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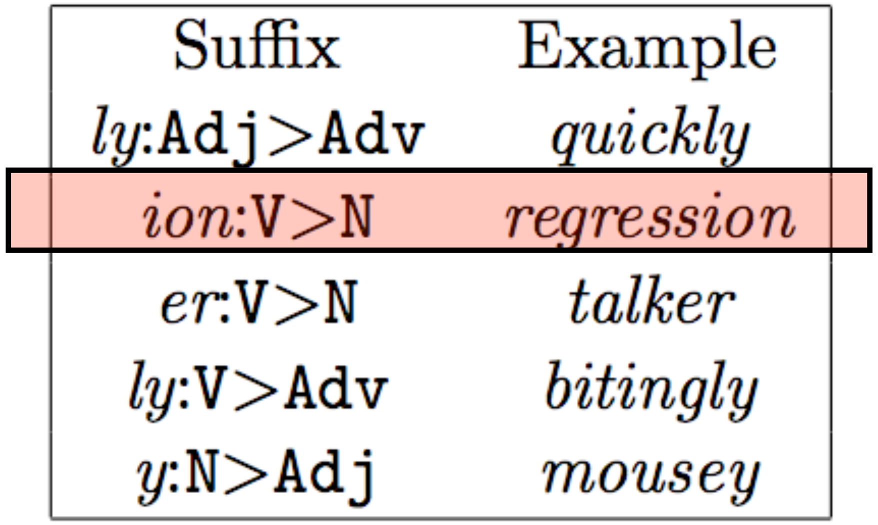
MAG *(Full-listing)*



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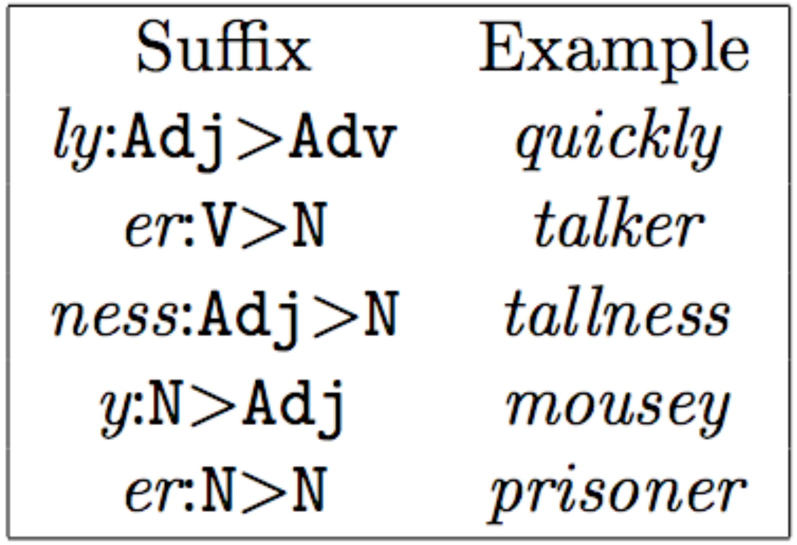
FG *(Inference-based)*



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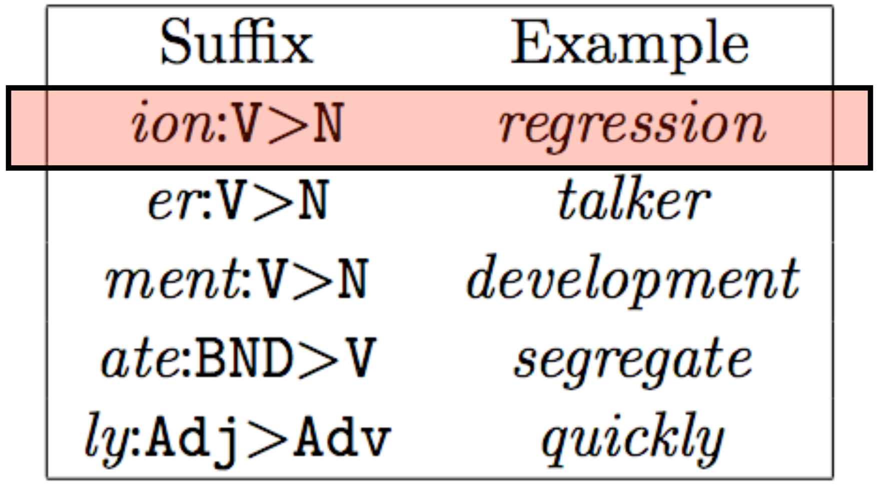

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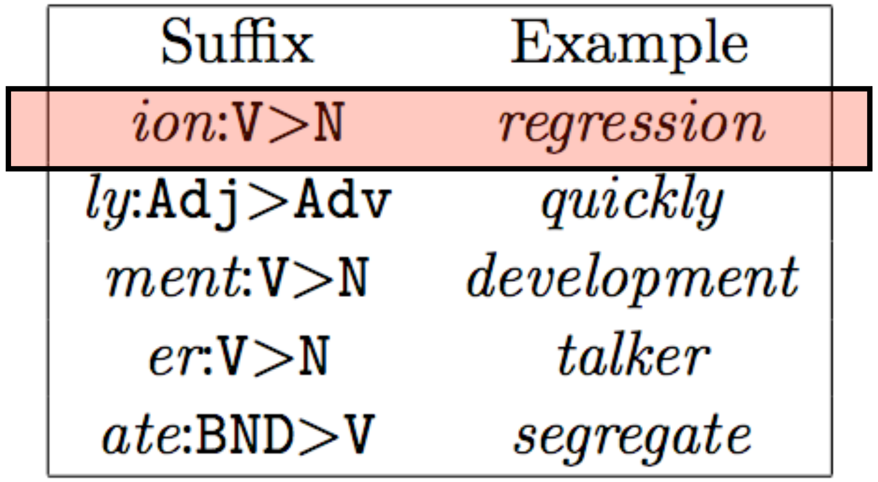
GDMN *(Exemplar)*



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er:V>N	*talker*
ate:BND>V	*segregate*

Productivity Analyses

1. Examples of highly productive suffixes.

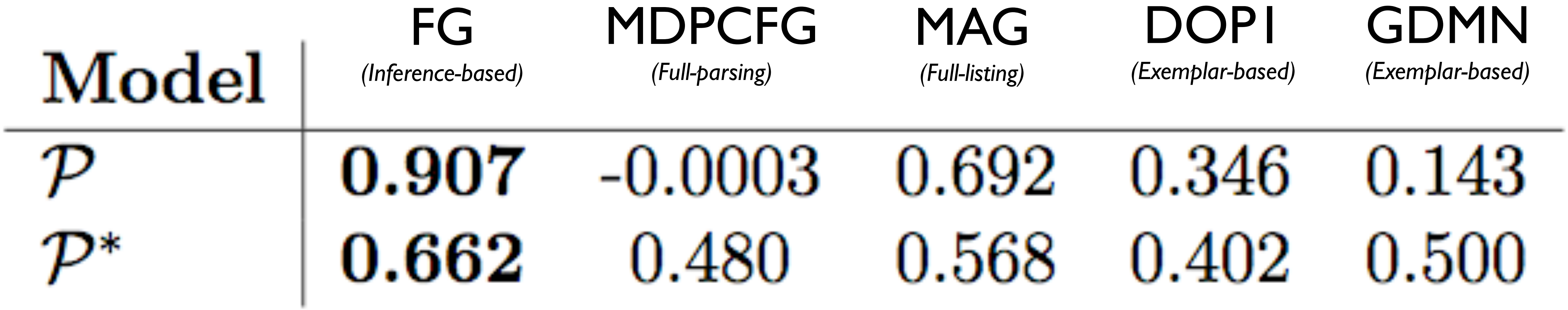
2. Convergence with other theoretical measures.

Baayen's Corpus-Based Measures

- Baayen's $\mathcal{P}$ / $\mathcal{P}^*$ (e.g., Baayen, 1992)
  - $\mathcal{P}$: Prob(NOVEL | SUFFIX) i.e. rate of growth of forms with suffix
  - $\mathcal{P}^*$: Prob(SUFFIX | NOVEL) i.e. rate of growth of vocabulary due to suffix

Productivity Correlations

($\mathcal{P}/\mathcal{P}^*$values from Hay & Baayen, 2002)



Model	FG *(Inference-based)*	MDPCFG *(Full-parsing)*	MAG *(Full-listing)*	DOP1 *(Exemplar-based)*	GDMN *(Exemplar-based)*
$\mathcal{P}$	**0.907**	-0.0003	0.692	0.346	0.143
$\mathcal{P}^*$	**0.662**	0.480	0.568	0.402	0.500

English Derivational Morphology

Productive	+ness *(goodness)*, +ly *(quickly)*
Semi-productive	+ity *(ability)*, +or *(operator)*
Unproductive	+th *(width)*

1. Individual suffix productivity differences (-ness/-ity/-th).
2. Suffix combinations.

Suffix Combinations

1. Suffix Ordering.

2. Generalization of Suffix Combinations.

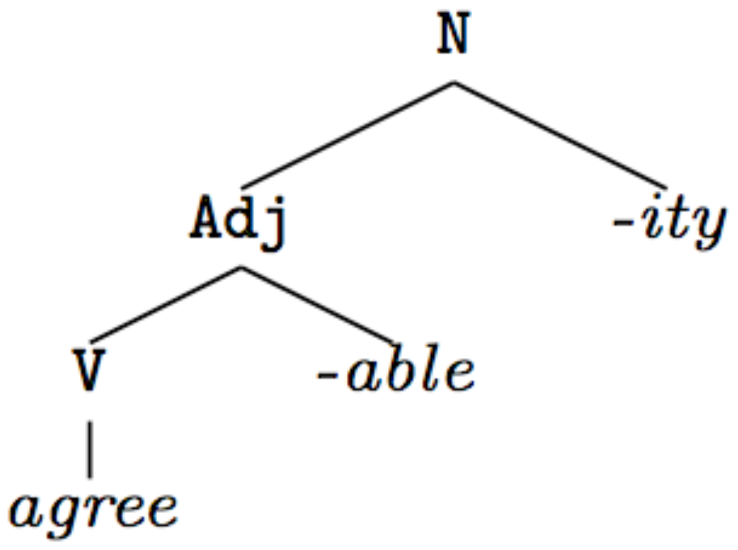
Suffix Combinations

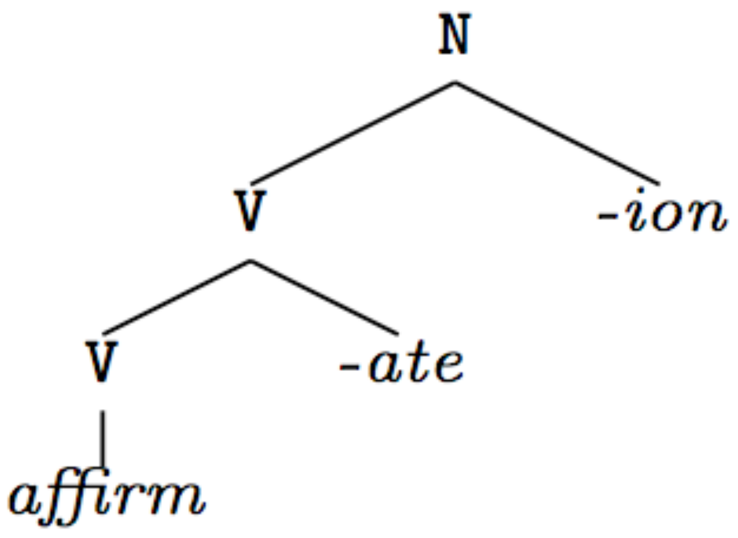
➡ 1. Suffix Ordering.

2. Generalization of Suffix Combinations.

Suffix Ordering

- Derivational morphology hierarchical and recursive.
  - Multiple suffixes can appear in a word.





Suffix Combinations

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- Many, many combinations of suffixes do not appear in words (even taking into account categories).

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    - 663 possible pairs.

Suffix Combinations

- Many, many combinations of suffixes do not appear in words (even taking into account categories).
  - Fabb (1988).
    - 43 suffixes.
    - 663 possible pairs.
    - Only 50 exist.

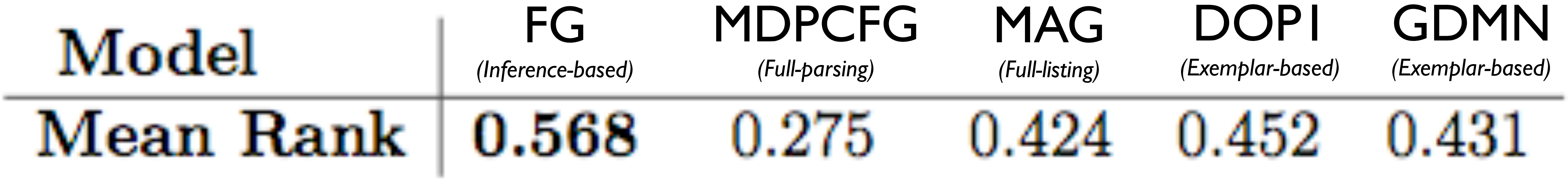
Complexity-Based Ordering (Hay, 2002)

On average, more productive suffixes appear after (outside of) less productive suffixes.

Measuring Ordering

- Examine attested orderings in corpus.
- *Mean rank* of each affix (Plag and Baayen, 2009).
  - Graph-theoretic statistic.
  - Measures degree to which each suffix tends to occur after other suffixes (on average).
- Compute *log odds* of suffix appearing second versus first for each model.

Mean Rank Correlations



Model	FG *(Inference-based)*	MDPCFG *(Full-parsing)*	MAG *(Full-listing)*	DOP1 *(Exemplar-based)*	GDMN *(Exemplar-based)*
Mean Rank	**0.568**	0.275	0.424	0.452	0.431

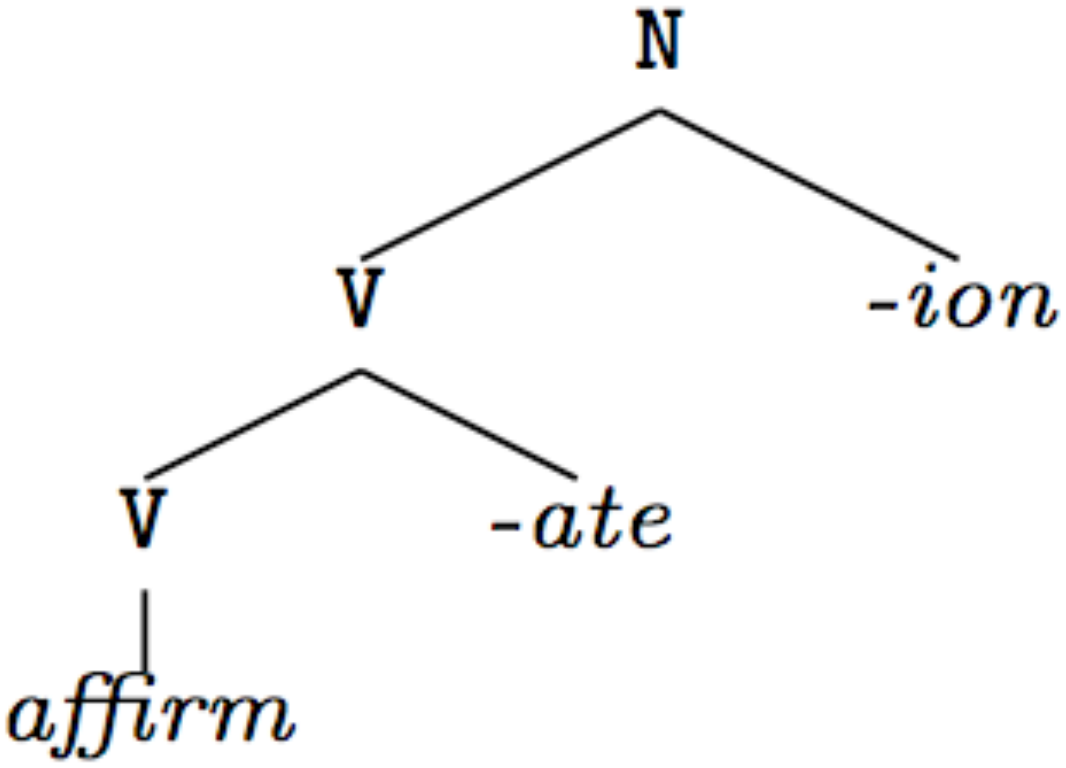
Suffix Combinations

1. Suffix Ordering.

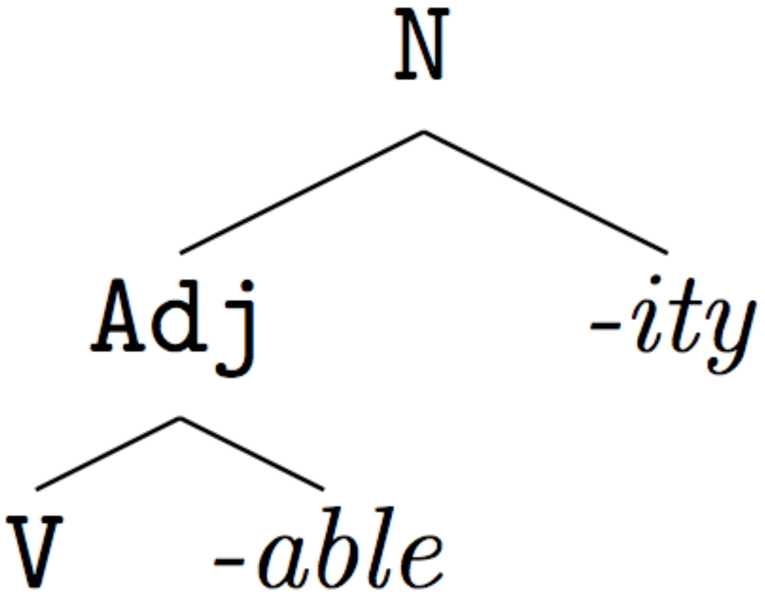
2. Generalization of Suffix Combinations.

Generalizable Combinations

Frozen Combinations

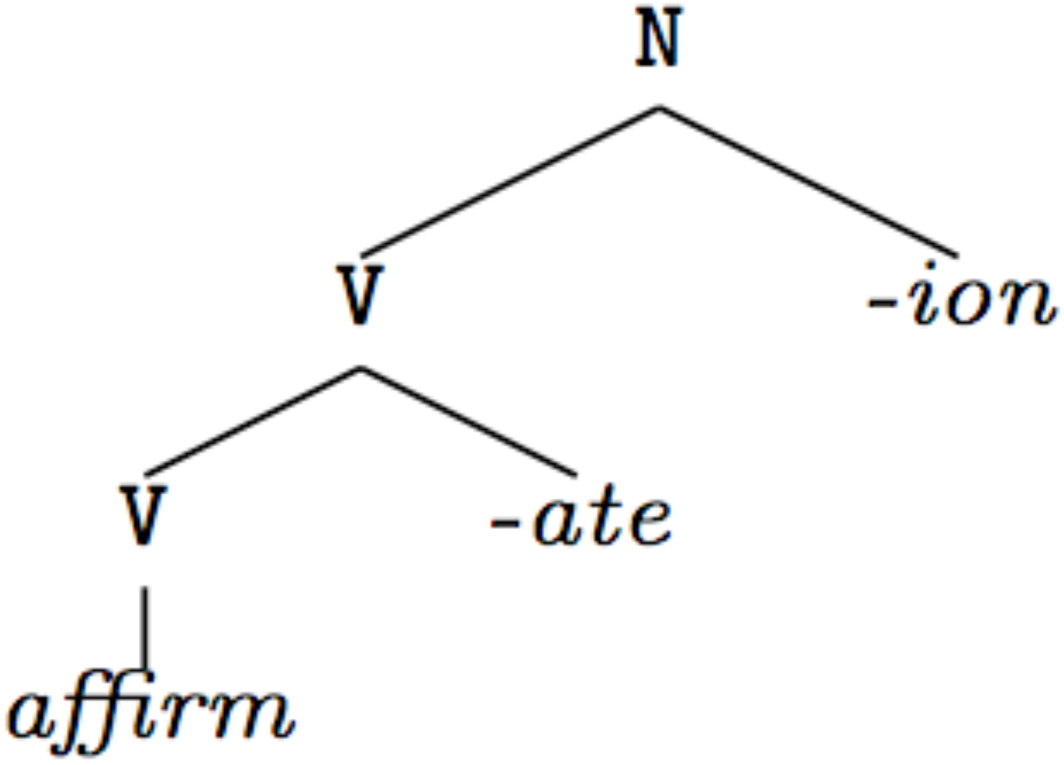


Generalizable Combinations

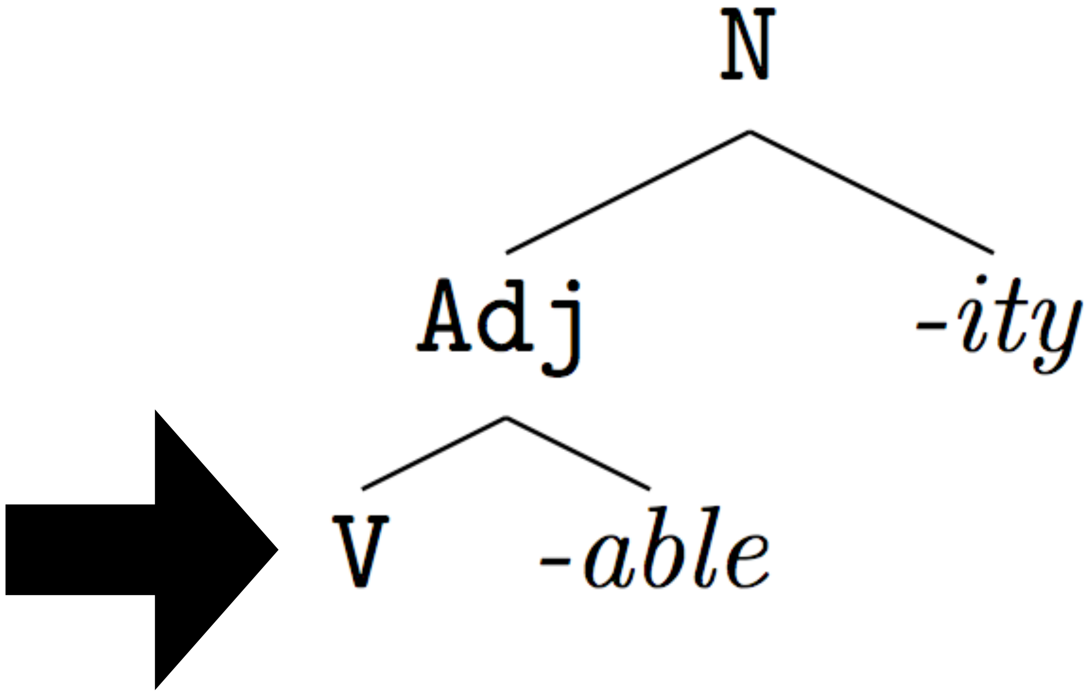


Generalizable Combinations

Frozen Combinations



Generalizable Combinations



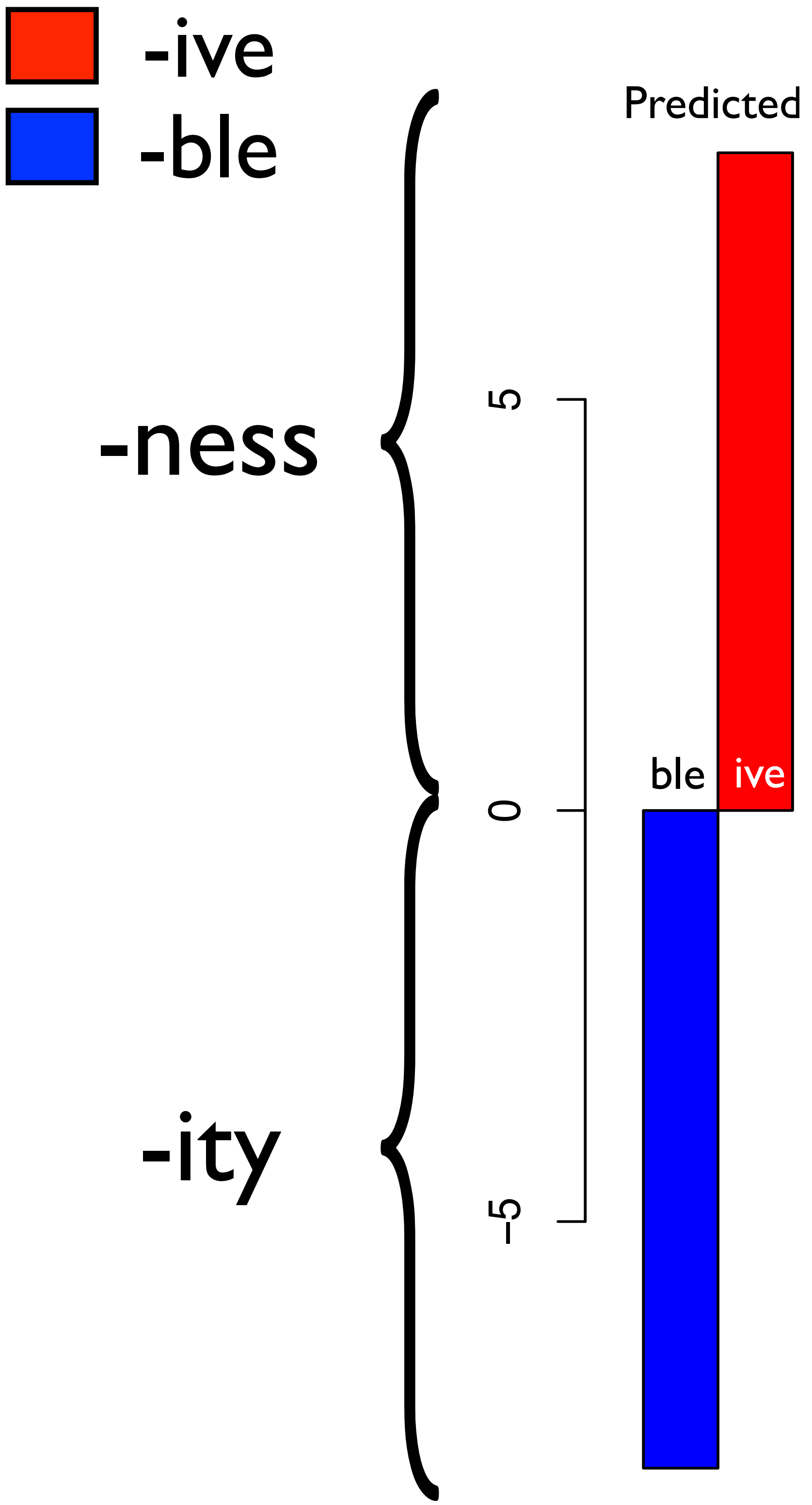
-ity v. -ness

- -ness more productive than -ity.
- -ity more productive than -ness after: -ile, -able, -(i)an, -ic.
  (Anshen & Aronoff, 1981; Aronoff & Schvaneveldt, 1978; Cutler, 1980)

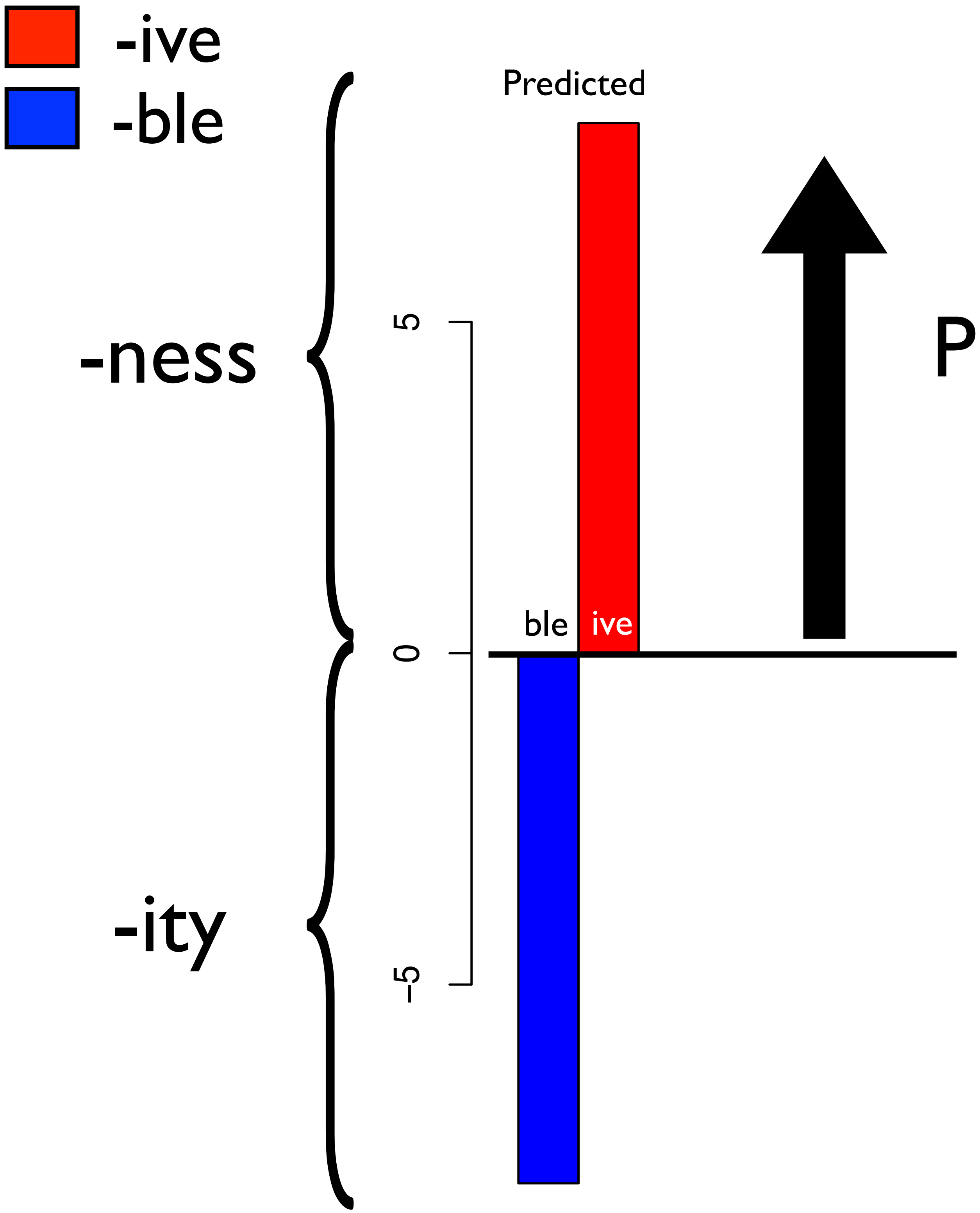
Two Frequent Combinations: -ivity v. -bility

- -ive + -ity: ***-ivity*** (e.g., selectivity).
  - Speaker prefer to use -ness with novel words (Aronoff & Schvaneveldt, 1978).
  - depulsiveness > depulsivity.
- -ble + -ity: ***-bility*** (e.g., sensibility).
  - Speakers prefer to use -ity with novel words (Anshen & Aronoff, 1981).
  - remortibility > remortibleness.

-ivity v. -bility

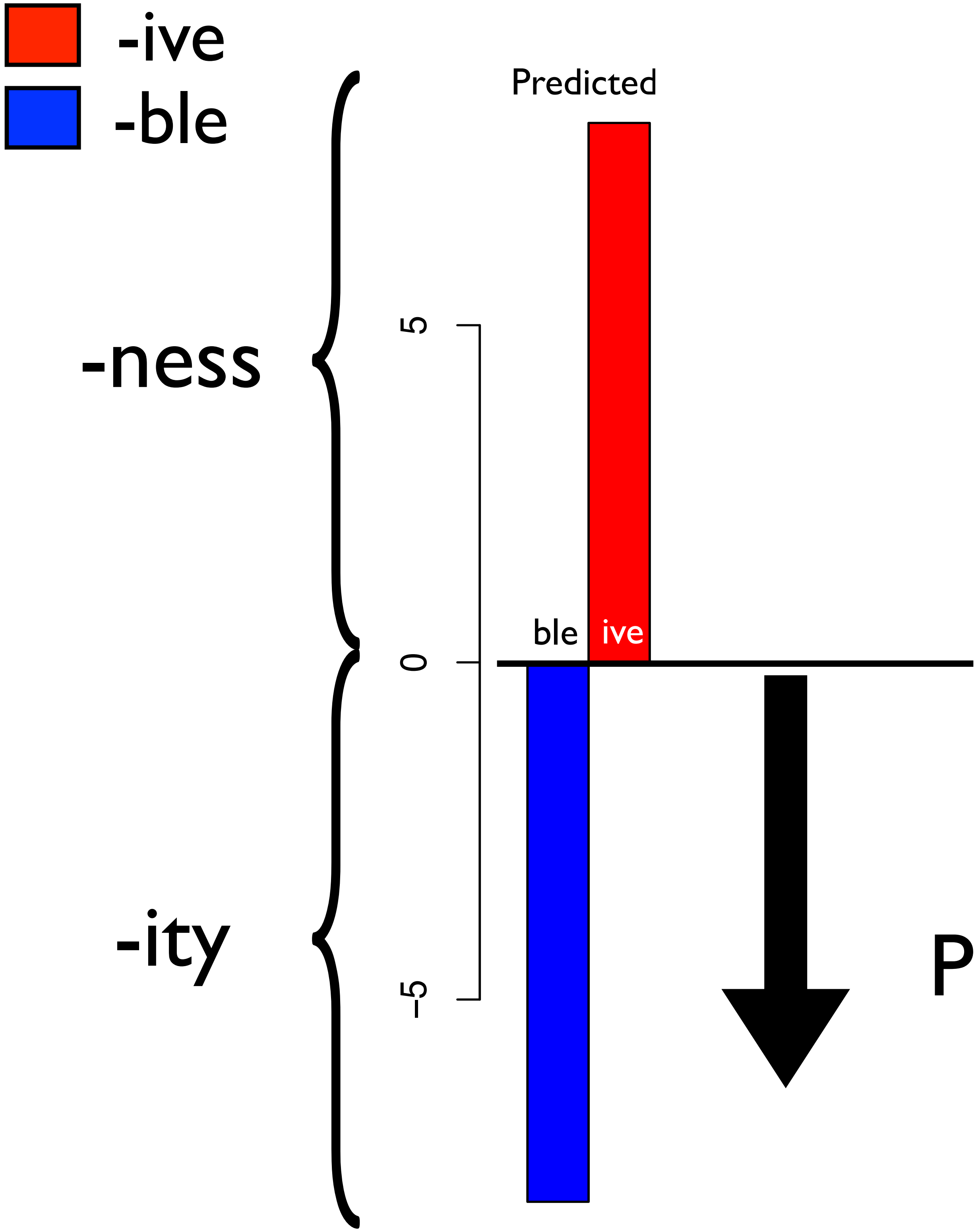


-ivity v. -bility



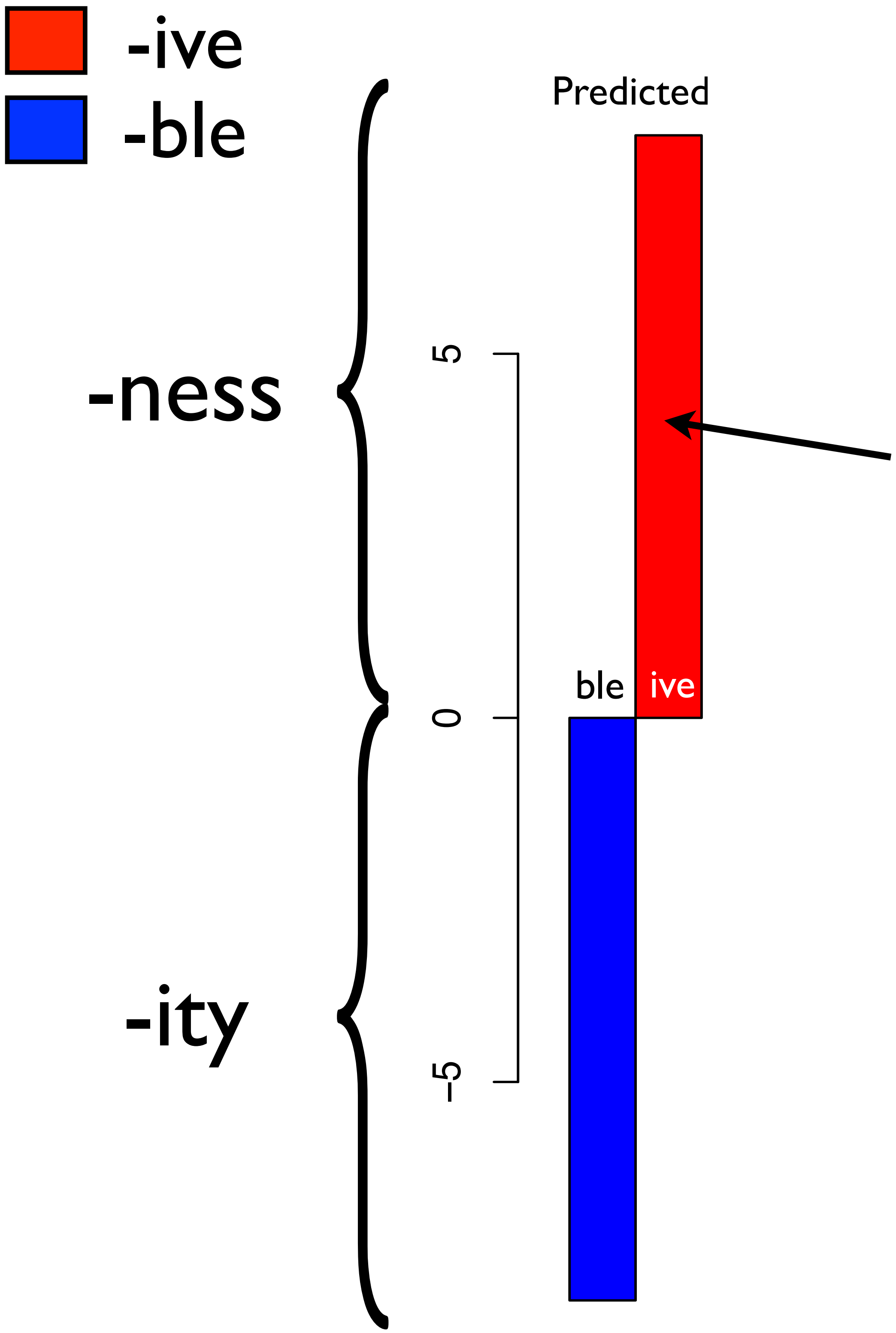
Preference for -ness

-ivity v. -bility



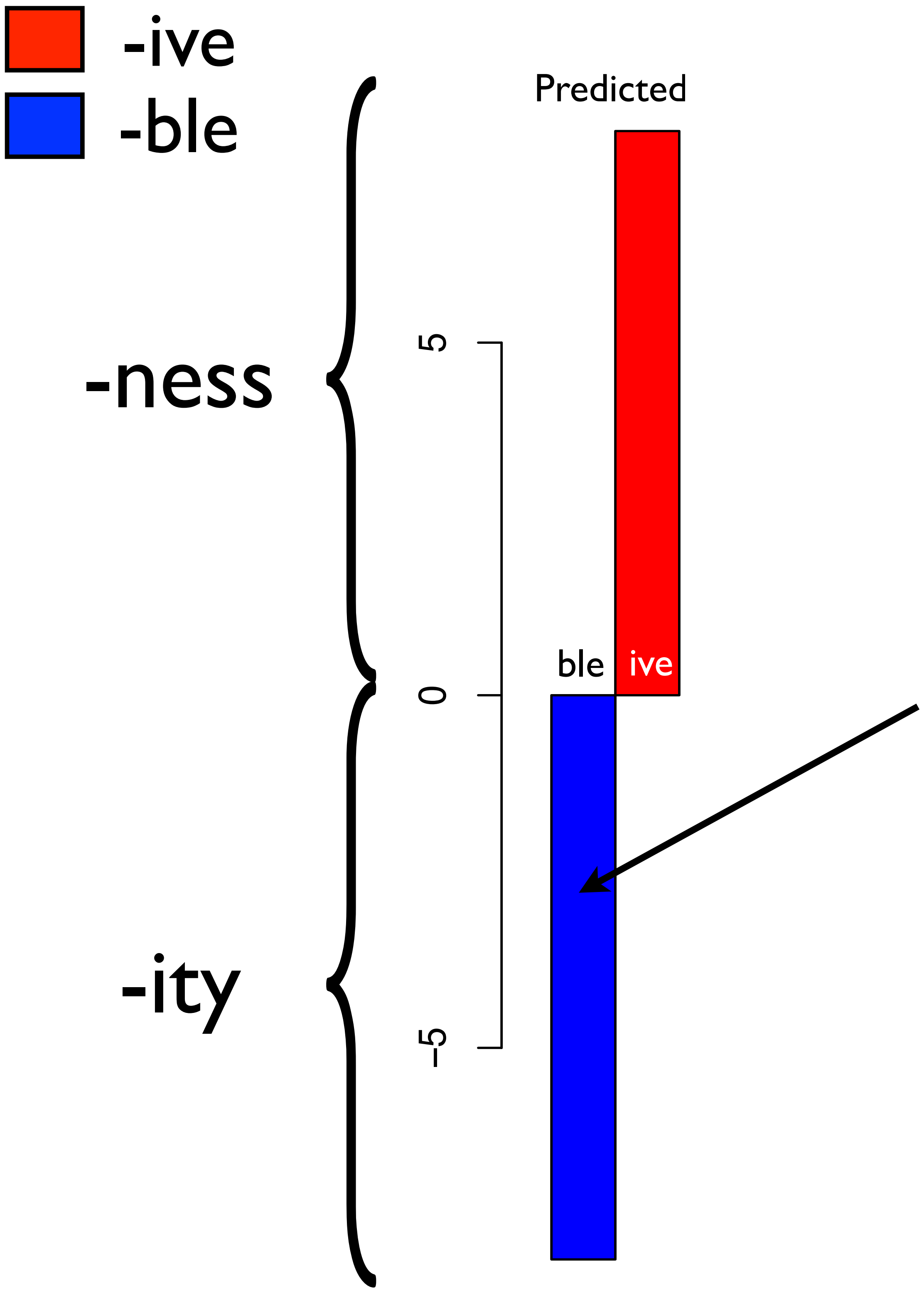
Preference for -ity

-ivity v. -bility



Preceding suffix -ive

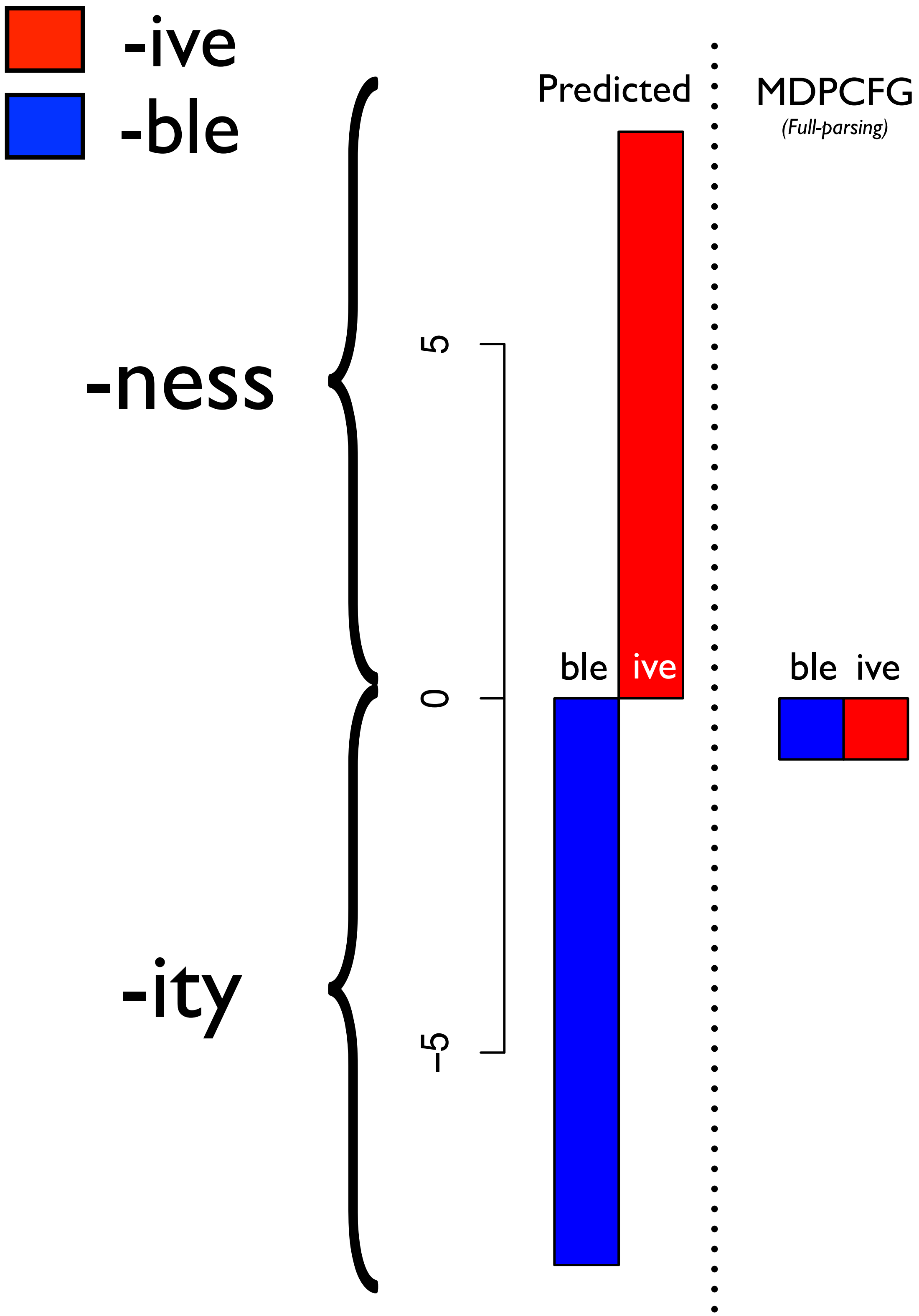
-ivity v. -bility



Preceding suffix -ble

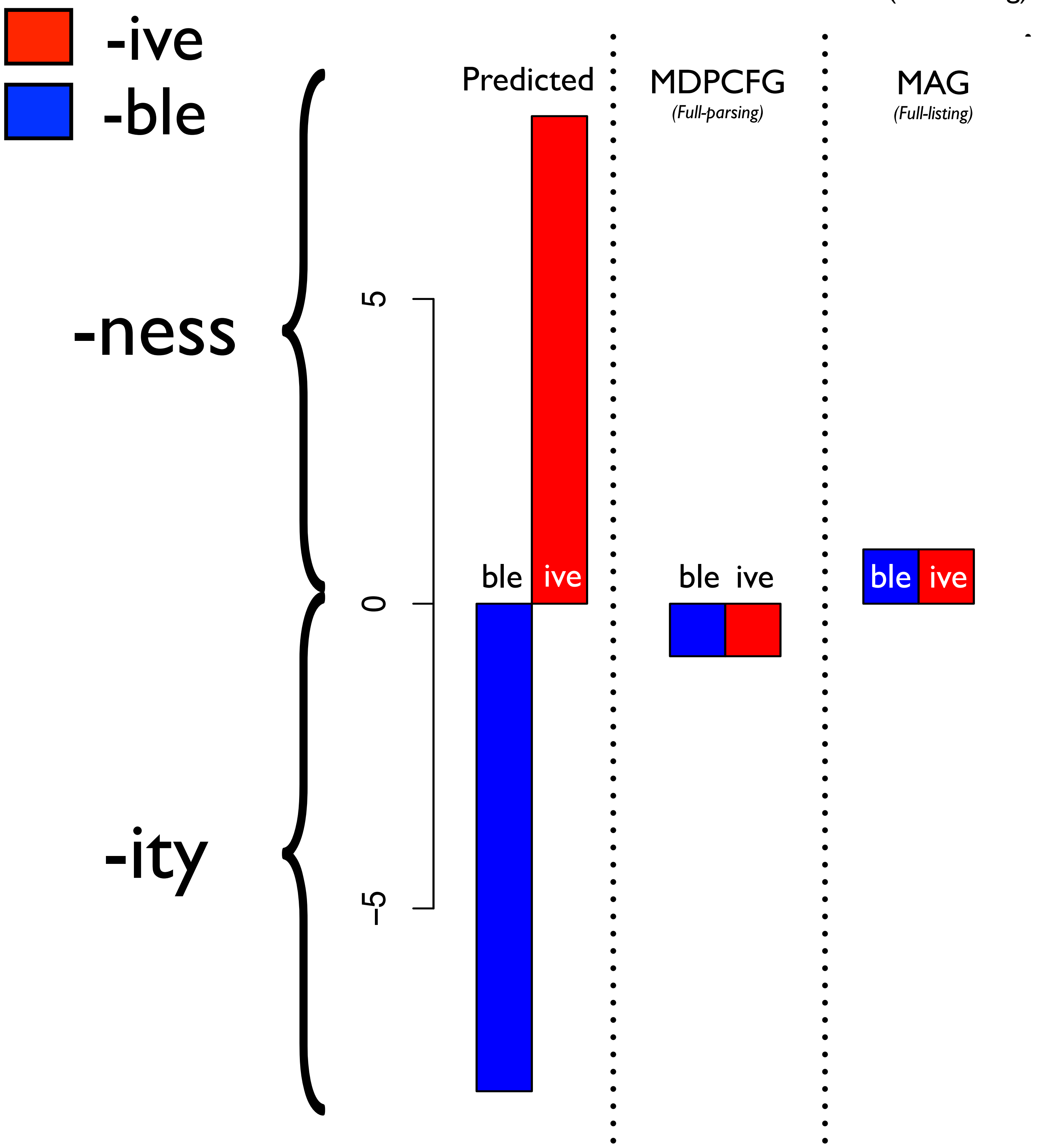
MDPCFG

(Full-parsing)



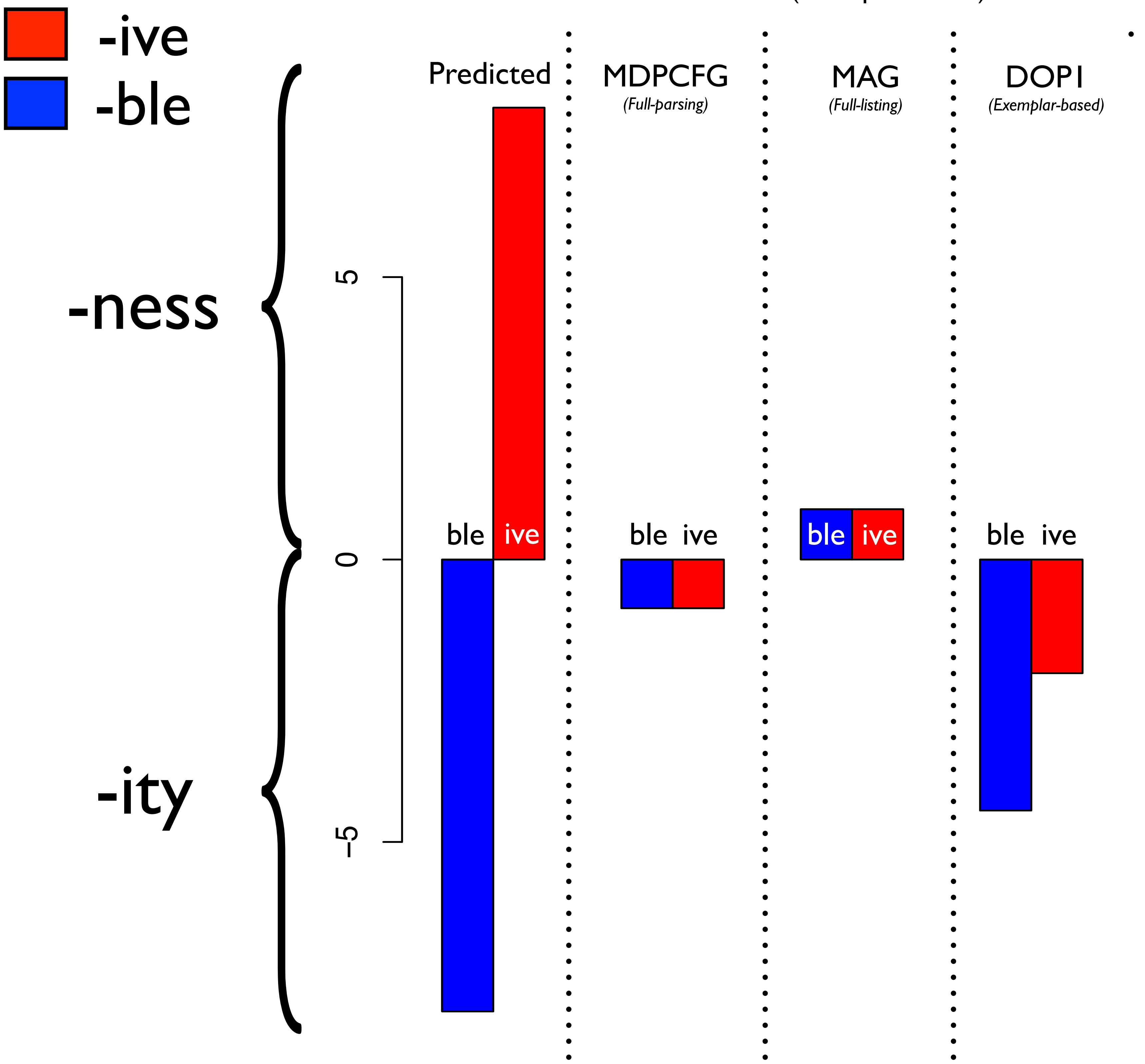
MAG

(Full-listing)



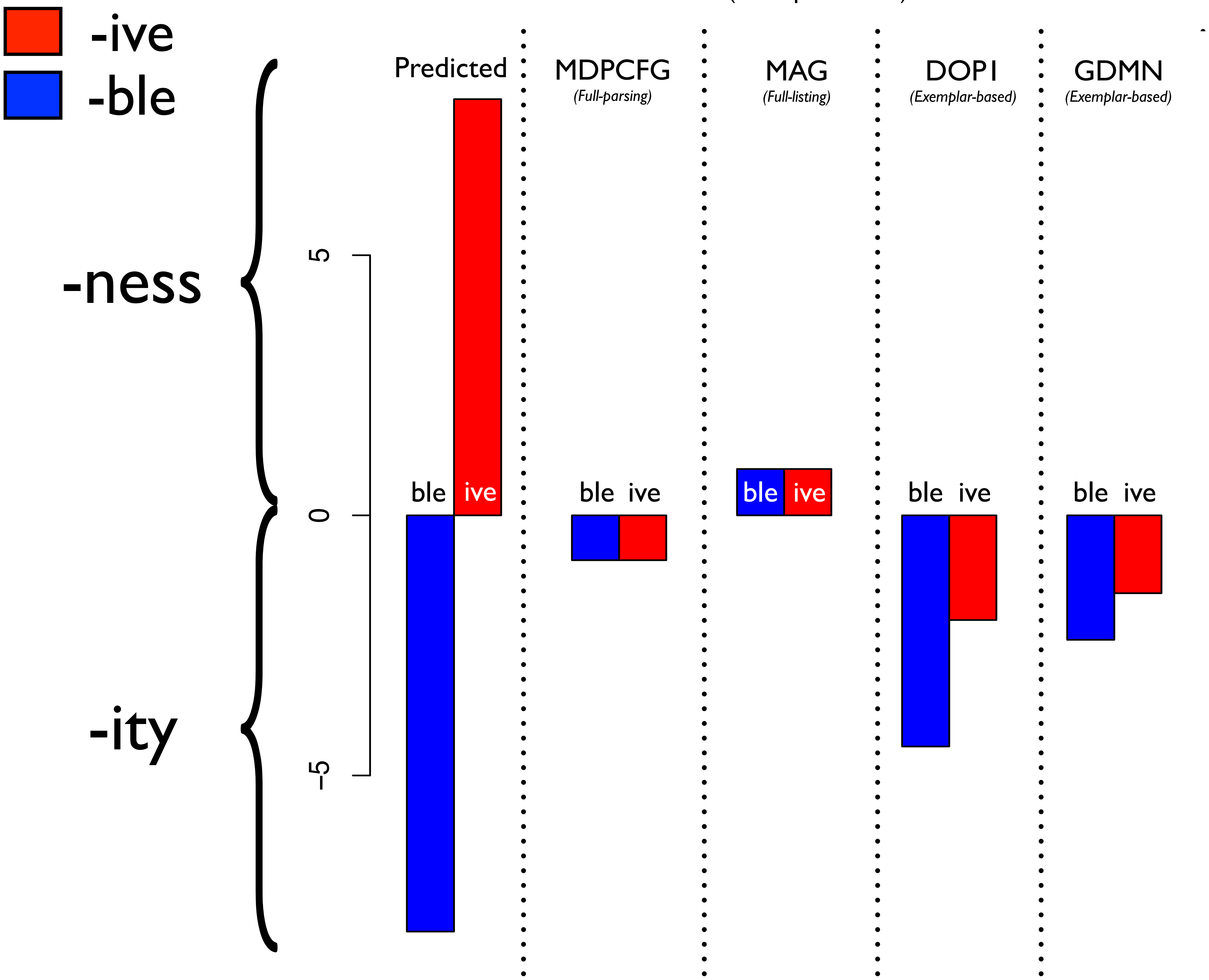
DOP1

(Exemplar-based)



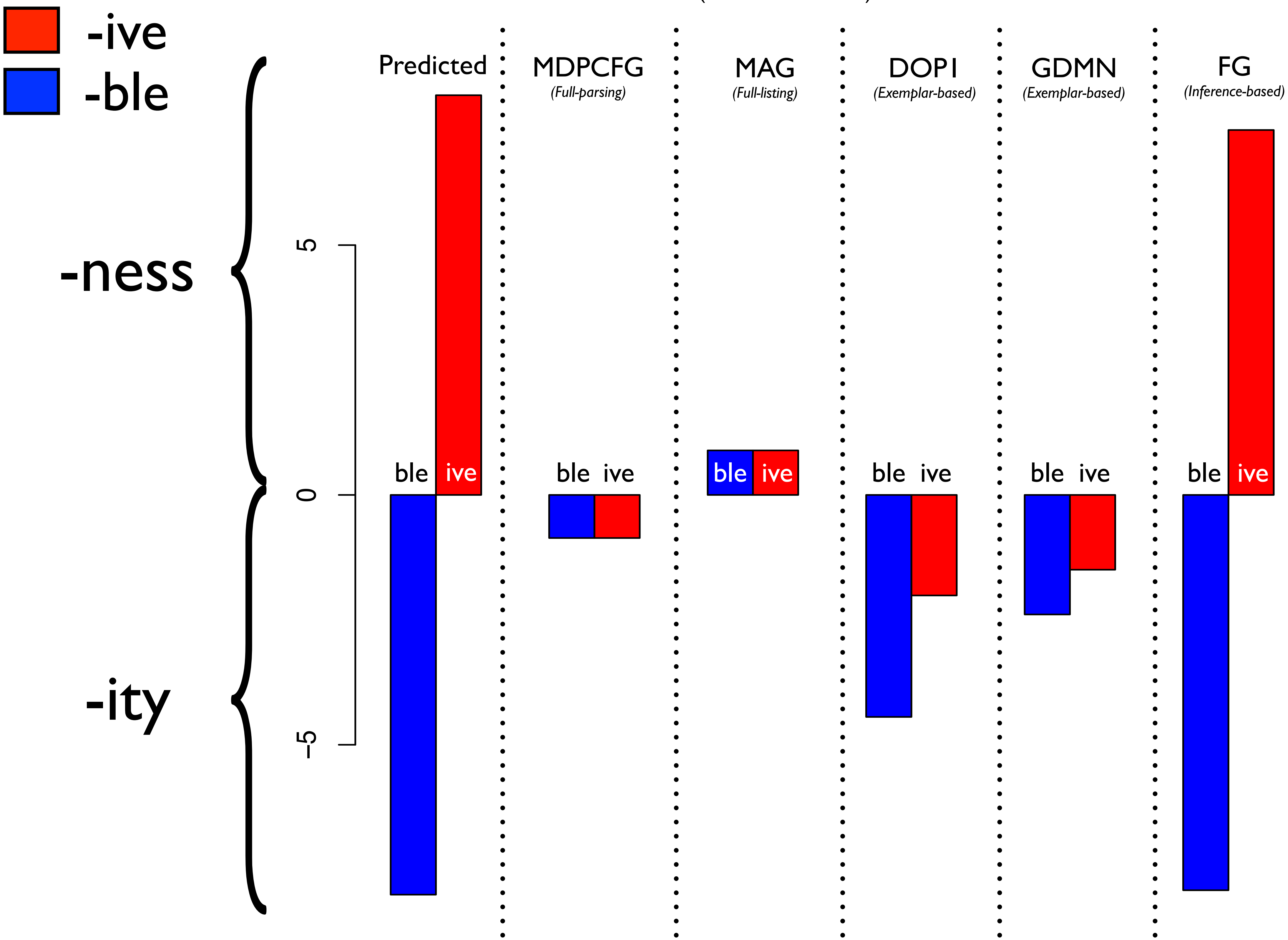
GDMN

(Exemplar-based)



FG

(Inference-based)



Discussion

- Inference-based approach able to correctly ignore high token frequency of -ivity because it balances a **tradeoff**.
- Other models use type or token frequencies.

Outline

1. The Proposal.
2. Five Models of Productivity and Reuse.
3. Empirical Evaluation

   The English Past Tense

   English Derivational Morphology
4. Conclusion

Conclusion

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- View productivity and reuse as an inference.

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- Link between theory of programming languages and Bayesian models.
- Able to capture dominant patterns **without** semantic and phonological structure.
    - Future work...

Thanks!