Structural Analysis of Tile Vaulting: Method and Variables.

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Keywords: catalan vault, graphic statics, finite elements.

Abstract

Although the interest in tile vaulting -Guastavino vaulting- is growing worldwide both in the academic and professional field, there are not few unknowns that remain unresolved regarding their structural behavior. Despite the publications concerning this issue -including the one by Guastavino-, there is still an open debate in the academic domain about the treatment of these vaults in their structural assessment and the decision of the structural analysis method.

Our research presents the analysis of six tile barrel vaults. Each vault has a variable incorporated -height (20-30 cm), number of layers (2-3) or existence of buttressing walls- or a combination of variables, which make it different. The vaults are assessed by the method of graphical analysis and finite element method, using uniform and asymmetric loads. At the same time, the six vaults are built to perform load tests on them and the material is studied in the laboratory to collect the data of its physical and mechanical features to be introduced in the calculation models. The construction of the vaults with the exact knowledge of its geometry, boundary conditions and material allows virtual models to be remarkably true to life, which rarely happens when we face the restoration of these vaults.

The comparison of the results is made with two objectives:

- To assess the suitability of the analytical methods used. To check its accuracy in predicting the real behavior of the vaults and to reveal the information that each provides.

- To evaluate the contribution of the selected variables to the stability of the vault. The introduction of these variables keeping the other parameters constant, allows the comparison of results with the corresponding knowledge of the contribution of the parameter that has been varied.
Prior approach

The calculation of the “catalan vault”:

This research aims to quantify the contribution of resistance and/or balance of the elements that determine the bearing capacity of the tile vault as shape, span, thickness (number of layers), existence of spandrel walls, filling, etc...

Definition:

Catalan vaults are masonry vaults, made of brick and binder (plaster, cement, mortar) and generally thin bricks are used, with the peculiarity of being placed flat. They can have one or more layers: the first, at least, is built with plaster performed without centering, and the consecutive ones are joined with mortar. They are built with very small thicknesses. Typically, they are two sheets (about 10 cm. overall, including the intermediate layer of mortar and coatings), but also one-layer vaults can be found (about 5 cm.). Their slenderness, ratio of the radius of curvature and span, is often near 100, but there are much more slender ones.

The construction process is simple and inexpensive in the context of pre-industrial techniques: a vault without centering and quick execution that could be built easily.

Some history and current structural theories:

Currently, “catalan vault” has the attention of many specialists scattered around the world (especially Europe and America).

Although it may seem obvious today that vaults have horizontal thrusts, even Rafael Guastavino defended their cohesive and monolithic behavior. However, demonstrating his ability as a constructor, he built them taking into account these thrusts.

Jacques Heyman set the modern basis for their graphic analysis. His famous hypothesis about masonry structures claimed that sliding between voussoirs is impossible, its tensile strength should be considered zero and its compressive strength is infinite. Under these conditions, the limit analysis theorems would apply to brick structures. Ricardo Gulli, however claimed the Finite Element Method (FEM) as the best way of analysis.

From this time, the work of Angel Truño in 1950 should also be highlighted. He wrote a treatise on the vault, studying exhaustively this building technique and the way it is constructed.

Ignacio Bosch i Reig defends membrane analysis. He proposes a system for calculating the domes forming imagining virtual nerves that would be supported on both elementary arcs produced by cuts parallel to the contour arches.

Joan Bergós’s contribution was also important: he performed extensive analysis and load tests on catalan vaults, which are included in his books: "Materiales y elementos de construcción" and "Tabicados huecos".

Following the Heyman’s hypothesis, Santiago Huerta from the School of Architecture of Madrid states that catalan vaults have little tensile strength, they crack and have horizontal thrusts. He recommends vaults to be calculated with equilibrium analysis, as any masonry structure. He rejects analysis by the finite element method.
In Barcelona, the architect José Luis González, is one of the leading specialists in restoration of Catalan vaults. He continued the work by Bassegoda conducting a comprehensive study of examples built on the 15th century. He defends preservation whenever possible and recommends load testing as the most reliable method to test its strength.

Pere Roca, an engineer from Barcelona also, recommends limit analysis and macromodelation by MEF, considering the following:

- A precise macromodelation of the geometry has to be made.
- Consider the material nonlinearity.
- Consider limited compressive strength
- Possible consideration of tensile non-zero (but very limited)
- Consider geometric nonlinearity.

The research group of John Ochsendorf (MIT) with Philippe Block (ETH) has developed the Thrust Network Analysis, an equilibrium analysis method in three dimensions. It is “a new methodology for generating compression-only vaulted surfaces and networks” (Block & Ochsendorf, 2007), which allows designing forms using the minimum compressive material.
The BLOCK Research Group at ETH Zurich University, led by Philippe Block, develops new software tools and CNC manufacturing. They explore the traditional technique combined with the new software and low-tech materials such as cardboard. The tile vault construction offers a little material that does not need a strong formwork for construction.

Megan L. Reese was a master student at MIT and her dissertation won the prize awarded by the Guastavino Biennial. She analyzed by static graphics and FEM Guastavino vaults and domes, and she recommended graphical statics for restoration.

**Methodology**

The research will be addressed from three perspectives: historical, analytical, and experimental.

Historically, to understand different “catalan vault” construction and analytical techniques employed throughout history.

Analytically, graphic and computer models are made to assess vaults’ behavior and failure.

Experimentally, we have built vaults for load tests in the future and have been performed in the laboratory specimens to characterize the material. The experimental results will be contrasted with those got from the theoretical models, thereby assessing their degree of accuracy and suitability as calculation procedures.

| **1. “Sencillado”:** Building the first layer with plaster. |
| **2. “Ladreado”:** Applying the binder on the brick before placing it. |
| **3. Placing the bricks:** With a single diagonal rap on the brick with the trowel. |
| **4. Scratching the joints:** To obtain more adherence between the layers. |
| **5. “Doblado”:** Building the second (or posterior) layer with mortar. |
| **6. Spandrel walls:** To stiffen the structure. |

Table 1. Commissioning work.
Object of study

There are many factors that determine the bearing capacity of “catalan vaults” such as its shape, its span and its thickness but also the mastery of the placing.

In order to quantify the contribution of resistance and/or balance of these elements, six vaults are modeled, each of which have a variation in thickness, height or can have, or not, spandrel walls. That will allow us to compare results for the evaluation of the impact of each element in the bearing capacity of the whole.

As shown in the figures below [fig.3 to 8], the vaults have a span of 3m and a width of 1m. The different parameters are: thickness (2 or 3 layers), the existence of spandrel walls and the height (20 or 30 cm). Load variables are: uniform loads, eccentric loads, and uniform + eccentric loads.

Types of vaults:

![Fig. 3. Type 01](image)

![Fig. 4. Type 02](image)

![Fig. 5. Type 03](image)
Fig. 6. Type 04

Fig. 7. Type 05

Fig. 8. Type 06
ANALYSIS

*Material characterization*

The first step to define the material in the program SAP2000 was to know its characteristics, which we also needed to apply the density on the graphic approach. The material characterization tests will be undertaken on the laboratory to provide the necessary data, once this is done, we will adjust the models to match virtual calculations with load tests.

This research is a preview of a doctoral thesis, not a finished research. At this point in the investigation, in which laboratory tests are not yet performed, the conclusions will only be obtained from the results of graphic FEM analysis.

The data used for the definition of the material have been taken from the book by Megan L. Reese "Structural analysis and evaluation of the Guastavino vaults", which in turn borrowed them from Atamturktur and Guastavino (Tables 2 and 3). We are aware that in order to compare the results with the load tests, we will have to modify the models according to the new material data obtained after the laboratory tests.

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<tr>
<td>Density (ρ)</td>
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<td>kN/m³</td>
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*Table 2. Results of materials tests, (Atamturktur 2006, 119)*
Compressive Strength, 5-day       14,19  N/mm²
Compressive Strength, 360-day     22,67  N/mm²
Tensile Strength                   1,98   N/mm²
Transverse (Bending) Strength      0,62   N/mm²

Table 3. Test results obtained by Guastavino (1892, 58-59)

Models

The ability to perform three-dimensional models is a plus for the FEM versus traditional limit analysis; we used three-dimensional models of each of the vaults. However, to obtain information about the section of the vault and compare it more directly with the graphic analysis, we have also used flat models (which get their third dimension in defining the finite element thickness).
**Loads**

The definition of load cases was determined by the graphical analysis. We introduced loads gradually, both uniform and eccentric, proving each time its stability.

Finally, different combinations of the following load cases are used in each model:

- **Uniform loads**: 2KN, 5KN, 10KN

  ![Fig. 23. Uniform load](image)

- **Eccentric punctual loads**: 3KN and three 1KN loads.

  ![Fig. 24-27. Punctual loads](image)
Comparing results

Now we will pay attention to the different methods of calculation watching any of the models, in this case, vault type 4:

The thrust line stays within the vault section which ensures its stability.

The thrust at the supports is similar in both calculations. It is noted that there are slight differences which may be due to the layout of the polygon of forces which may be variable.
We can see that the stresses reached in both FEM and graphical analysis are very low in relation to the ones that the material can resist. The stresses observed in the computer calculation vary along the thickness of the section (corresponding quite accurately with the upper and lower sides’ values of the 3-D model). If we make an average of the stress values in the section thickness, average stresses are around 0.2 N/mm², which coincide with those obtained in the graphical analysis.

Increasing the uniform load, the thrust line fits quite naturally into the vault section. This will occur in the graphical analysis of these vaults with all uniform load however great. The stability is ensured as long as the material resists. As we have already seen, the tensions remain very small in relation to the resistance of the material. When vaults are subjected to heavy loads, we will have to pay close attention to the movements at the supports, since the thrusts are considerably high.

Although in this example there seems to be a perfect correlation between the methods used, the analysis of the other types of vaults showed uneven results with the two different methods and with the two models of computation by FEM.

Proven the stability of the vault subjected to distributed loads, we now introduce a punctual load:

![Fig. 30. Vault type 4 with 3KN eccentric punctual load.](image)

Although the vault bears without any problem a uniform load of 10KN/m², the thrust line does not fit the section introducing only a punctual load of 3kN. It is observed in the finite element analysis that the value of the tension stresses obtained are almost equal to compressions, which would not be resisted by the material (remember that some authors recommend taking tensile strength as zero).

By adding to the system a uniform load of 2KN/m², stability improves. However, there are still significant tension stresses (observed in the models by FEM) and the thrust line doesn’t stay within the central third of the section, which causes a non-regular distribution of stresses.

Using the static and kinematic approach and the uniqueness theorem, we found that the punctual load causing collapse was 3,9KN with a uniform load of 2KN/m².
Fig. 31. Data for the kinematic analysis of vault type 4.

The addition of more uniform load would make the vault more stable, provided we have controlled the thrust at the supports of course. Regarding the graphical analysis (considering infinite compression strength), a very large uniform load would lead to a state of compression so that tensions caused by a punctual load of 3KN would seem insignificant.

A thicker vault also helps to the stability under these conditions. Thus, in model 5, -a vault with three layers of brick- stability is not committed for the same punctual load of 3kN.

Fig. 32. Vault type 4 with 2KN uniform load and 3 eccentric punctual loads of 1KN.
The use of a punctual load in the virtual models will not correspond exactly to the actual load that the vaults will suffer when performing the load tests. In reality the vaults are loaded with sand bags of 25 kg each. It is impossible that these bags transferred his weight as a punctual load at a point so precise, so the load will be distributed, but in a small space. Therefore, another load case has been created in the virtual model, transforming the punctual load of 3KN in three loads of 1KN, which is more real, and also more stable.

Fig. 33. Load test performed on vault type 4.

### Displacements

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Table 4. Displacements (in mm)

Being aware of the risk of obtaining quantitative values from a linear analysis by FEM of a masonry structure, this table can give some interesting conclusions by comparing the results:

**Graphs 1 and 2 (displacements in mm)**

Construction History Society of America
3rd Biennial Meeting American Construction History, 1850-1950
Cambridge MA, November 2-3, 2012
Comparing the deformations between the 6 different vaults, we observed that obviously deformations with uniform loads decrease slightly with a third layer of bricks, but that does not seem to affect vaults with spandrel walls in deformations with uniform loads, which are exactly the same in types 1 and 3 and types 4 and 6 (types of vaults with and without walls). This doesn’t happen when applying eccentric loads. The walls are able to reduce the deformations in these cases usually around 30 or 40%.

![Graph 3 and 4 (displacements in mm)]

It is also noteworthy the difference in the results between 3D and 2D models in the vault types 2 and 5 (with three layers of brick). Clearly one of the two models is not appropriate, since the results are very different. When adding a third layer of bricks, deformation decreases much more in 2D models than in 3D ones. In the flat model, the thickness of the three layers of brick is drawn directly while drawing the finite elements, and is the assignment of the “thickness value” of that element which gives depth to the arch drawn. However in the 3D model the number of layers of brick is defined by assigning the thickness to the finite element. This difference in the binding between the finite element’s nodes and the definition of its third dimension seems to be the key of this inconsistency.

![Graphs 5 and 6 (displacements in mm)]

As for the height of the vault, there is a big difference between the deformations of the vaults of 20 cm and 30 cm. Under uniform loads lower vaults deform twice. With the existence of eccentric loads 20 cm-high vaults also deform more, but this difference becomes narrower, with a deformation of 10 to 30% larger.
Conclusions

This paper presents the first part of an investigation that is still ongoing, therefore, the conclusions derived from this first phase of research are partial conclusions, they will also have to be qualified and expanded as we get further results.

The study has shown uneven results with different methods of calculation. We understand the limitations and existing error by proposing a linear elastic analysis by finite elements for masonry vaults. The quantitative data obtained have been used for comparison and as a mere auxiliary diagnostic tool, since we are aware of the inadequacy of the application of linear elastic analysis because of the fact of not considering the low tensile strength of the material. This type of analysis can only provide qualitative information, identifying high compression points or associating tension to cracks or fissures (Pere Roca, 2012).

However, the fact that seems more relevant is the discrepancy between the data obtained with different models (2D or 3D) of the same type of vault using FEM. While in some of the analyzed types of vault the convergence of results (with an admissible error) was a fact, in others, the data obtained were far from representative. The variation in the results can naturally vary depending on the type of mesh or finite element used, but in this case the error exceeds what would be logically acceptable.

In this incipient thesis a nonlinear analysis by FEM will be performed in order to demonstrate the suitability or inadequacy of this method for such structures and the data will be corroborated with the results given by real load tests. However, we can already conclude that whereas the equilibrium analysis using static and kinematic approach yields reliable data on the structural behavior of the vault, a simple linear analysis by the finite element method has been shown in the case studies to be unreliable. It is also doubtful that a more laborious adjusted model could solve the problem, but anyway, we discourage this job considering the relatively small information that we get with this type of analysis.
## Bibliography

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