Two-stage Estimation of Real Estate Price Movements for High Frequency Tradable Indexes in a Scarce Data Environment

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Abstract

Indexes of commercial property prices face much scarcer transactions data than housing indexes, yet the advent of tradable derivatives on commercial property places a premium on both high frequency and accuracy of such indexes. The dilemma is that with scarce data a low-frequency return index (such as annual) is necessary to accumulate enough sales data in each period. This paper presents an approach to address this problem using a two-stage procedure with frequency conversion, by first estimating lower-frequency indexes staggered in time, and then applying a generalized inverse estimator to convert from lower to higher frequency return series. The two-stage procedure can improve the accuracy of high-frequency indexes in scarce data environments. In this paper the method is demonstrated and analyzed by application to empirical commercial property repeat-sales data.

Key Words:
Real estate price indexes; Frequency-conversion; Transactions-based-index estimation; Derivatives.

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1. Introduction & Background

In the world of transaction price indexes used to track market movements in real estate, it is a fundamental fact of statistics that there is an inherent trade-off between the frequency of a price-change index and the amount of "noise" or "error" in the individual periodic price-change or "capital return" estimates.[1] Geltner & Ling (2006) discussed the trade-off that arises, as higher-frequency indexes are more useful, but *ceteris paribus* are more noisy and noise makes indexes less useful. More generally, the fundamental problem is transaction data scarcity for index estimation, and this is a particular problem with commercial property price indexes, because commercial transactions are much scarcer than housing transactions. Also, sufficient good quality hedonic data is particularly lacking for most commercial properties, making repeat-sales indexes the only practical option in many circumstances, which can further reduce the sample size especially in the early years of an index. But the greater utility of higher frequency indexes has recently come to the fore with the advent of tradable derivatives based on real estate price indexes.[2] Tradability increases the value of frequent, up-to-date information about market movements, because the lower transactions and management

[1] The terms "noise" and "error" are used more or less interchangeably in this paper. They are placed in quotes because they are used in their sense as statistical terms of art, not with the same meaning as they are often used in common parlance. "Error" does not imply that there is any computational mistake or that anyone has done anything wrong, and "noise" does not imply any sound is made (obviously).

[2] Over-the-counter trading of the IPD Index of commercial property in the UK took off in 2004 and has been growing strongly since then. The S&P/Case-Shiller House Price Index (CSI) has been traded on the Chicago Mercantile Exchange since early 2006, while trading on the appraisal-based NCREIF Property Index (NPI) of commercial property in the US commenced in the summer of 2007. The Moody's/REAL Commercial Property Price Index, launched in September 2007 based on Real Capital Analytics Inc (RCA) data, is also designed to be a tradable index and is, like the CSI, a repeat-sales transaction price-based index.

costs of the derivatives compared to direct cash investment in physical property allows profit to be made synthetically based on the market movements tracked by the index.[3] Higher-frequency indexes also allow more frequent "marking" of derivatives contracts, which in turn allows smaller margin requirements, which increases the utility of the derivatives.[4]

Goetzmann (1992) introduced into the real estate literature what is perhaps the major approach to date for addressing small-sample problems in price indexes, the use of biased ridge or Stein-like estimators in a Bayesian framework. Other approaches that have been explored in recent years include various types of parsimonious regression specifications that effectively parameterize the historical time dimension, as well as procedures that make use of temporal and spatial correlation in real estate markets.[5] Some such techniques show promise, but are perhaps more appropriate in the housing market than in commercial property markets. Spatial correlation is more straightforward in housing markets, and the need for transparency in a tradable index can make it problematical to estimate the index on sales outside of the subject market segment. Another concern that is of particular importance in indexes supporting derivatives trading

[3] This is so even if the index underlying the derivative is lagged, as the derivative contracts' prices can reflect the effect of the lag. Thus, for example, movements in a high-frequency transactions price-based index can inform contemporaneous movements in the prices of continuously-traded derivatives based on a more sluggish or lower-frequency appraisal-based index such as the NPI. To illustrate, suppose the transaction index indicates the market dropped 4%. This might cause a 4% drop in the price of a one-year contract on the appraisal-based index if the lag in the index is less than one year, even though it might cause only a much smaller price drop in an appraisal-based contract that expires within a quarter or two (as the more sluggish index will take longer than that to register the full market movement).

[4] For example, margin requirements in a swap contract are dictated by the likely net magnitude of the next payment owed, which is essentially a function of the periodic volatility of the index, and volatility (per period) is a decreasing function of index frequency (simply because there is less time for market price change deviations around prior expectations to accumulate between index return reports that cover shorter time spans). Lower margin requirements allow greater use of synthetic leverage which facilitates greater liquidity in the derivatives market.

[5] See, for example, Schwann (1998), McMillen & Dombrow (2001), and Clapp (2004), among others. A recent overview is in Pace & LeSage (2004).

is that the index estimation procedure should minimize the constraints placed on the temporal structure and dynamics of the estimated returns series, allowing each consecutive periodic return estimate to be as independent as possible, in particular so as to avoid lag bias and to capture turning points in the market even if these are inconsistent with prior temporal patterns in the index.[6]

In the present paper we propose a two-stage estimation procedure in which, after a first-stage regression is run to optimize the index at a sufficiently *low frequency* to eliminate most noise (recognizing the scarce data environment), a second-stage regression is performed to convert a staggered series of such low-frequency indexes to a *higher-frequency index*. The proposed procedure is optimal in the sense that it minimizes additional (second stage) noise and does not introduce artificial index price lag bias or smoothing (as would be the case with simple smoothing or rolling techniques, and with some of the time-parameterization techniques noted earlier).[7]

While the resulting high-frequency index does not have as high a signal/noise ratio (SNR) as the underlying low-frequency indexes, it can have a higher SNR than direct high-frequency estimation, with the advantage of the greater utility of the higher frequency. This suggests that it may be useful in the information marketplace to publish both the staggered low-frequency indexes and the higher-frequency second-stage derived index as supplemental information, either alone or along with directly-estimated high-frequency indexes, at least in markets where data is too scarce to rely solely on directly-

[6] This is particularly important to allow the derivatives to hedge the type of risk that traders on the short side of the derivatives market are typically trying to manage. For example, developers or investment managers seek to hedge against exposure to unexpected and unpredictable downturns in the commercial property market.

[7] While in the present paper we apply the second-stage frequency conversion to traditional repeat-sales indexes estimated at the lower frequency, in principle the 2-stage procedure can be applied with any methodology for estimating the first-stage (lower-frequency) indexes as long as the lower-frequency estimation can be staggered regularly in time.

estimated high-frequency indexes. However, we also note some anecdotal evidence that annual indexes and 2-stage quarterly indexes based on them may have more difficulty than direct-quarterly estimation in fully tracking the early stages of a sharp market downturn.

2. Two-stage Estimation & the Moore-Penrose Pseudoinverse

In this section we will introduce the two-stage/frequency-conversion procedure for deriving a quarterly-frequency index from four underlying staggered annual-frequency indexes. This perspective is taken for illustrative purposes, as other frequency conversions are equally possible in principle (e.g., from quarterly to monthly).

As noted, commercial property transaction price data in particular is scarce (e.g., compared to housing data). To the extent the market wants to trade specific segments, such as, say, New York office buildings, the transaction sample becomes so small that we may need to accumulate a full year's worth of data before we have enough to produce a good transactions-based estimate of market price movement. This is the type of context in which we propose the following two-stage/frequency-conversion procedure to produce a quarterly index.

2.1 The Proposed Methodology:

We begin by estimating annual indexes in four versions with quarterly staggered starting dates, beginning in January, April, July, and October. We label these four annual indexes as "CY", "FYM", "FYJ", and "FYS" to refer to "calendar years" and "fiscal years" identified by their ending months. Each index is a true annual index, not a rolling or moving average within itself, but consisting of independent consecutive annual returns.[8] The result will look something like what is pictured in Exhibit 1 for an example index based on the Real Capital Analytics repeat-sales database for New York City area office property. If properly specified, these annual indexes generally have essentially no lag bias.[9] Each of these indexes also has as little noise as is possible given the amount of data that can be accumulated over the annual spans of time.[10]

Insert Exhibit 1 about here.

Next, a frequency-conversion is applied to this suite of annual-frequency indexes to obtain a quarterly-frequency price index implied by the four staggered annual indexes. We want to perform this frequency conversion in the most accurate way possible, with as little additional noise and bias as possible. How can we use those staggered annual indexes to derive an up-to-date quarterly-frequency index? Looking at the staggered annual-frequency index levels pictured in Exhibit 1, one is tempted to try to construct a quarterly-frequency index by simply averaging across the levels of the four indexes at each point in time. (Try to fit a curve "between" the four index levels.) But such a process would entail a delay of three quarters in computing the most recent quarterly

[8] That is, independent *within* each index. Obviously, there is temporal overlap *across* the indexes.
[9] Time-weighted dummy variables are used to eliminate temporal aggregation. For example, for the calendar year (CY) index beginning January 1st, a repeat-sale observation of a property that is bought September 30 2004 and sold September 30 2007 has time-dummy values of zero prior to CY2004 and subsequent to CY2007, and dummy-variable values of 0.25 for CY2004, 1.0 for CY2005 and CY2006, and 0.75 for CY2007. This specification, attributable to Bryon & Colwell (1982), eliminates the averaging of the values across the years, and pegs the returns to end-of-year points in time. See Geltner & Pollakowski (2007) for more details. However, it should be noted that in the early stages of a sharp downturn in the market, loss aversion behavior on the part of property owners cause a data imbalance that can make it difficult for an annual-frequency index to fully register the downturn at first. (See subsequent discussion.)
[10] A noise filter such as ridge regression can be helpful in this regard (as introduced into the real estate literature by Goetzmann, 1992), and has in fact been used in the indexes shown. As noted, the two-stage procedure does not constrain the methodology used to estimate the lower-frequency indexes, and various procedures might be employed to optimize the estimation of the lower-frequency indexes.

return (while we accumulate all four annual indexes spanning that quarter), which for derivatives trading purposes would defeat the purpose of the higher-frequency index. Such a levels-averaging procedure would also considerably smooth the true quarterly returns (it would effectively be a time-centered rolling average of the annual returns).

The approach we propose to the frequency-conversion procedure is a second-stage "repeat-sales" regression at the quarterly frequency using the four staggered annual indexes as the input data.[11] Each annual return on each of the four staggered indexes is treated as a "repeat-sale" observation in this second-stage regression. If we have T years of history, we will have 4T-3 such "repeat-sales" observations (the row dimension of the second-stage regression data matrix), and we will have 4T quarters for which we have time-dummies (the columns dimension in the regression data matrix, the quarters of history for which we want to estimate returns). We are missing "1st-sales" observations for the first three quarters of the history, the quarters that precede the starting dates for all of the annual indexes other than the one that starts earliest in time (the CY index in our present example), as the staggered annual indexes each must start one quarter after the previous. Obviously, with fewer rows than columns in the estimation data matrix, our regression is "under-identified", that is, the system has fewer equations than unknowns.[12] Basic linear algebra tells us that such a system has an infinite number of exact solutions (that is, quarterly index return estimates that will cause the predicted values to exactly match the "repeat-sales" observations on the left-hand-side of the second-stage regression, i.e., a regression R^2 of 100%, a perfect fit to the data, which is the low-

[11] As noted, the input annual indexes from the first stage do not need to be repeat-sales indexes, in principle. They could be any good transactions-price based type of index, such as a hedonic index.
[12] We cannot simply drop out the first three quarters from the second-stage index, as that will then impute the first three annual returns entirely to the first quarter (only) of the index history and thereby bias the estimation of all of the quarterly returns.

frequency index returns). However, of all of those infinite solutions, there is a particular solution that minimizes the variance of the estimated parameters, i.e., that minimizes the additional noise in the quarterly returns, noise added by the frequency-conversion procedure. This solution is obtained using what is called the "Moore-Penrose pseudoinverse" matrix of the data. This solution is "best" in the sense that it has the least variance (is most "precise") and the least bias possible for a linear estimator. We shall refer to this frequency-conversion method as the "Moore-Penrose Generalized Inverse Estimator", or MPGIE for short.[13]

How good is the MPGIE as a frequency-conversion method? For practical purposes, it is quite good. It adds effectively very little noise or bias to the annual returns. This can be seen by numerical simulation. Exhibit 2 depicts a typical randomly-generated history of true quarterly market values (the thick black line, which in the real world would be unobservable), the corresponding staggered annual index levels (thin, dashed lines, here without any noise, to reveal the noise added purely by the frequency-conversion second stage), and the resulting second-stage MPGIE-estimated quarterly index levels (thin red line with triangles, labeled "ATQ" for "Annual-to-Quarerly").[14] Clearly, the derived MPGIE quarterly index almost exactly matches the true quarterly market value levels. Numerous simulations of random histories and varying market patterns over time give similar results to those depicted in Exhibit 2. The MPGIE-based

[13] See Appendix A for an introduction to the Moore-Penrose pseudoinverse and its role in solving the under-identified system. See Appendix B for a discussion of bias in the resulting estimator. As noted in Appendix A, it can be shown that, while the Moore-Penrose pseudoinverse estimator of the under-identified system is biased, the bias is minimized among the class of linear estimators (Chipman, 1964). That is, the Moore-Penrose is "Best Linear Minimum Bias Estimator" (BLMBE).

[14] In Exhibit 2 the first (CY) annual index starts arbitrarily at a value of 1.0, and the subsequent three staggered annual indexes are pegged to start at the interpolated level of the just-prior annual index at the time of the subsequent index's start date. This is merely a convention and does not impact the quarterly return estimates, as all indexes are only indicators of relative price movements across time, not absolute price levels.

frequency-conversion in itself appears to add only a little noise or bias to the staggered annual indexes.

Insert Exhibit 2 about here.

2.2 An Illustration of the Setup for the 2^{nd} Stage Regression

To clarify and summarize the proposed procedure, consider this illustration. Suppose the following staggered annual returns were estimated from the 1^{st} stage regression:

Annual Period	CY-Returns	Annual Period	FYM-Return	Annual Period	FYJ-Return	Annual Period	FYS-Return
1Q00-4Q00	a	2Q00-1Q01	b	3Q00-2Q01	c	4Q00-3Q01	d
1Q01-4Q01	e						

As shown below, the left-hand side variable for the 2^{nd} stage regression will be the stack of annual returns (left-most column in the data table) and the right-hand side variables would be time-dummies that are set equal to 1 for the four quarters that make up a particular annual return observation (remaining columns in the table below).

Stacked-Returns	1Q 2000	2Q 2000	3Q 2000	4Q 2000	1Q 2001	2Q 2001	3Q 2001	4Q 2001
a	1	1	1	1	0	0	0	0
b	0	1	1	1	1	0	0	0
c	0	0	1	1	1	1	0	0
d	0	0	0	1	1	1	1	0
e	0	0	0	0	1	1	1	1

As seen above, there are more quarterly returns to estimate than there are staggered annual return observations. Specifically, for the ATQ frequency conversion, there are always 3 extra parameters to estimate. Appendix A outlines the method used for this estimation and Appendix B shows a way to see that the bias resulting from such an estimation decreases as T becomes large. Indeed, intuitively, the reader can convince oneself that as T becomes large, the percentage difference between T and T + 3 decreases. Hence, the system gets closer to being effectively identified and thus the bias goes down over time. From the simulation analysis in Exhibit 2, it can be seen that with even small values of T, the amount of the bias is small. It should also be noted that since the 2nd-stage regression fits the stacked returns observations exactly, any noise in the estimation of the staggered annual returns gets carried over to the estimated quarterly returns. Thus, there is a direct relationship between statistically reliable 1st and 2nd stage estimations.

2.3 General Characteristics of the Resulting Derived Quarterly Index (ATQ):

Based on the foregoing argument, the 2-stage derived quarterly index (which we shall refer to here as the "ATQ", for "annual-to-quarterly") offers the prospect of being more precise than a directly-estimated single-stage quarterly index. However, in general it cannot be as "good" as the annual-frequency indexes if we define the index quality by the signal/noise ratio. However, the ATQ will provide information more frequently than the annual indexes, and this may make the trade-off worthwhile. To see this, consider the following.

In the signal/noise ratio (SNR) the numerator is defined theoretically as the periodic return volatility (longitudinal standard deviation) of the (true) market price changes, and the denominator is defined as the standard deviation of the error in the estimated periodic returns.[15] The MPGIE frequency-conversion procedure gives a SNR denominator for the ATQ which is not much larger than that of the underlying annual-frequency indexes (the standard deviation of the error in the second-stage quarterly return estimates is not much larger than that in the first-stage annual return estimates, as evident in the simulation depicted in Exhibit 2 by the fact that the ATQ adds very little error). But the numerator of the SNR is governed by the fundamental dynamics of the (true) real estate market. These dynamics dictate that the periodic return volatility will be smaller for higher frequency returns. For example, if the market follows a random walk (serially uncorrelated returns), the quarterly volatility will be 1/SQRT(4) = 1/2 the annual volatility. This means that, even if the SNR denominator did not increase at all, the SNR in the ATQ would be one-half that in the underlying annual indexes. If the market has some sluggishness or inertia (positive autocorrelation in the quarterly returns) then the SNR will be even more reduced in the ATQ below that in the annual indexes.

Importantly, the SNR of the 2-stage ATQ can still be greater than that of a directly-estimated (single-stage) quarterly index. To see this, suppose price observations occur uniformly over time. Then there will be four times as much data for estimating the typical annual return in the annual-frequency indexes compared to the typical quarterly return in the directly-estimated quarterly-frequency index. By the basic "Square Root of

[15] The theoretical SNR cannot be observed or quantified in the real world, where the true market returns cannot be observed, and hence the true market volatility (SNR numerator) cannot be observed. Empirical estimates of the theoretical SNR are confounded by the fact that the volatility of any empirically estimated index will itself be "contaminated" by the noise in the estimated index. Furthermore, the denominator of the theoretical SNR should equal the theoretical cross-sectional standard deviation in the return estimates, which is not exactly what is measured by the regression's standard errors of its coefficients. To see this, consider conceptually a "perfect" index whose return estimates always exactly equal the unobservable true market returns each period. The regression producing such an index would have zero in the denominator of its SNR and yet would still have positive standard errors for its coefficients for any finite estimation sample, as there is noise in the estimation database, causing the regression to have non-zero residuals in the data. In spite of these practical limitations, the theoretical SNR is a useful construct for conceptual analysis purposes (and also in simulation analysis, where "true" returns can be simulated and observed).

N Rule" of statistics, this implies that the directly-estimated quarterly index will tend to have SQRT(4) = 2 times greater standard deviation of error in its (quarterly) return estimates than the annual indexes have in their (annual) return estimates. Thus, the SNR for the direct quarterly index will have a denominator twice that of the annual indexes. This compares to the 2-stage ATQ whose SNR denominator may be only slightly greater than in the annual indexes (as was suggested in the earlier section, depicted in Exhibit 2). Of course, either way of producing a quarterly index will still be subject to the same numerator in the theoretical SNR, which is purely a function of the true market volatility. Thus, while the 2-stage ATQ will have a lower SNR than the underlying annual-frequency indexes, it may have a higher SNR than a directly-estimated (1-stage) quarterly index. In data-scarce situations, this can make an important difference.[16]

Importantly, while the ATQ does not have as good a SNR as the annual indexes, it does provide more frequent returns than the annual indexes (quarterly instead of annual), and thereby does provide additional information.[17] Thus, there is a useful trade-off between the staggered annual indexes and the derived ATQ. The ATQ gives up some SNR information usefulness in the accuracy of its return estimates, but in return provides higher frequency return information.

[16] Formal definition and computation of the "standard error" for the MPGIE second-stage regression is not straightforward, and is not attempted here. As noted, the regression is under-identified, which means there are no "residuals" in the second stage. It is not clear how to define a "standard error" metric in a manner that would be practical for empirical computation in the real world and that would also be meaningful for our present purposes. A key problem is that the standard deviation of the regression residuals does not generally correspond to the type of "error" that is meaningful in a price index, which is the standard deviation of the error in the estimate of the coefficient (each individual return).

[17] Among the four staggered annual indexes we do get new information every quarter, but that information is only for the entire previous 4-quarter span, which is not as useful as information about the most recent quarter itself, which is what is provided by the ATQ. For example, a turning point in the most recent quarter will not necessarily show up in the most recent annual index, as the latter is still influenced by market movement earlier in the 4-quarter span it covers.

This trade-off suggests that it may make sense in practical applications to produce and publish *both* the staggered annual indexes and their implied MPGIE-based ATQ quarterly index. Whether the real estate derivatives market will want to actually trade contracts written on the ATQ is a question that only the market can decide. But in any case the ATQ will provide information useful to market participants and analysts.

2.4 An Illustrative Example of Annual-to-Quarterly Derivation in Data-Scarce Markets:

To gain a more concrete feeling for the above-described methodology and application, consider one of the smaller (and therefore more data-scarce) markets among the 29 Moody's/REAL Commercial Property Price Indexes that are based on the RCA repeat-sales database: New York City metro area office properties.[18]

Insert Exhibit 3 about here.

First consider a specific example of how the ATQ works. In Exhibit 3, note how the ATQ is generally consistent with the annual returns that span each quarter. However, the quarterly index picks up the changes implied by *changes* in the staggered annual indexes. For example, while the FYJ annual index ending at the end of 2008Q2 was positive (up 1.6%), it was less positive than the immediately preceding FYM annual index ending in 2008Q1 (up 12.2%). The resulting derived ATQ quarterly index indicated a downturn in 2008Q2 (-4.3%).

[18] The Moody's/REAL Commercial Property Price Index is produced by Moody's Investor Services under license from Real Estate Analytics LLC (REAL). During the 2006-08 period the New York Office index averaged 24 repeat-sales transaction price observations (second-sales) per quarter, and in the most recent quarter (3Q08) there were only 11 observations.

At first it may seem odd that the derived quarterly index can be negative when the annual index that underlies it is positive. The intuition behind a result such as the above example is that an annual index could still be increasing as a result of rises during the earlier quarters of its 4-quarter time-span, with a drop in the last quarter that does not wipe out all of the previous three quarters' gains. For example, suppose the following are the true quarterly changes during the past 5 quarters: 07Q2 = +3%, 07Q3 = +3%, 07Q4 = +3%, 08Q1 = +3%, 08Q2 = -4%. Then the FYM08 annual index covering 07Q2-08Q1 would show +12% (ending 3/31/08), and the FYJ08 annual index covering 07Q3-08Q2 would show +5% (ending 6/30/08), even though the 08Q2 quarterly return is negative. When the most recent annual index is rising at a lower rate than the next-most-recent annual index, it can (although does not necessarily) indicate that the most recent quarter was negative. The derived quarterly return (ATQ) methodology is designed to discover and quantify such situations as best we can. As noted, simple curve-fitting of the annual indexes introduces smoothing, and will not be able to pick up in a timely manner the kind of turning point just described.

On the other hand, as we previously noted, the ATQ will have a lower signal/noise ratio than the underlying staggered annual return indexes from which it is derived. The result can be that each individual quarterly return is more prone to noise, possibly less accurate, than is the case for each of the annual returns in the annual-frequency indexes. For example, in Exhibit 3, look at the ATQ return for 2008Q1. The ATQ indicates an uptick in the that quarter of 4.4%. It seems unlikely that market values really rose that quarter, which was well into the credit crunch that began in late 2007 (though still prior to the more serious financial and economic events that occurred later in

2008). The ATQ estimate of +4.4% may indeed make mathematical sense given that the FYM08 annual index ending at the end of 2008Q1 rose 12.2% after the previous CY07 annual index ending at the beginning of 2008Q1 rose by only 10.6%. But those annual indexes are not without noise, and the effect of that noise may appear exaggerated in the ATQ estimate for 2008Q1.

The New York office index is also a good index in which to see the difference that can occur between direct quarterly estimation and the 2-stage ATQ. Exhibit 3 shows these two alternative ways to estimate a quarterly-frequency index, a comparison of which suggests both the strengths and weaknesses of the ATQ approach.

Even though noise filtering is used in the direct quarterly estimation, this index (indicated by the green line with diamonds) is relatively noisy, and differs importantly from the staggered annual indexes estimated from the same repeat-sales data.[19] Arguably, the direct quarterly index does not as well represent what was going on in the New York metro region office market during much of the 2001-2008 period, especially in the index's downtick in 2003Q2 and its price plateau from 2005Q2 through 2006Q3, a period which casual observation and the trade press would seem to indicate was one of robust growth in the market for New York office buildings. The strong uptick in the most recent 2008Q3 is also problematical in the direct-quarterly estimated index. The staggered annual indexes and the ATQ seem to better represent the strong bull market of the 2004-07 period, and indeed in this particular case the ATQ appears visually to be about as good

[19] In Exhibit 3 the staggered annual indexes are all presented with the convention of starting at a value of unity (as in Exhibit 1, but in contrast to Exhibit 2). All indexes in Exhibit 3 (as within the subsequent exhibits) are estimated from the same repeat-sales database. It should be noted that the indexes analyzed in the present paper (and shown in the exhibits) do not exactly equal the "official" indexes published by Moody's, as the indexes used for academic research purposes in the present paper are based entirely on the most recent transactions prices dataset (that is, the indexes are fully "backward-adjusted").

as the annual indexes (by the smoothness of the index lines' appearance), in addition to being more frequent.[20] Note in particular the downturn in 2007Q4 picked up by the ATQ but not yet apparent in the annual indexes, a downturn consistent with the credit crunch of late 2007. More recently, the ATQ matches the 1.9% price decline of the latest annual-frequency index covering the four quarters ending September 30, 2008. The directly-estimated quarterly index has nearly twice the quarterly volatility of the ATQ, a likely indication of greater noise in the former index.

On the other hand, in spite of the anomalous uptick in the most recent quarter, the directly-estimated quarterly index shows some sign of slightly temporally leading the ATQ and the annual-frequency indexes. This is most notable in the direct quarterly index's more definitive downturn after 2007, ending almost 5% down during the 2007Q3-2008Q3 period. Arguably, this better reflects the recent market turning point than the ATQ which is only down less than 2% over the same period.

3. Hypothesized Strengths & Weaknesses of the 2-stage Approach

The preceding section presented a concrete example of both the strengths and weaknesses of the 2-stage/frequency-conversion procedure for providing supplemental higher-frequency market information in small markets. The suggestion is that the advantage for the 2-stage approach over direct (single-stage) high-frequency estimation would lie in the ATQ's greater precision (less noise). This would occur if the second stage of regression adds less noise (and bias) than is removed by the effective increase in

[20] However, in the analysis to be presented in Exhibit 5 in Section 4 below, there is evidence from annual-frequency volatility comparison that the ATQ adds some 350 basis-points of error standard deviation to that of the direct annual-frequency index estimation (however, this is much less than the approximately 800 basis-points of error which Exhibit 5 indicates was added by the direct-quarterly estimation).

sample size presented by the longer return period intervals in the staggered lower-frequency estimation in the first of the two stages. This may ultimately be an empirical question, and even if the 2-stage procedure does tend generally to be more accurate than direct high-frequency estimation, either procedure may be more accurate in a given specific empirical instance. In any case, the specific "errors" will differ across the two techniques, which suggests that there may be benefit in the information marketplace in producing and publishing both types of high-frequency indexes.

While precision is a potential strength of the ATQ, there may be a weakness as well. The preceding examination of the New York office index during the 2007-08 market downturn suggested that perhaps direct single-stage quarterly estimation is better at capturing the early stages of a sharp downturn in the market. Recall that the directly-estimated index fell more than twice as fast as the ATQ in the New York office market during the first four quarters of the downturn (4Q07-3Q08). In other words, the hypothesis would be that direct quarterly estimation might show a slight temporal lead ahead of annual estimation (and the resulting ATQ) in such market circumstances. This could result from the effect of loss aversion behavior on the part of property owners during the early stages of a sharp market downturn. Property owners react conservatively, not revising their reservation prices downward (perhaps even ratcheting them upwards, effectively pulling out of the asset market). Unless and until property owners are under pressure to sell in a down market, the result is a sharp drop-off in trading volume. This has two impacts relevant for transaction price index estimation. First, the relatively few transactions that do clear during the early stages of the downturn reflect relatively positive or eager buyers. This dampens the price reduction actually realized in the market (as reflected in the market-clearing prices). But it does not prevent a directly-estimated high-frequency index from reflecting that market price reduction (such as it is), as best such an index can do so (given the data scarcity, which increases the noise in the index), in the sense that the index does not have a lag bias.

The second effect of the fall-off in sales volume, however, poses a particular issue for annual-frequency indexes as compared to higher-frequency directly-estimated indexes. An annual index reflects an entire 4-quarter span of time in each periodic return, and in the downturn/loss-aversion circumstance just described the most recent part of that 4-quarter time span has markedly fewer transaction observations than the earlier part of the span. Thus, the data used to estimate the annual index's most recent annual return is dominated by the earlier, pre-downturn sales transactions. Even though the annual index uses Bryon-Colwell-type time-weighted dummy variables (as described in a previous footnote), the sparser data in the more recent part of the time span may make it difficult for the annual index to fully reflect the recent market movement. Such a difficulty in the annual indexes would then carry over into the quarterly ATQ indexes derived from them.

4. An Empirical Comparison of 2-stage versus Direct Estimation in Data-Scarce Markets

The RCA repeat-sales database and the Moody's/REAL Commercial Property Price Indexes based on that data present an opportunity to begin an empirical comparison of the two approaches. As noted, computation of index estimated returns standard errors is not straightforward for the ATQ, and "apples-to-apples" comparisons of estimated standard errors across the two procedures is not attempted in the present paper[21]. However, there are two statistical characteristics of an estimated real estate asset market price index that can provide practical, objective information about the quality of the index. These two characteristics are the volatility and the first-order autocorrelation of the index's estimated returns series. Based on statistical considerations, we know that noise or error in the index returns will tend to increase the observed volatility in the index returns. And we know that noise or error will also tend to drive the index returns' first-order autocorrelation down, toward negative 50%.[22] Based on economic considerations, we know that the true quarterly volatility in commercial property market prices tends to be fairly low, and the true first-order autocorrelation tends to be at least slightly positive.[23]

For example, the quarterly S&P/Case-Shiller home price index has volatility of only 1.7% per quarter, and first-order autocorrelation of 66%, from 1987 through 2007. While this may reflect some smoothing and/or greater sluggishness and "price stickiness" in housing markets compared to commercial property markets, and the NCREIF Property Index's quarterly volatility of 1.7% and first-order autocorrelation of 80% (1984-2007) may reflect appraisal smoothing, the transaction-price-based version of the NCREIF index produced by the MIT Center for Real Estate still only shows 3.8% quarterly

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[21] For one thing, consider that the second-stage regression itself has no residuals, as it makes a perfect fit to the staggered lower-frequency indexes that are its dependent variable. Furthermore, as noted, the objective of a price index regression is not the minimization of transaction price residuals *per se*, but rather the minimization of error in the coefficient estimates (the index's periodic returns).

[22] These are basic characteristics of the statistics of indexes. (See, e.g., Geltner & Miller *et al* (2007), Chapter 25.)

[23] Fundamentally, this is due to the relatively conservative, "cash-cow" nature of stabilized commercial property (the RCA-based indexes exclude leverage and development projects), and the relatively illiquid, sluggish nature of the private, search market in which property assets trade.

volatility (with only 5% first-order autocorrelation).[24] This compares to approximately 8% quarterly volatility over the same time-period in large-cap stock indexes and REIT indexes.

Considering the foregoing, it would seem reasonable to compare the two index estimation methodologies based on the volatilities and first-order autocorrelations of the resulting estimated historical indexes. Lower volatility, and higher first-order autocorrelation, would be indicative of an index that is likely to have less noise or error in its individual periodic returns. For example, in the New York Office index that we considered previously in Exhibit 3, the 2-stage ATQ index has 3% quarterly volatility, versus 6% in the directly-estimated ("DirQ") index that seemed more noisy.

Among the Moody's/REAL Commercial Property Price Indexes there are 16 indexes (including the New York Office index we have previously examined) that are currently published at only the annual frequency (with four staggered versions, as described above), because the available transaction price data is deemed to be insufficient to support quarterly estimation. An examination of the relative values of the quarterly volatilities and first-order autocorrelations resulting from estimation of quarterly indexes by the two alternative procedures across these 16 market segments can provide the beginnings of a quantitative comparison of the two procedures.

The 16 annual-frequency Moody's/REAL indexes include eight at the MSA level and eight at the multi-state regional level. The eight MSA-level indexes are: four different property sectors (apartment, industrial, office, retail) for Southern California (Los Angeles and San Diego combined), three other MSA-level office indexes (New

[24] At the annual frequency, during 1984-2007, the volatility of the transactions-based version of the NPI is 9%, and the first-order autocorrelation is +40%.

York, Washington DC, and San Francisco), and one apartment index (for Southern Florida, which combines Miami, Ft Lauderdale, West Palm Beach, Tampa Bay, and Orlando). The eight multi-state regional indexes include the four property sectors each within each of two NCREIF-defined regions: the East and the South.[25]

Insert Exhibit 4 about here.

Exhibit 4 summarizes the comparison of the precision of the two approaches based on a volatility and autocorrelation comparison of the two quarterly index procedures (labeled “ATQ” and “DirQ” in the exhibit). The volatility test is defined by the ratio of the ATQ quarterly volatility divided by the DirQ quarterly volatility. The autocorrelation test is defined by the difference between the ATQ first-order autocorrelation minus that of the DirQ. Both tests are applied separately to the entire 31-quarter available history 2001-2008Q3 and to the more recent 15-quarter period 2005-08Q3. The RCA repeat-sales database “matured” to a considerable degree by 2005, with many more repeat-sales observations available since that time. The comparison is made for each of the 16 indexes and also averaged across the eight MSA-level and eight regional-level indexes.

The overall impression left by this comparison is that the 2-stage ATQ approach seems to provide generally lower volatility and higher 1st-order autocorrelation in most cases, suggesting that this approach is more precise (less noisy). However, in many cases

[25] The East Region includes all the 15 states north and east of Georgia, Tennessee, and Ohio. The South Region includes the 9 states encompassed inclusively between and within Florida, Georgia, Tennessee, Arkansas, Oklahoma, and Texas. There is thus some geographical overlap between the MSA-level and regional-level indexes, in the sense that three of the eight MSA-level indexes are also within two of the regional-level indexes. The New York and Washington DC office indexes are within the East Office regional index, and the South-Florida Apartments index is within the South Apartment regional index.

the two procedures are similar and the advantage of the ATQ seems to be slight. It must also be noted that the available history is still rather short. As we saw in the case of the New York office index, which by our test comes out in favor of the ATQ, even where the 2-stage procedure is more accurate, the alternative procedure of direct quarterly estimation can provide useful perspective, if for no other reason than that the noise in the two procedures is different noise, hence, the two procedures can provide a “cross-check” on each other.

It is also possible to make a more quantitative comparison of the amount of extra noise or random error added by either the ATQ or direct-quarterly estimation compared to direct annual-frequency estimation, by modeling the longitudinal moments of the resulting annual-frequency returns series, the volatilities and the first-order autocorrelations. The method of the analysis is explained in Appendix C, and the results are shown in Exhibit 5. The exhibit shows, for the ATQ and direct-quarterly estimation, the implied standard deviation of the extra noise in the quarterly index (at the annual frequency) compared to direct single-stage annual-frequency index estimation. The comparison is shown based on both the volatility and the first-order autocorrelation (at the annual frequency). The comparisons are averaged across the four staggered annual starting dates (Jan, Apr, Jul, Oct). Out of the 64 possible cells of comparison (16 indexes X 2 moments of comparison X 2 high-frequency methodologies), the ATQ is superior to the direct-quarterly estimation (smaller implied added noise) in 46 of the cells, the direct-quarterly estimation is superior to the ATQ in 12 cells, and the two are equal in the remaining 6 cells. Thus, the analysis in Exhibit 5 essentially confirms the previous indication in Exhibit 4 that the ATQ method tends to be superior to direct-quarterly estimation in terms of precision or noise reduction. However, Exhibit 5 also provides a quantitative estimate of the amount of extra noise is added by going from the annual to quarterly-frequency estimation. The extra noise is seen to lie on average roughly in the range of 100 to 400 basis-points in the case of the ATQ, and in the range of 400 to 700 basis-points in the case of direct-quarterly estimation.

Insert Exhibit 5 about here.

How important this much extra noise is depends upon the usage of the indexes. We noted in our discussion of the New York office index in Section 2.4 that that ATQ index "looked pretty good" (in particular by the smoothness of its appearance and in comparison with the direct-annual indexes). Yet Exhibit 5's volatility comparison suggests that the ATQ adds some 350 basis-points of extra noise standard deviation. One way to interpret this is that an annual standard deviation of 3.5% is not that great in a returns series of a risky investment asset class (e.g., blue chip stock market indexes have annual volatility in the 15% to 20% range).[26] It is also important to remember that noise does not accumulate in the index, and therefore its practical impact diminishes over longer investment or trading horizons. Derivative contracts of several years duration, especially if settlement is based on an average across perhaps three individual quarters, could effectively mitigate the excess noise for many trading purposes. On the other hand,

[26] The fact that the standard deviation of the added noise is 3.5% does not imply that the annual volatility of the quarterly index is 3.5% greater than the annual index volatility. For example, the New York office annual index has an average volatility of 6.6% (averaged over the four staggered starting dates), while the corresponding ATQ has an average annual volatility of 7.5%. Noise, by definition, is uncorrelated with the underlying true volatility. Thus, the volatility of the high-frequency index is the square root of the some of the underlying low-frequency index variance and the variance of the added noise:

$$7.5\% = \text{SQRT}[(6.6\%)^2 + (3.5\%)^2].$$

For practical purposes it is not clear that 7.5% volatility is effectively much different from 6.6% volatility.

in any given quarterly return, 350 basis-points is a lot of error standard deviation and certainly would pull down the signal-noise ratio considerably. Given that the major purpose of the quarterly estimation over the annual-frequency estimation is to provide individual quarterly return estimates, this would seem to take some of the wind out of the sails of the quarterly estimation.

Finally, while the ATQ comes out an apparent winner against the DirQ in the precision comparison, recall that we raised a possible weak point about the ATQ in its ability to quickly and fully reflect the early stages of a sudden and sharp market downturn, such as occurred during 2007-08 in the U.S. commercial property markets. We suggested that during such times property-owner loss-aversion behavior could cause the underlying annual-frequency indexes to experience difficulty fully reflecting a late-period price drop in the market.

Insert Exhibit 6 about here.

Exhibit 6 presents some empirical evidence relevant to this point from the same 16 Moody's/REAL market-segment indexes examined previously. The exhibit shows the percentage price change during the four quarters from 4Q07 through 3Q08 inclusive as tracked in each market by the 2-stage ATQ index and the directly-estimated quarterly index. The idea is that the index showing a larger price decline during that period is arguably more accurately capturing the downturn in its initial stages. While the evidence in Exhibit 6 in this regard is mixed (especially at the regional level), the overall impression is that the directly-estimated indexes generally captured a larger downturn. In the last column, Exhibit 6 also shows the difference between the one-quarter leading versus lagging cross-correlation between the two types of indexes within each market. If this measure is negative, it indicates that the correlation of the direct-quarterly index with the ATQ one quarter later is greater than the correlation of the converse, in other words. This is taken to be some evidence that the direct-quarterly index tends to show a slight temporal lead over the 2-stage ATQ. In summary, while the evidence in Exhibit 6 is admittedly weak (especially given the short history), it suggests some support for the hypothesis that annual-frequency indexes and the ATQ based on them may be slightly lagged compared to direct quarterly estimation.

We conclude this analysis of the MPGIE-based 2-stage/frequency-conversion procedure application to data-scarce markets with the general suggestion that the procedure can add value. What we were here labeling the ATQ index appears to be generally at least as precise as direct quarterly estimation. The ATQ indexes track very closely the staggered annual-frequency indexes on which they are based and add generally only modest volatility, while providing quarterly instead of annual-frequency returns. Thus, the ATQ indexes are capable of adding information that the marketplace may be able to use. However, in data-scarce environments such as examined here (e.g., second-sales observational frequency averaging in the mid-20s per quarter), the ATQ arguably is best used as a supplement to annual-frequency indexes, not a replacement, as the higher-frequency indexes do have slightly greater noise than the annual-frequency indexes. Indeed, the ATQ may itself also be supplemented by direct quarterly estimation, as the two procedures will tend to display "different noise", and the direct quarterly estimation may be more effective at capturing the early stages of market downturns.

Therefore, the two quarterly estimation procedures will tend to complement each other, each serving as a "cross-check" on the other.

5. Conclusion

This paper has described a methodology for estimating higher frequency (e.g., quarterly) price indexes from staggered lower-frequency (e.g., annual) indexes. The application examined here is to provide supplemental higher-frequency information about market movements in data-scarce environments that require low-frequency indexes. The 2-stage approach takes advantage of the lower frequency to, in effect, accumulate more data over the longer-interval time periods which can be used to estimate returns with less error. Then a frequency conversion procedure is applied using the Moore-Penrose pseudoinverse matrix in an under-identified second-stage "repeat-sales" regression in which the staggered low-frequency indexes provide the "repeat-sales" data inputs. Linear algebra theory establishes that this frequency conversion procedure is optimal in the sense that it minimizes the variance and bias added in the second stage. Numerical simulation and empirical comparisons suggest that the noise and bias added in the second stage may indeed be small. The result is a higher-frequency index that, while it has a signal/noise ratio lower than the underlying low-frequency indexes, nevertheless adds higher frequency information that may be useful in the marketplace, especially in the context of tradable derivatives.

Empirical analysis suggests that the 2-stage procedure will often produce indexes that have lower volatility and/or higher first-order autocorrelation than directly-estimated high-frequency indexes, suggesting that the 2-stage procedure tends to be more accurate and therefore can provide useful supplemental information in the marketplace. In a comparison across 16 indexes in data-scarce environments the 2-stage procedure outperforms direct estimation in almost all cases. The ATQ indexes track very closely the staggered annual-frequency indexes on which they are based and add generally only modest volatility, while providing quarterly instead of annual-frequency returns. The annual-frequency correlation coefficient between the ATQ and the corresponding underlying direct-annual indexes exceeds 99% in all 64 cases examined here (16 indexes X 4 staggered starting dates).[27] Moment comparison analysis suggests that the ATQ typically adds 100-400 bps of extra noise (measured by standard deviation of the noise) to that in the underlying annual-frequency indexes, but this typically results in less than 100 bps of extra volatility in the quarterly index (at the annual frequency). In markets where the true volatility is typically in the neighborhood of 6% to 10% per annum, this is not too much extra volatility (combined with the high correlation). However, there is also evidence that annual-frequency based indexes, including the ATQ quarterly index derived from them, may have difficulty fully and immediately reflecting the early stages of a sharp market downturn such as has occurred in U.S. commercial property markets during 2007-08. Direct quarterly estimation may add information in this regard, although with greater noise than the ATQ procedure.

Appendix D presents charts of the ATQ and direct-quarterly index for all 16 annual-frequency Moody's/REAL Indexes.

[27] This is a mechanical result of the "perfect fit" in the under-identified second-stage regression.

Appendix A:

The Moore-Penrose Pseudoinverse or the Generalized Inverse

The Moore-Penrose pseudoinverse is a general way of solving the following system of linear equations:

$$y = \mathbf{X}b, \qquad y \in \mathbf{R}^n \;;\; b \in \mathbf{R}^k \;;\; \mathbf{X} \in R^{n\times k} \qquad (1)$$

It can be shown that there is a general solution to these equations of the form:

$$b = \mathbf{X}^{\dagger}y \qquad (2)$$

The $\mathbf{X}^{\dagger}$ matrix is the unique Moore-Penrose pseudoinverse of X that satisfies the following properties:

1. $\mathbf{X}\,\mathbf{X}^{\dagger}\,\mathbf{X} = \mathbf{X}$ ($\mathbf{X}\,\mathbf{X}^{\dagger}$ is not necessarily the identity matrix)
2. $\mathbf{X}^{\dagger}\,\mathbf{X}\,\mathbf{X}^{\dagger} = \mathbf{X}^{\dagger}$
3. $(\mathbf{X}\,\mathbf{X}^{\dagger})^T = \mathbf{X}\,\mathbf{X}^{\dagger}$ ($\mathbf{X}\,\mathbf{X}^{\dagger}$ is Hermitian)
4. $(\mathbf{X}^{\dagger}\,\mathbf{X})^T = \mathbf{X}^{\dagger}\,\mathbf{X}$ ($\mathbf{X}^{\dagger}\,\mathbf{X}$ is also Hermitian)

The solution given by equation (2) in the previous Appendix is a minimum norm least squares solution. When **X** is of full rank (i.e., rank is at most min(n, k)), the generalized inverse can be calculated as follows:

Case 1: When n = k (same number of equations as unknowns) : $\mathbf{X}^{\dagger} = \mathbf{X}^{-1}$
Case 2: When n < k (fewer equations than unknowns) : $\mathbf{X}^{\dagger} = \mathbf{X}^T(\mathbf{X}\,\mathbf{X}^T)^{-1}$
Case 3: When n > k (more equations than unknowns) : $\mathbf{X}^{\dagger} = (\mathbf{X}^T\,\mathbf{X})^{-1}\,\mathbf{X}^T$

In the application for deriving quarterly indexes from staggered annual indexes, Case 2 provides the relevant calculation. Furthermore, it should be noted that when the rank of **X** is less than k, no unbiased linear estimator, b, exists. However, for such a case, the generalized inverse provides a minimum bias estimation.[28] For the basic references on the Moore-Penrose pseudoinverse see the references by Penrose (1955, 1956), Chipman (1964), and Albert (1972) in the bibliography.

[28] Properties of the generalized inverse can be found in Penrose (1954) and equation (2) first appeared in Penrose (1956). Proofs of Cases 1 – 3 can be found in Albert (1972) and a proof of minimum biasedness is given in Chipman (1964).

Appendix B:
A Note on Bias in the Moore-Penrose Frequency Conversion

Here we consider the case relevant to our present purposes, i.e. where $\mathbf{X}^{\dagger} = \mathbf{X}^{T} (\mathbf{X}\,\mathbf{X}^{T})^{-1}$. Therefore, in our application, the solution (or estimation) of the second-stage regression (equation (2) of Appendix A) can be re-written as:

$$b = \mathbf{X}^{T} (\mathbf{X}\,\mathbf{X}^{T})^{-1} y$$

Considering that the true value of the predicted variable (y) is by definition: $\mathbf{X}b_{True}$, therefore the expected value of b is:

$$E[b \mid X] = \mathbf{X}^{T} (\mathbf{X}\,\mathbf{X}^{T})^{-1} \mathbf{X}\, b_{True}$$

Let $\mathbf{R} = \mathbf{X}^{T} (\mathbf{X}\,\mathbf{X}^{T})^{-1} \mathbf{X}$ be the "resolution" matrix, which would have otherwise been the k by k identity ($\mathbf{I}$) matrix if $\mathbf{X}$ had been of full column rank. In our case, the resolution matrix is instead a symmetric matrix describing how the generalized inverse solution "smears" out the b_{True} into a recovered vector b. The bias in the generalized inverse solution is

$$\begin{aligned} E[b \mid X] - b_{True} &= \mathbf{R}\, b_{True} - b_{True} \\ &= (\mathbf{R} - \mathbf{I})\, b_{True} \end{aligned}$$

We can formulate a bound on the norm of the bias:

$$\| E[b \mid X] - b_{True} \| \leq \| \mathbf{R} - \mathbf{I} \| \, \| b_{True} \|$$

Computing $\| \mathbf{R} - \mathbf{I} \|$ can give us an idea of how much bias has been introduced by the generalized inverse solution. However, the bound is not very useful since we typically have no knowledge of $\| b_{True} \|$.

In practice, we can use the resolution matrix, **R**, for two purposes. First, we can examine the diagonal elements of **R**. Diagonal elements that are close to one correspond to coefficients for which we can expect good resolution. Conversely, if any of the diagonal elements are small, then the corresponding coefficients will be poorly resolved. Secondly, we can multiply **R** times a particular test coefficient vector b_{test} to see how the vector would resolve in the inverse solution b.[29]

[29] This strategy is called the "resolution test". One commonly used test in the geophysics literature is a "spike model", which is a vector of coefficients with all zero elements, except for one single entry, which is one. Multiplying **R** times a spike coefficient vector effectively picks out the corresponding column of the resolution matrix.

Appendix C:

Quantitative Estimation of Added Noise at High Frequency

Let A_a be the value level (in logs) of annual-frequency index at the end of year "a", and I_q be the (log) value level for the corresponding ATQ index in quarter "q" where q = 4a, and Q_q be the value of the corresponding directly-estimated quarterly-frequency index. Then:

$$I_q = A_a + \varepsilon^I_q \qquad \text{(C1)}$$

$$Q_q = A_a + \varepsilon^Q_q \qquad \text{(C2)}$$

where the ε terms are independent random errors (in log levels) that represent the added error relative to the single-stage annual-frequency estimated index by the 2-stage ATQ and the direct-quarterly estimation respectively. Then in returns, let r_t be the annual-frequency return estimated from the single-stage annual-frequency index in year "t" and r^*_t be the annual-frequency return for the same year estimated from one of the quarterly-frequency indexes (ATQ or direct-quarterly which we will label DirQ):

$$r_t = A_a - A_{a-1} \qquad \text{(C3)}$$

$$r^{*I}_t = I_{4a} - I_{4a-4} = r_t + (\varepsilon^I_{4a} - \varepsilon^I_{4a-4}) \;=\; r_t + \eta^I_t \qquad \text{(C4)}$$

$$r^{*Q}_t = Q_{4a} - Q_{4a-4} = r_t + (\varepsilon^Q_{4a} - \varepsilon^Q_{4a-4}) \;=\; r_t + \eta^Q_t \qquad \text{(C5)}$$

Define "σ" to be the volatility (longitudinal standard deviation of the returns) of the annual index:

$$\sigma = STD[r_t]$$

Then by basic statistics the volatilities of the ATQ and DirQ respectively are:

$$\sigma^I = STD[r^{*I}_t] \;=\; SQRT[\sigma^2 + VAR[\eta^I_t]\,] \qquad \text{(C6)}$$

$$\sigma^Q = STD[r^{*Q}_t] \;=\; SQRT[\sigma^2 + VAR[\eta^Q_t]\,] \qquad \text{(C7)}$$

And hence we can derive the standard deviation of the extra index return noise compared to single-stage annual-frequency estimation as a function of the annual and quarterly volatilities as:

$$STD[\eta^I_t] \;=\; SQRT[(\sigma^I)^2 - \sigma^2] \qquad \text{(C8a)}$$

$$STD[\eta^Q_t] \;=\; SQRT[(\sigma^Q)^2 - \sigma^2] \qquad \text{(C8b)}$$

The standard deviation of the extra return noise (in annual-frequency returns) compared to direct single-stage annual-frequency estimation is simply the square root of the variance difference: the quarterly index annual return variance minus the single-stage annual index variance.

Now consider the index serial correlation. Similar to our derivation above, using the fact that the extra noise in the quarterly indexes compared to the annual index is uncorrelated with anything, the quarterly index annual returns first-order auto-covariance is related to the annual index returns auto-covariance as follows:*

$$COV[r^*_t, r^*_{t-1}] = COV[r_t, r_{t-1}] - \sigma_\eta^2/2 \qquad (C9)$$

Labeling the direct annual index returns first-order auto-correlation as: ρ, and the quarterly index annual returns first-order auto-correlation as ρ_{r^*} , we see that the annual and quarterly first-order autocorrelations are related as:†

$$\rho_{r^*} = (\rho\sigma^2 - \sigma_\eta^2/2)/(\sigma^2 + \sigma_\eta^2) \qquad (C10)$$

which approaches -50% as the added noise dominates over the direct-annual returns in the quarterly index (if σ_η is very large relative to σ).

We can invert (C10) to derive the magnitude of the standard deviation of the extra noise added in the quarterly index (at the annual return frequency) compared to the direct 1-stage annual-frequency index, as implied by the relationship between the quarterly and direct-annual autocorrelation in the annual frequency returns:‡

$$\sigma_\eta = SQRT[(\rho - \rho_{r^*})\sigma^2/(\rho_{r^*} + ½)] \qquad (C11)$$

Or more specifically for each quarterly methodology:

$$STD[\eta^I_t] = SQRT[(CORREL[r_t, r_{t-1}] - CORREL[r^{*I}_t, r^{*I}_{t-1}])\sigma^2/(CORREL[r^{*I}_t, r^{*I}_{t-1}] + ½)]$$

$$STD[\eta^Q_t] = SQRT[(CORREL[r_t, r_{t-1}] - CORREL[r^{*Q}_t, r^{*Q}_{t-1}])\sigma^2/(CORREL[r^{*Q}_t, r^{*Q}_{t-1}] + ½)]$$

Thus, both the volatilities, and the first-order autocorrelations, at the annual frequencies, provide indications of the magnitude of extra noise added in going from direct 1-stage annual-frequency index estimation to quarterly-frequency estimation. In principle these indications should be the same, but for short finite empirical samples they may differ. These are the moment comparisons used in Exhibit 5 in the paper.

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* Derivation: $COV[r^*_t, r^*_{t-1}] = COV[r_t + \eta_t , r_{t-1} + \eta_{t-1}]$
$= COV[r_t, r_{t-1}] + COV[\eta_t, \eta_{t-1}] + COV[r_t, \eta_{t-1}] + COV[\eta_t, r_{t-1}]$
$= COV[r_t, r_{t-1}] + COV[(\varepsilon_t - \varepsilon_{t-1}), (\varepsilon_{t-1} - \varepsilon_{t-2})] + COV[r_t, (\varepsilon_{t-1} - \varepsilon_{t-2})] + COV[(\varepsilon_t - \varepsilon_{t-1}), r_{t-1}]$
$= COV[r_t, r_{t-1}] - COV[\varepsilon_{t-1}, \varepsilon_{t-1}] = COV[r_t, r_{t-1}] - VAR[\varepsilon_t]$
$= COV[r_t, r_{t-1}] - \sigma_\varepsilon^2 = COV[r_t, r_{t-1}] - \sigma_\eta^2/2$

† Derivation: $\rho_{r^*} = CORREL[r^*_t, r^*_{t-1}] = COV[r^*_t, r^*_{t-1}]/(\sigma_{r^*}^2) = (COV[r_t, r_{t-1}] - \sigma_\varepsilon^2)/(\sigma^2 + 2\sigma_\varepsilon^2)$
$= (\rho\sigma^2 - \sigma_\varepsilon^2)/(\sigma^2 + 2\sigma_\varepsilon^2) = (\rho\sigma^2 - \sigma_\eta^2/2)/(\sigma^2 + \sigma_\eta^2)$.

‡ Derivation: $\rho_{r^*} = (\rho\sigma^2 - \sigma_\eta^2/2)/(\sigma^2 + \sigma_\eta^2)$, ➔ $(\sigma^2 + \sigma_\eta^2)\rho_{r^*} = \rho\sigma^2 - \sigma_\eta^2/2$, ➔ $\rho_{r^*}\sigma^2 + \rho_{r^*}\sigma_\eta^2 + \sigma_\eta^2/2 = \rho\sigma^2$,
➔ $(\rho_{r^*} + ½)\sigma_\eta^2 = \rho\sigma^2 - \rho_{r^*}\sigma^2 = (\rho - \rho_{r^*})\sigma^2$, ➔ $\sigma_\eta^2 = (\rho - \rho_{r^*})\sigma^2/(\rho_{r^*} + ½)$.

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Exhibits

Exhibit 1: (CY is the calendar year index ending December 31 each year, FYM is the index ending March 31 each year, FYJ is the index ending June 30 each year, and FYS is the index ending September 30 each year.)

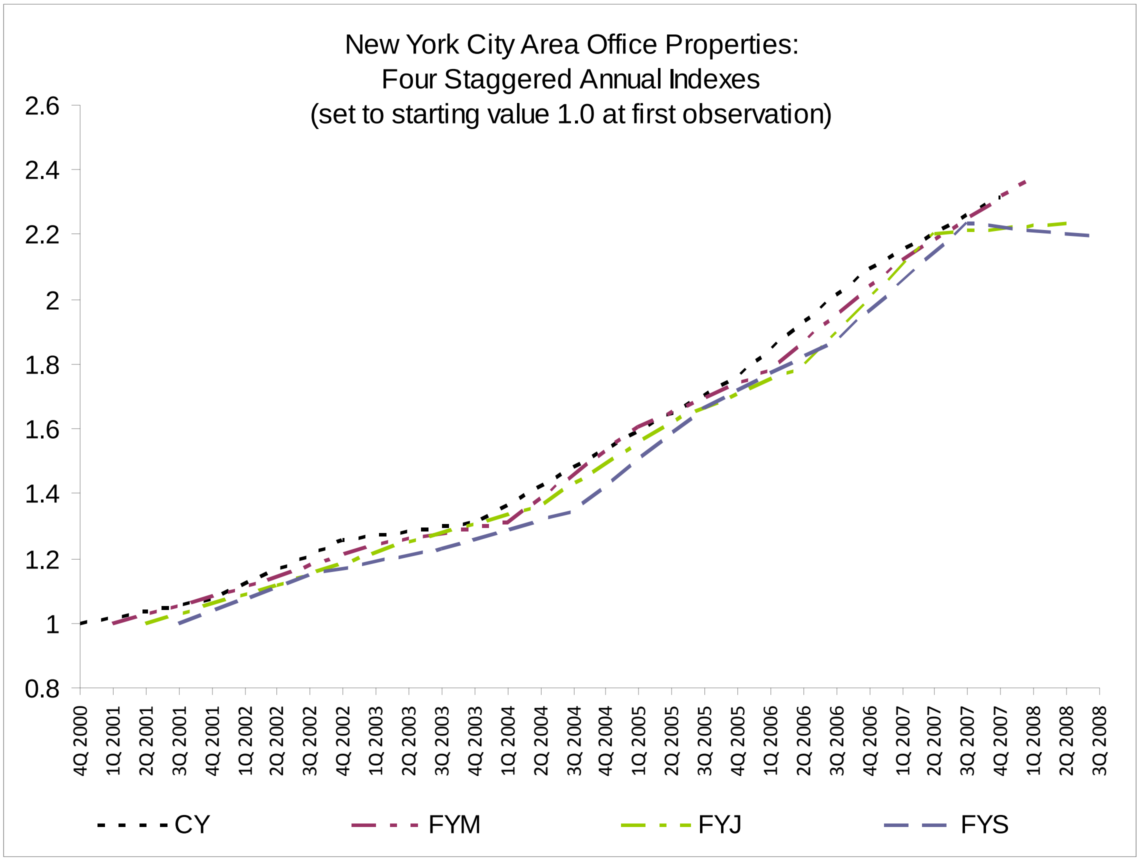


Exhibit 2: True vs Estimated Underlying Staggered Annual Indexes (simulation)

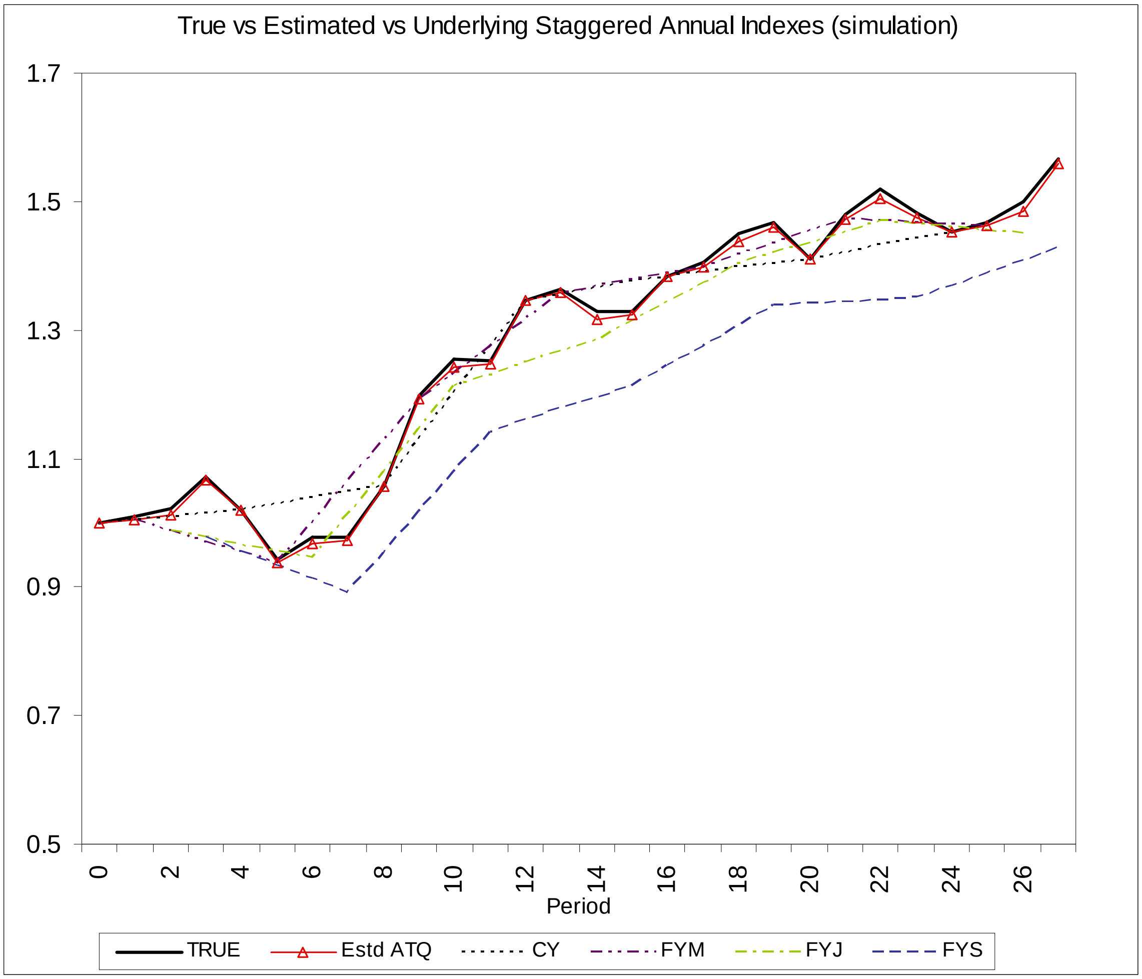


Exhibit 3: New York office property price index based on RCA repeat-sales database, directly estimated (green diamonds) and derived ATQ (red triangles), together with the staggered annual indexes: 2001Q1-2008Q3.

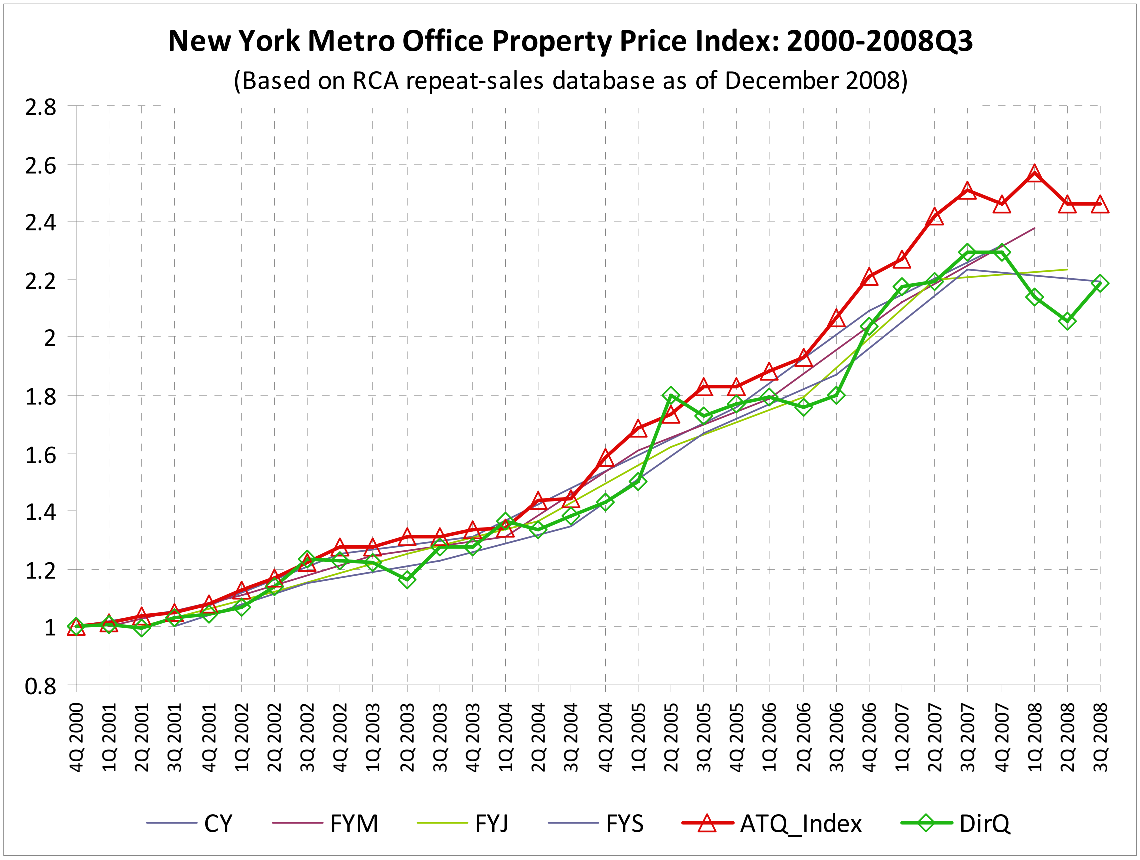


Exhibit 4: Comparison of 16 quarterly indexes in data-scarce markets: 2-stage (“ATQ”) versus direct quarterly (“DirQ”), based on Volatility and Autocorrelation Tests.

Index Comparison: 2-stage vs direct quarterly estimation: Evidence on Index Precision.					
	Data*:	Vol Test**:		AC(1) Test***:	
MSA-level indexes:					
Index:		2005-08	2001-08	2005-08	2001-08
NY Office	24 (11)	**0.49**	**0.54**	**2%**	**6%**
DC Office	17 (7)	**0.91**	**0.88**	*-36%*	*-23%*
SF Office	15 (8)	**0.85**	**0.87**	*-9%*	*-4%*
SC Office	33 (21)	**0.63**	**0.69**	**4%**	**3%**
SC Industrial	35 (26)	**0.72**	**0.54**	*-9%*	**15%**
SC Retail	19 (10)	**0.62**	**0.61**	*-13%*	**16%**
SC Apts	50 (43)	**0.73**	**0.57**	**42%**	**67%**
FL Apts	22 (13)	**0.74**	**0.71**	**68%**	**54%**
Average:	27 (17)	**0.71**	**0.68**	**6%**	**17%**
Regional-level indexes:					
Index:		2005-08	2001-08	2005-08	2001-08
E Office	66 (33)	**0.70**	**0.68**	**4%**	*-3%*
S Office	47 (32)	**0.48**	**0.45**	**24%**	**24%**
E Industrial	44 (26)	**0.55**	**0.72**	**10%**	**23%**
S Industrial	36 (31)	**0.74**	**0.73**	**56%**	**34%**
E Retail	32 (16)	**0.91**	**0.82**	**18%**	**8%**
S Retail	47 (23)	**0.63**	**0.61**	**74%**	**66%**
E Apts	66 (31)	**0.63**	**0.88**	**52%**	**18%**
S Apts	70 (40)	**0.69**	**0.63**	**67%**	**46%**
Average:	51 (29)	**0.67**	**0.69**	**38%**	**27%**
*Avg number of 2nd-sales obs/qtr 2006-08. Database was “immature” with considerably fewer 2^{nd}-sales observations prior to 2005. (In parentheses number of obs in most recent 3Q08 qtr.)					
** Ratio of ATQ volatility/DirQ volatility: <1 ==> **ATQ better**; >1 ==> *DirQ better*.					
*** Difference: AC(1)ATQ - AC(1)DirQ. >0 ==> **ATQ better**; <0 ==> *DirQ better*.					

Exhibit 5: Implied Extra Noise Standard Deviation (Annual Frequency) in Quarterly Compared to Annual Index Added By:				
	ATQ, based on comparison of:		**DirQ, Based on comparison of:**	
Index	**Volatility**	**AC(1)**	**Volatility**	**AC(1)**
NY Office	**3.51%**	0.00%	**8.18%**	0.00%
DC Office	**2.37%**	**0.00%**	**5.16%**	**23.08%**
SF Office	**3.36%**	*1.51%*	**4.91%**	*0.00%*
SC Office	**4.13%**	**1.93%**	**7.42%**	**7.51%**
SC Industrial	**3.99%**	**1.31%**	**7.80%**	**5.64%**
SC Retail	**3.53%**	0.00%	**6.57%**	0.00%
SC Apts	**4.02%**	**0.76%**	**5.08%**	**3.80%**
FL Apts	*9.31%*	**4.09%**	*2.59%*	**18.99%**
AVERAGE	**4.28%**	**1.20%**	**5.96%**	**7.38%**
Regional-level indexes:				
E Office	**2.74%**	**0.00%**	**5.85%**	**7.99%**
S Office	**2.87%**	**0.74%**	**6.11%**	**5.48%**
E Industrial	*4.34%*	**0.00%**	*2.86%*	**4.07%**
S Industrial	**3.69%**	**1.11%**	**7.30%**	**9.00%**
E Retail	**3.21%**	*1.11%*	**3.65%**	*0.00%*
S Retail	**3.16%**	**0.00%**	**6.05%**	**6.35%**
E Apts	*4.35%*	0.00%	*0.00%*	0.00%
S Apts	*6.67%*	**2.04%**	*2.47%*	**18.51%**
AVERAGE	**3.88%**	**0.63%**	**4.29%**	**6.43%**

Reported moments are based on the average across the four staggered annual start-dates (CY, FYM, FYJ, FYS).

Note: Implied extra noise is set to zero (by construction) if the quarterly volatility (at annual frequency) is less than the annual index volatility, or if the quarterly first-order autocorrelation (at the annual frequency) is higher than the annual index first-order autocorrelation.

Bold type indicates a cell in which the comparison indicates the ATQ adds less extra noise than the DirQ does.
Italics indicates a cell in which the comparison indicates the DirQ adds less extra noise than the ATQ does.

Exhibit 6: Comparison of 16 quarterly indexes in data-scarce markets: 2-stage ("ATQ") versus direct quarterly ("DirQ"), based on 2007-08 Downturn Magnitude & Lead-Lag Cross-Correlation.

Index Comparison: 2-stage vs direct-quarterly estimation: Capturing 2007-08 Downturn, & Lead-Lag Evidence.

		Index Return 4Q07-3Q08**:		
Index:	Data*:	ATQ	DirQ	Lead – Lag Correlation***:
NY Office	24 (11)	*-1.89%*	*-4.79%*	**5%**
DC Office	17 (7)	*-1.87%*	*-2.26%*	*-86%*
SF Office	15 (8)	*-4.69%*	*-6.63%*	*-28%*
SC Office	33 (21)	*-6.47%*	*-17.53%*	**48%**
SC Industrial	35 (26)	*-6.02%*	*-9.60%*	*-50%*
SC Retail	19 (10)	*-3.20%*	*-9.59%*	*-22%*
SC Apts	50 (43)	*-7.67%*	*-9.78%*	*-35%*
FL Apts	22 (13)	**-7.13%**	**-1.59%**	*-18%*
Average:	27 (17)	*-4.87%*	*-7.72%*	*-23%*
Index:	Data*:	ATQ	DirQ	Lead-Lag
E Office	66 (33)	**-5.20%**	**-3.48%**	*-87%*
S Office	47 (32)	*-3.01%*	*-4.92%*	*-22%*
E Industrial	44 (26)	**-2.85%**	**-1.26%**	*-34%*
S Industrial	36 (31)	*-10.28%*	*-12.13%*	*-38%*
E Retail	32 (16)	*-6.08%*	*-6.95%*	*-5%*
S Retail	47 (23)	**-7.03%**	**-6.48%**	*-46%*
E Apts	66 (31)	*-9.80%*	*-14.17%*	**4%**
S Apts	70 (40)	**-3.14%**	**4.42%**	*-1%*
Average:	51 (29)	**-5.92%**	**-5.62%**	*-29%*

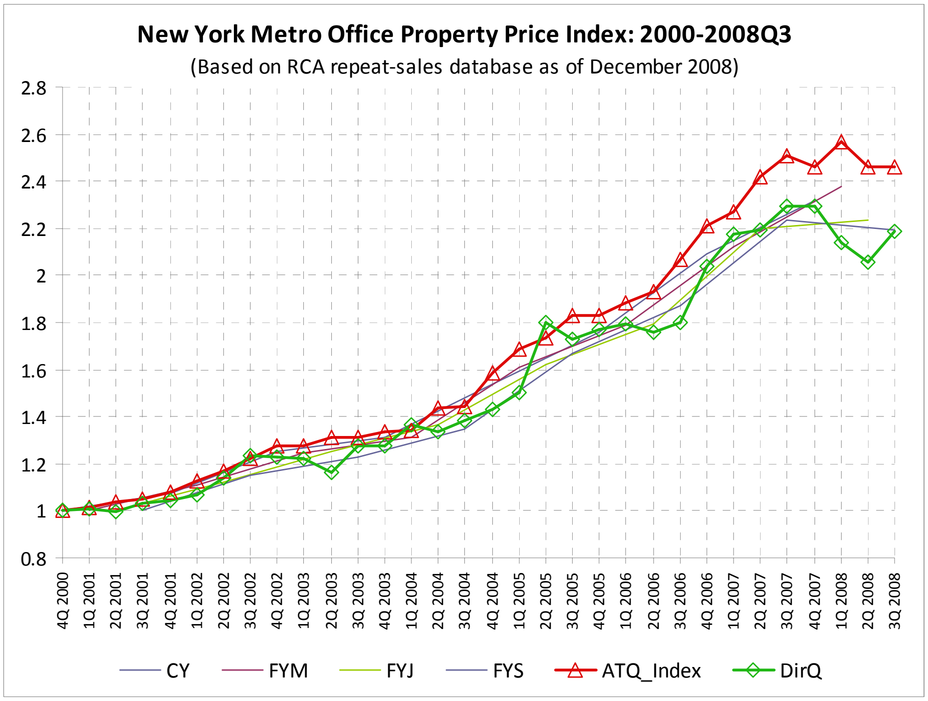
*Avg number of 2nd-sales obs/qtr 2006-08. Database was "immature" with considerably fewer 2nd-sales observations prior to 2005. (In parentheses number of obs in most recent 3Q08 qtr.)

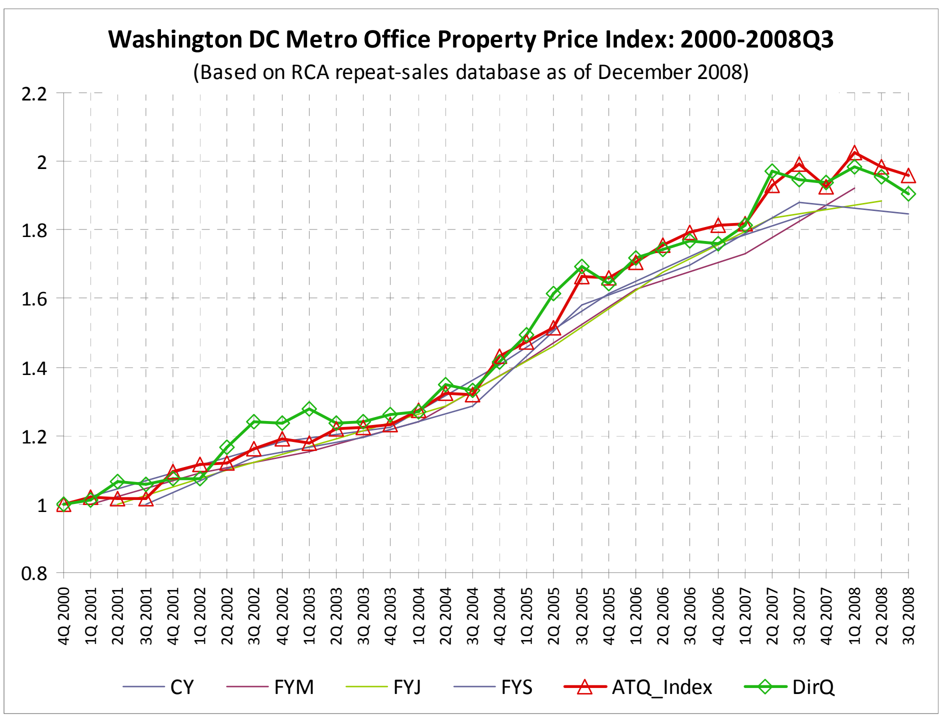
** Assuming actual market downturn was large: Greater negative magnitude of return is better (**Bold** ==> **ATQ better**; *Italic* ==> *DirQ better*).

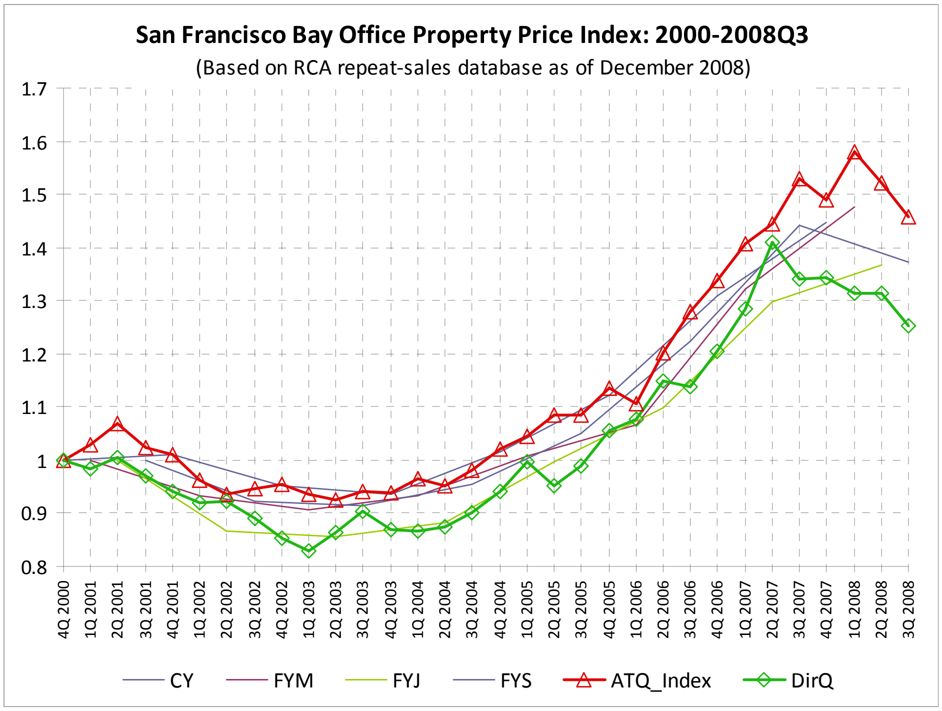
*** Difference: Correl(ATQ(t),DirQ(t+1)) – Correl(DirQ(t),ATQ(t+1)). **Positive** ==> **ATQ better**; *Negative* ==> *DirQ better*.

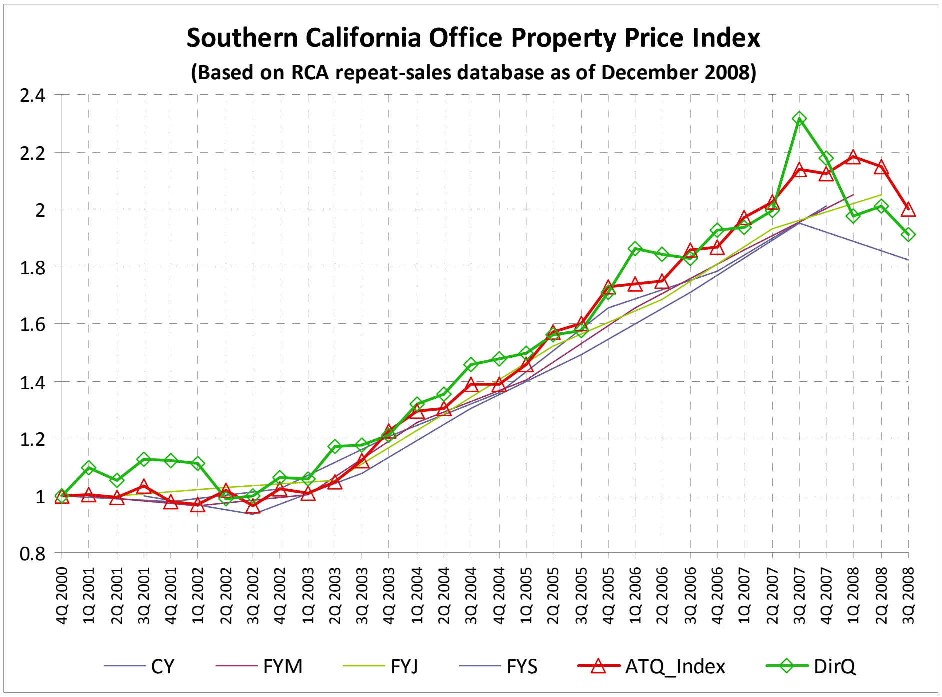
Appendix D:

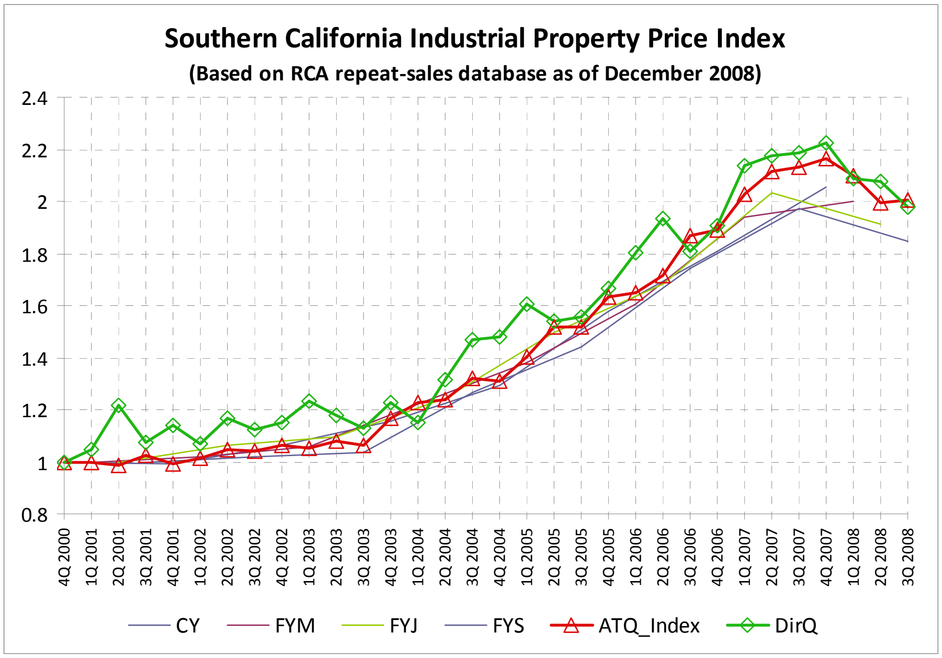
Charts of all 16 Moody's/REAL Annual Index Markets Showing ATQ & DirQ Indexes

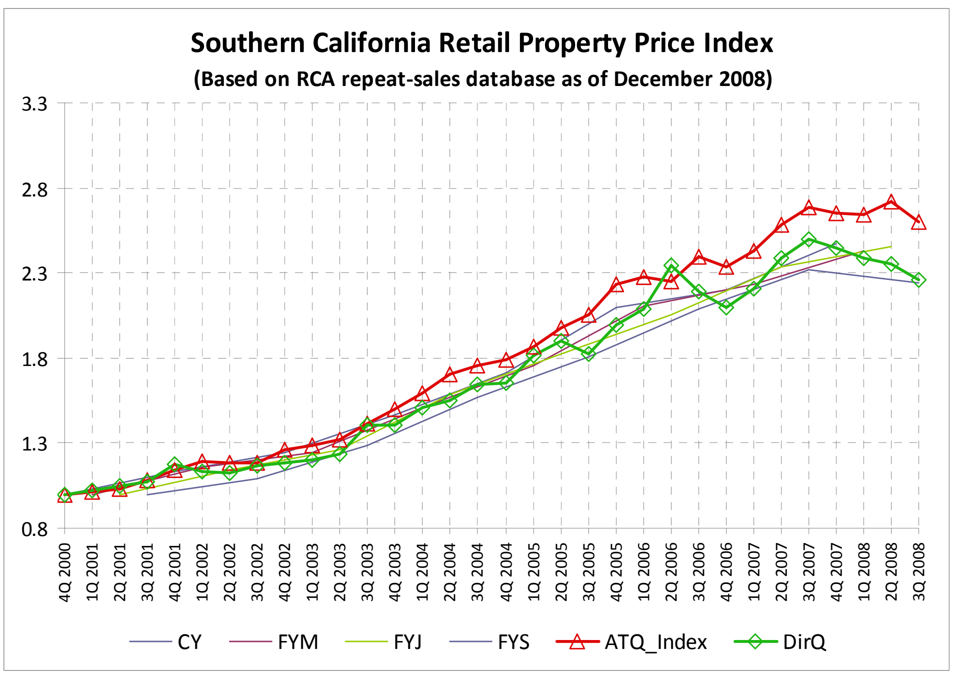


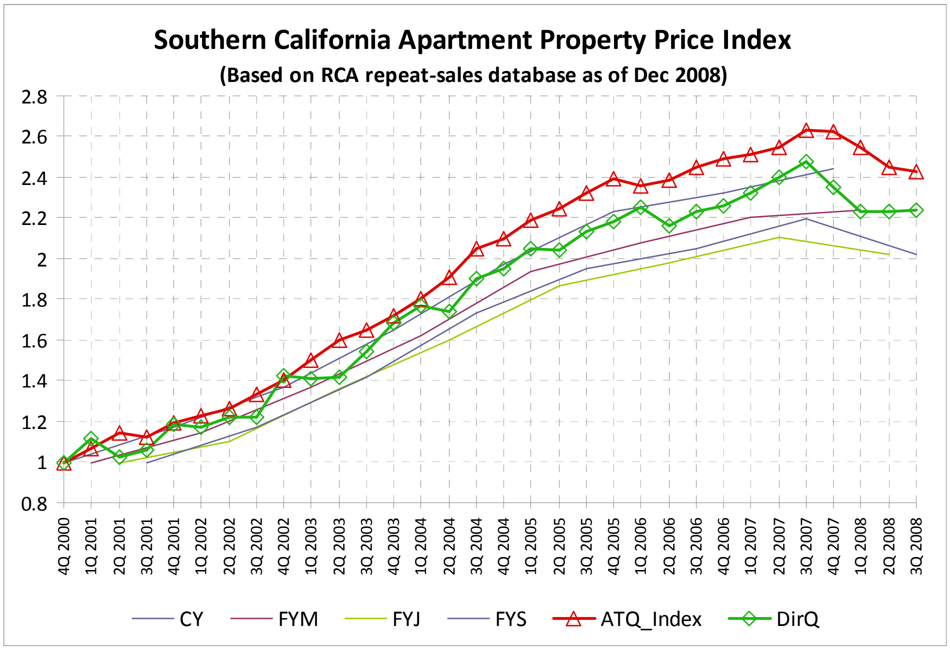


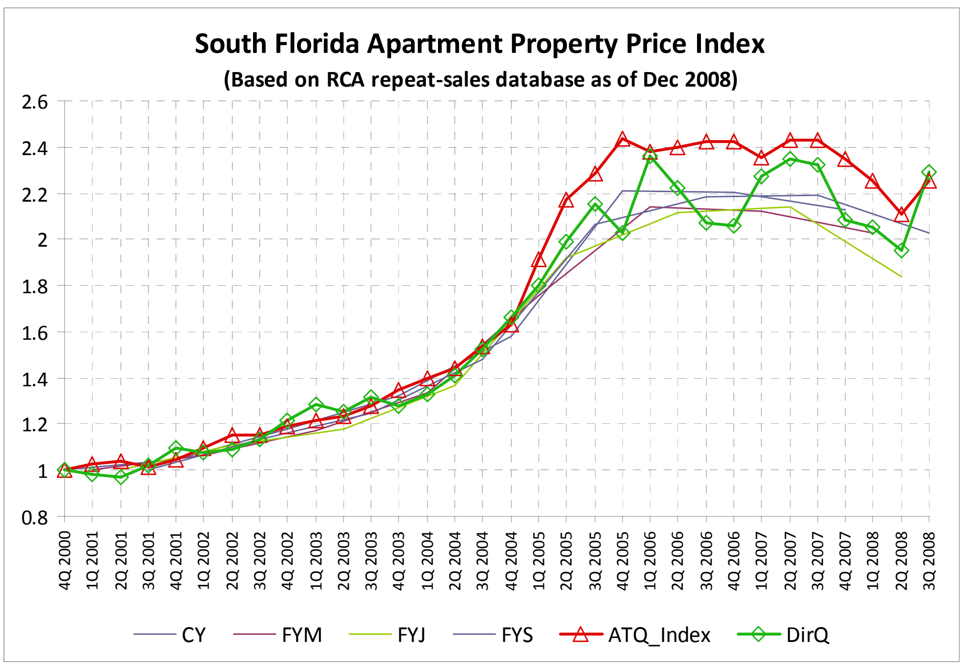

San Francisco Bay Office Property Price Index: 2000-2008Q3
(Based on RCA repeat-sales database as of December 2008)
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
4Q 2000 1Q 2001 2Q 2001 3Q 2001 4Q 2001 1Q 2002 2Q 2002 3Q 2002 4Q 2002 1Q 2003 2Q 2003 3Q 2003 4Q 2003 1Q 2004 2Q 2004 3Q 2004 4Q 2004 1Q 2005 2Q 2005 3Q 2005 4Q 2005 1Q 2006 2Q 2006 3Q 2006 4Q 2006 1Q 2007 2Q 2007 3Q 2007 4Q 2007 1Q 2008 2Q 2008 3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ

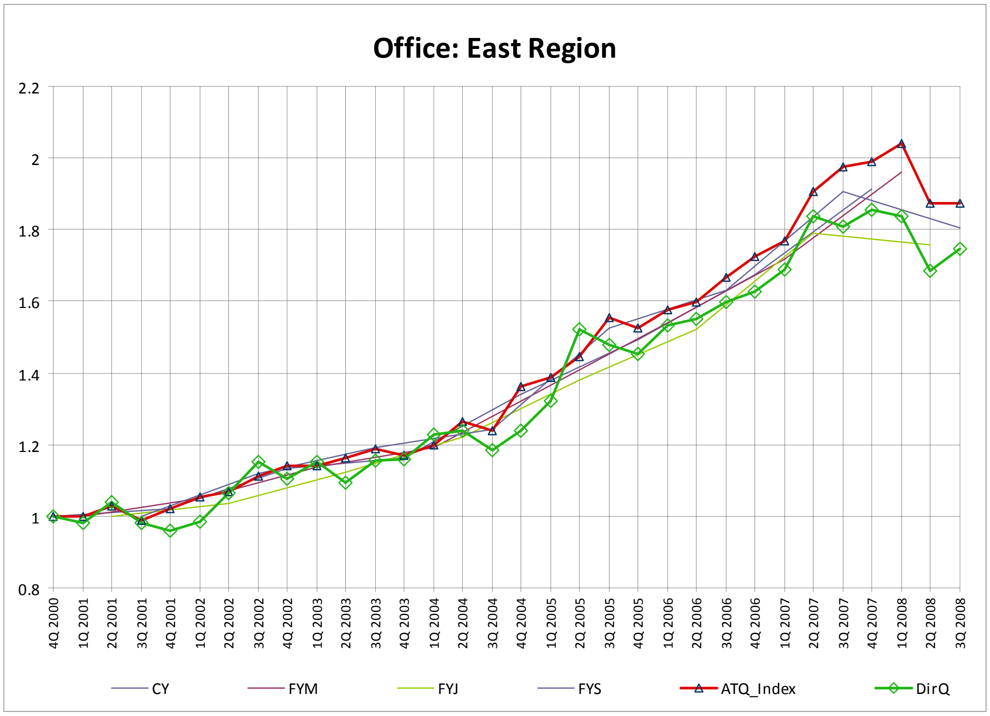

Southern California Office Property Price Index
(Based on RCA repeat-sales database as of December 2008)
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
4Q 2000 1Q 2001 2Q 2001 3Q 2001 4Q 2001 1Q 2002 2Q 2002 3Q 2002 4Q 2002 1Q 2003 2Q 2003 3Q 2003 4Q 2003 1Q 2004 2Q 2004 3Q 2004 4Q 2004 1Q 2005 2Q 2005 3Q 2005 4Q 2005 1Q 2006 2Q 2006 3Q 2006 4Q 2006 1Q 2007 2Q 2007 3Q 2007 4Q 2007 1Q 2008 2Q 2008 3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ


Southern California Industrial Property Price Index
(Based on RCA repeat-sales database as of December 2008)
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
4Q 2000
1Q 2001
2Q 2001
3Q 2001
4Q 2001
1Q 2002
2Q 2002
3Q 2002
4Q 2002
1Q 2003
2Q 2003
3Q 2003
4Q 2003
1Q 2004
2Q 2004
3Q 2004
4Q 2004
1Q 2005
2Q 2005
3Q 2005
4Q 2005
1Q 2006
2Q 2006
3Q 2006
4Q 2006
1Q 2007
2Q 2007
3Q 2007
4Q 2007
1Q 2008
2Q 2008
3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ

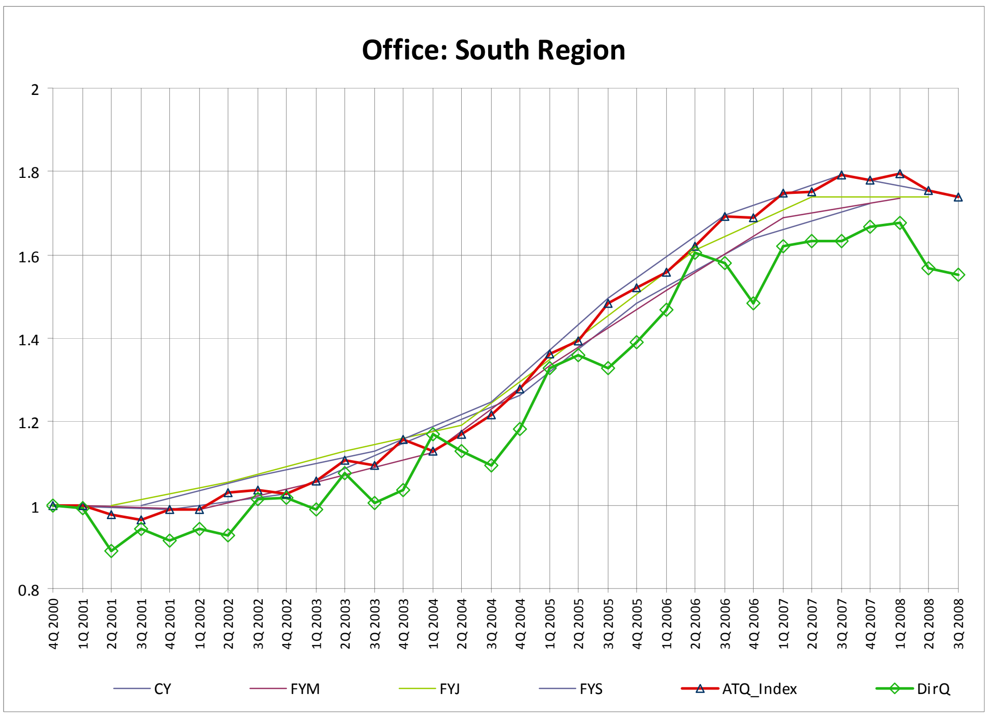

Southern California Retail Property Price Index
(Based on RCA repeat-sales database as of December 2008)
3.3
2.8
2.3
1.8
1.3
0.8
4Q 2000
1Q 2001
2Q 2001
3Q 2001
4Q 2001
1Q 2002
2Q 2002
3Q 2002
4Q 2002
1Q 2003
2Q 2003
3Q 2003
4Q 2003
1Q 2004
2Q 2004
3Q 2004
4Q 2004
1Q 2005
2Q 2005
3Q 2005
4Q 2005
1Q 2006
2Q 2006
3Q 2006
4Q 2006
1Q 2007
2Q 2007
3Q 2007
4Q 2007
1Q 2008
2Q 2008
3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ


Southern California Apartment Property Price Index
(Based on RCA repeat-sales database as of Dec 2008)
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
4Q 2000
1Q 2001
2Q 2001
3Q 2001
4Q 2001
1Q 2002
2Q 2002
3Q 2002
4Q 2002
1Q 2003
2Q 2003
3Q 2003
4Q 2003
1Q 2004
2Q 2004
3Q 2004
4Q 2004
1Q 2005
2Q 2005
3Q 2005
4Q 2005
1Q 2006
2Q 2006
3Q 2006
4Q 2006
1Q 2007
2Q 2007
3Q 2007
4Q 2007
1Q 2008
2Q 2008
3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ

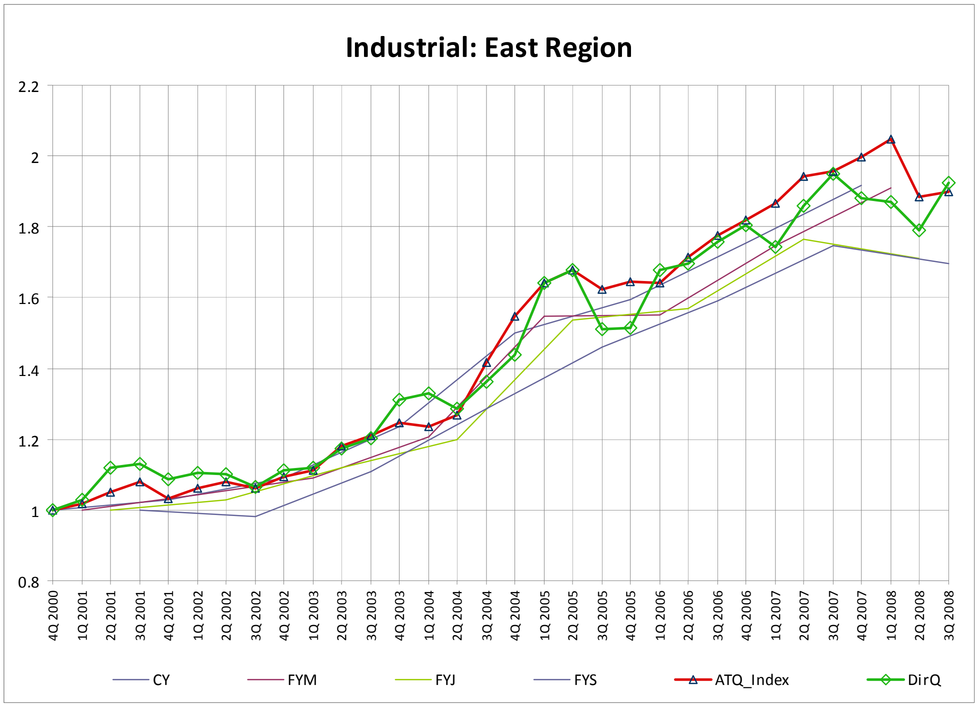

South Florida Apartment Property Price Index
(Based on RCA repeat-sales database as of Dec 2008)
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
4Q 2000
1Q 2001
2Q 2001
3Q 2001
4Q 2001
1Q 2002
2Q 2002
3Q 2002
4Q 2002
1Q 2003
2Q 2003
3Q 2003
4Q 2003
1Q 2004
2Q 2004
3Q 2004
4Q 2004
1Q 2005
2Q 2005
3Q 2005
4Q 2005
1Q 2006
2Q 2006
3Q 2006
4Q 2006
1Q 2007
2Q 2007
3Q 2007
4Q 2007
1Q 2008
2Q 2008
3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ

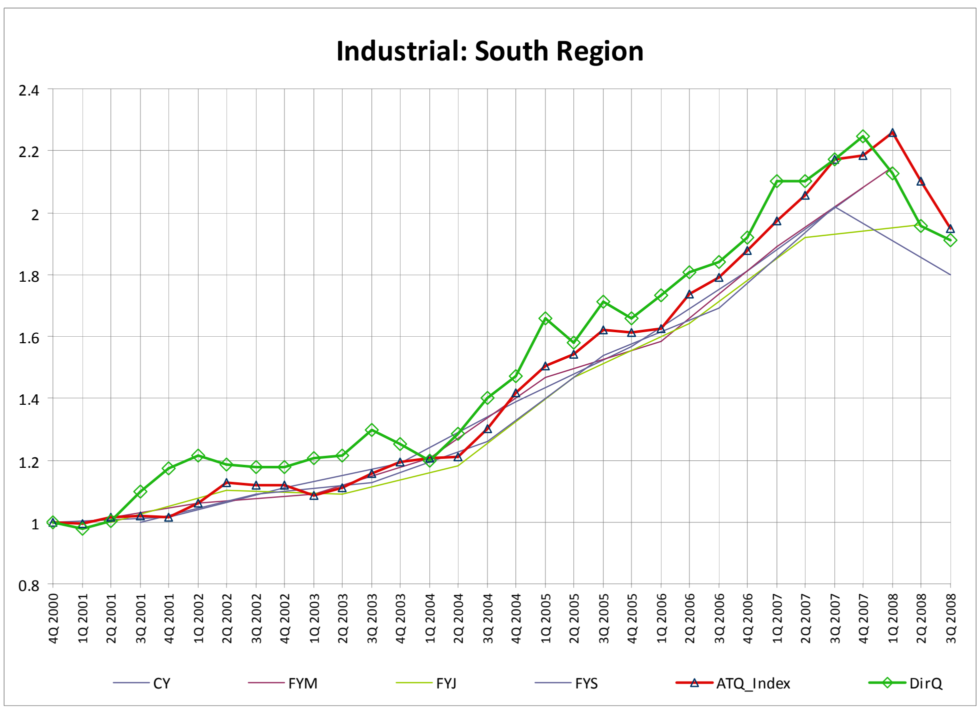


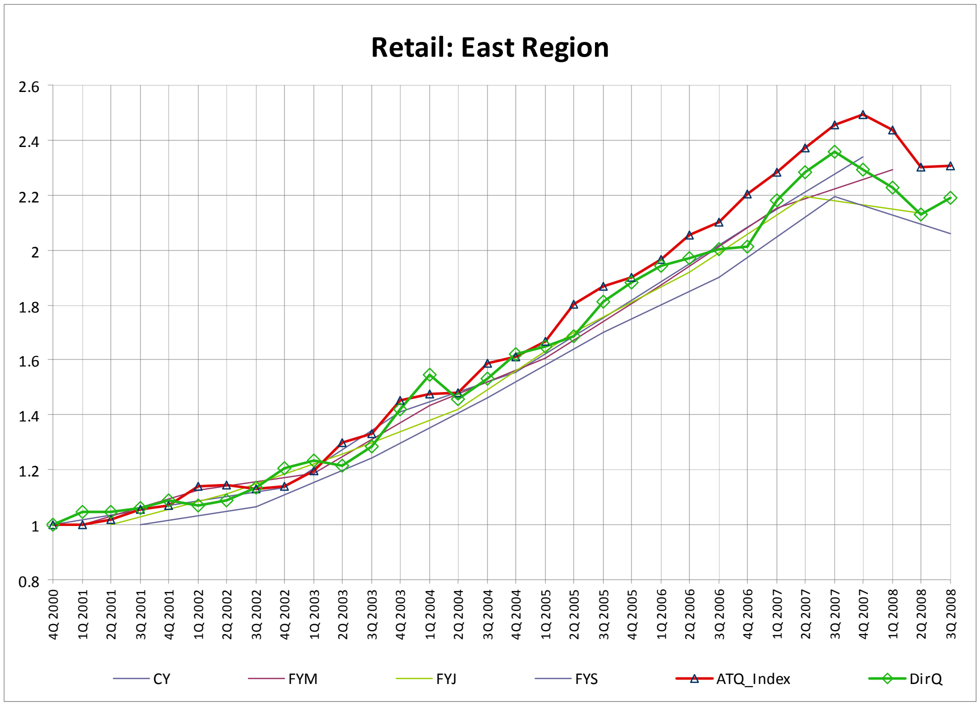
Office: East Region
2.2
2
1.8
1.6
1.4
1.2
1
0.8
4Q 2000
1Q 2001
2Q 2001
3Q 2001
4Q 2001
1Q 2002
2Q 2002
3Q 2002
4Q 2002
1Q 2003
2Q 2003
3Q 2003
4Q 2003
1Q 2004
2Q 2004
3Q 2004
4Q 2004
1Q 2005
2Q 2005
3Q 2005
4Q 2005
1Q 2006
2Q 2006
3Q 2006
4Q 2006
1Q 2007
2Q 2007
3Q 2007
4Q 2007
1Q 2008
2Q 2008
3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ



Office: South Region
2
1.8
1.6
1.4
1.2
1
0.8
4Q 2000
1Q 2001
2Q 2001
3Q 2001
4Q 2001
1Q 2002
2Q 2002
3Q 2002
4Q 2002
1Q 2003
2Q 2003
3Q 2003
4Q 2003
1Q 2004
2Q 2004
3Q 2004
4Q 2004
1Q 2005
2Q 2005
3Q 2005
4Q 2005
1Q 2006
2Q 2006
3Q 2006
4Q 2006
1Q 2007
2Q 2007
3Q 2007
4Q 2007
1Q 2008
2Q 2008
3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ


Industrial: East Region
2.2
2
1.8
1.6
1.4
1.2
1
0.8
4Q 2000
1Q 2001
2Q 2001
3Q 2001
4Q 2001
1Q 2002
2Q 2002
3Q 2002
4Q 2002
1Q 2003
2Q 2003
3Q 2003
4Q 2003
1Q 2004
2Q 2004
3Q 2004
4Q 2004
1Q 2005
2Q 2005
3Q 2005
4Q 2005
1Q 2006
2Q 2006
3Q 2006
4Q 2006
1Q 2007
2Q 2007
3Q 2007
4Q 2007
1Q 2008
2Q 2008
3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ


Industrial: South Region
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
4Q 2000
1Q 2001
2Q 2001
3Q 2001
4Q 2001
1Q 2002
2Q 2002
3Q 2002
4Q 2002
1Q 2003
2Q 2003
3Q 2003
4Q 2003
1Q 2004
2Q 2004
3Q 2004
4Q 2004
1Q 2005
2Q 2005
3Q 2005
4Q 2005
1Q 2006
2Q 2006
3Q 2006
4Q 2006
1Q 2007
2Q 2007
3Q 2007
4Q 2007
1Q 2008
2Q 2008
3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ


Retail: East Region
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
4Q 2000
1Q 2001
2Q 2001
3Q 2001
4Q 2001
1Q 2002
2Q 2002
3Q 2002
4Q 2002
1Q 2003
2Q 2003
3Q 2003
4Q 2003
1Q 2004
2Q 2004
3Q 2004
4Q 2004
1Q 2005
2Q 2005
3Q 2005
4Q 2005
1Q 2006
2Q 2006
3Q 2006
4Q 2006
1Q 2007
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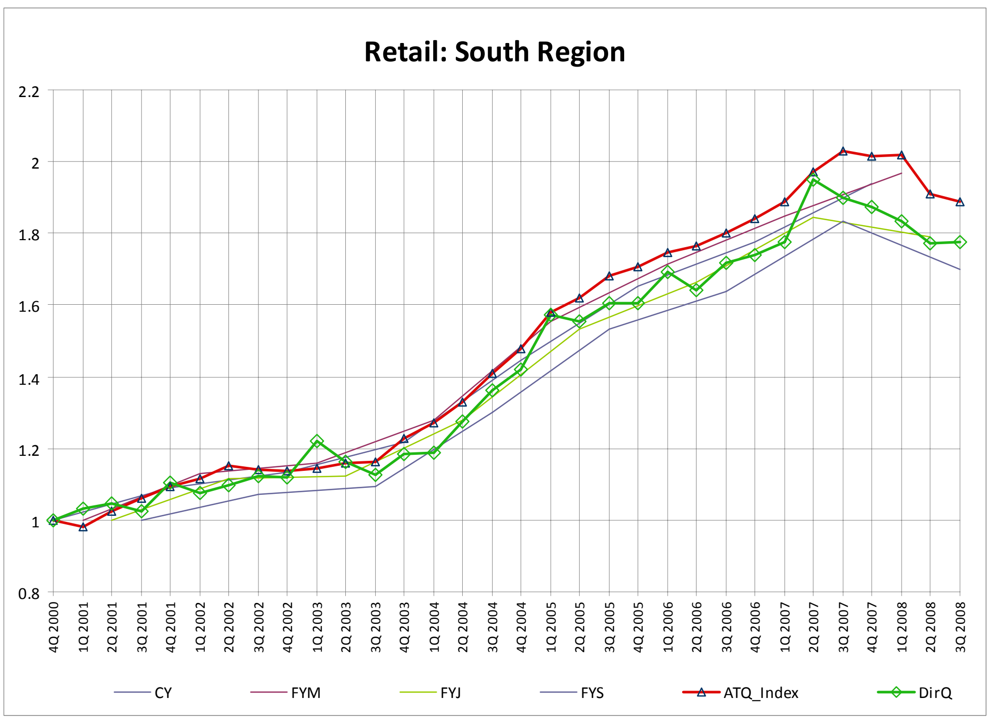
2Q 2008
3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ


Retail: South Region
2.2
2
1.8
1.6
1.4
1.2
1
0.8
4Q 2000
1Q 2001
2Q 2001
3Q 2001
4Q 2001
1Q 2002
2Q 2002
3Q 2002
4Q 2002
1Q 2003
2Q 2003
3Q 2003
4Q 2003
1Q 2004
2Q 2004
3Q 2004
4Q 2004
1Q 2005
2Q 2005
3Q 2005
4Q 2005
1Q 2006
2Q 2006
3Q 2006
4Q 2006
1Q 2007
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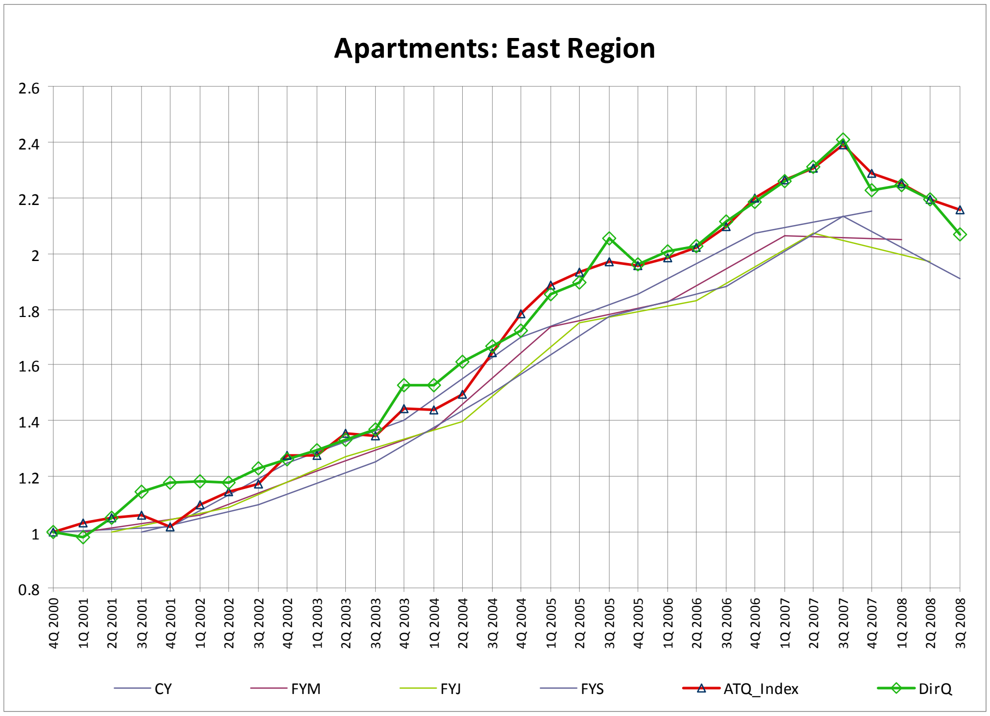
4Q 2007
1Q 2008
2Q 2008
3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ


Apartments: East Region
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
4Q 2000
1Q 2001
2Q 2001
3Q 2001
4Q 2001
1Q 2002
2Q 2002
3Q 2002
4Q 2002
1Q 2003
2Q 2003
3Q 2003
4Q 2003
1Q 2004
2Q 2004
3Q 2004
4Q 2004
1Q 2005
2Q 2005
3Q 2005
4Q 2005
1Q 2006
2Q 2006
3Q 2006
4Q 2006
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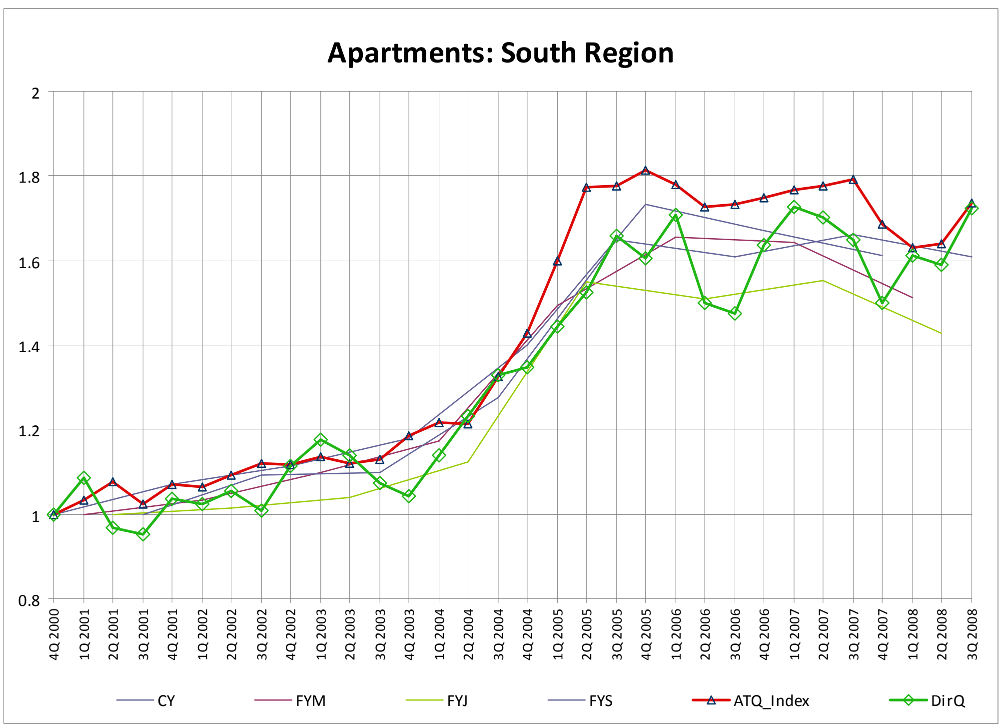
3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ


Apartments: South Region
2
1.8
1.6
1.4
1.2
1
0.8
4Q 2000
1Q 2001
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3Q 2001
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1Q 2002
2Q 2002
3Q 2002
4Q 2002
1Q 2003
2Q 2003
3Q 2003
4Q 2003
1Q 2004
2Q 2004
3Q 2004
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3Q 2006
4Q 2006
1Q 2007
2Q 2007
3Q 2007
4Q 2007
1Q 2008
2Q 2008
3Q 2008
CY
FYM
FYJ
FYS
ATQ_Index
DirQ