Comment

Road Congestion: The Diagrammatic Analysis

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I. Introduction

In a recent article, De Meza and Gould (1987) discussed the change in the income distribution resulting when a common property resource, to which everyone formerly had free access, was taken into private ownership. They produced several analytically distinct examples to demonstrate that it is not necessarily true that all those who formerly had free access would be made worse off by the change even if no compensating transfers were made. I would not wish to argue with most of the cases that they present, but one that relates to the use of a congested road is clearly incorrect.

In their analysis of the problem, De Meza and Gould use what appears to a reader to be the standard graphical analysis of a problem involving demand and supply. However, it is incorrect to apply the graphical approach in this form to the analysis of road congestion, and so, in showing that their conclusion is incorrect, I also show that the conventional graphical approach is incorrect. Although doubt has been expressed before (see Neuberger 1971), the correct approach to the diagrammatic analysis of road congestion has not previously been shown.

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II. Analysis

What causes confusion in the analysis of road congestion is the fact that the variable factor (vehicles plus drivers) is under the control of the ultimate consumer of the final product (journeys completed). In conventional analysis the demand for a factor of production (e.g., labor) is derived from the demand for the product. The consumer's decision to buy the product is separate from the laborer's decision to work for the wage offered. In the case of road congestion the decision to carry out a journey is inseparable from, indeed identical with, the decision to put a vehicle on the road.

When the road congestion problem has been analyzed graphically in the past, authors have represented demand as represented by traffic flow (e.g., Neuberger [1971] and De Meza and Gould [1987]; note that Walters [1961] avoids the pitfall). But whereas consumers choose whether to buy goods given the price, they do not choose a traffic flow given the price. The traffic flow is an endogenous variable resulting from the characteristics of the road and interactions among road users. They actually make a choice whether, given the cost of a journey, they should put their vehicles on the road. Thus the decision to undertake a journey affects the number of vehicles on the road, or density, directly and traffic flow only indirectly. So the true demand curve relates traffic density (the number of vehicles on the road) to the cost of the journey, including the time costs imposed by congestion. The higher the cost of the journey, the fewer the people willing to put their vehicles on the road, so that this demand curve, as shown by the line DD' in panel A at the bottom left in figure 1, slopes downward in the conventional way.

Furthermore, it is obvious that the larger the number of vehicles on the road, the greater the level of congestion, and so the greater the cost of a journey. It follows that potential road users both perceive themselves to be and are facing a conventionally upward-sloping supply curve. This supply curve is therefore shown by the line SS' in panel A of figure 1 as sloping upward in the conventional way.

As readers will be aware, however, a feature of the graphical analysis of road congestion is the backward-bending cost curve, displayed by Walters (1961), De Meza and Gould (1987), and most commentators in the years between. This curve is the line ss' in panel C of figure 1, derived from the curve in panel A using a construction due to Worcester (1969).

As stated above, the feasible traffic flow is a function of the number of vehicles on the road, that is, density, and is determined by the technical characteristics of the road and the interaction of road users. Because traffic speed decreases as density increases, traffic flow as a
function of density reaches a peak and then declines. Let $F$ be the flow, $Z$ the density, and $f(Z)$ the speed ($f' < 0$). Then $F = Z \cdot f(Z)$, and traffic flow reaches a maximum when $f + Zf' = 0$ and declines when $f + Zf' < 0$. In panel B at the top left in figure 1, the curve relating traffic flow to density is therefore shown as rising and then declining.

From the relationships shown in panels A and B and with a simple 45° line at the top right, the two curves shown in panel C can be derived showing the relationship between traffic flow and journey cost both on the supply side and on the demand side. The first of these, $sxs'$, will be the curve familiar to readers. It initially slopes upward from left to right, with cost being lowest with no other vehicles on the road; it becomes steeper as congestion increases, becomes vertical when traffic flow is at a maximum, and then bends backward sloping upward from right to left.
The other line, $dxd'$, will not be familiar. It shows the relationship derived from the demand curve in panel A: the way in which the "desired" traffic flow changes as the cost of travel changes because the number of vehicles put on the road changes. So if the cost of travel is high, the number of would-be travelers is low so that traffic density is low and the desired traffic flow is low. If the price or cost of travel falls, the number of cars on the road increases and so, therefore, does the desired traffic flow until it reaches a maximum. If the price or cost of travel were to fall further, however, still more vehicles would be put on the road and the increase in density would lead to a fall in the traffic flow. This curve is therefore shown as sloping downward from the upper left to the right and then bending back to slope down from right to left.

So this "derived demand curve" is backward bending just as the average cost curve is backward bending. The upper branch, $dv$, of the derived demand curve is associated with the lower branch, $sw$, of the cost curve since both result when traffic density is low and speeds are high. On the other hand, the lower branch, $vd'$, of the derived demand curve is associated with the upper branch, $ws'$, of the cost curve since both result when traffic density is high and speeds are low.

### III. Free Access, Private Ownership and Monopoly

With the correct approach to the graphical analysis of road congestion, it is easy to see the errors in De Meza and Gould's approach. They use the graphical model shown here as figure 2 to demonstrate their argument. The flow along a given stretch of road is indicated along the horizontal axis, and costs and prices are indicated on the vertical axis. They correctly show the average cost curve, $ss'$, rising as the traffic flow increases but with an increasing slope that becomes vertical as the maximum traffic flow is reached and then slopes upward and backward as the traffic flow decreases because of increasing congestion.

In the literature on road congestion, it is generally assumed that the price that people will be willing to pay to travel along a given road is fixed by the costs of traveling along alternative routes and by the costs of alternative means of transport. The demand curve is then horizontal, and as readers can confirm, it matters little whether the approach used in panel A or panel C of figure 1 is used: the same conclusions will be reached. De Meza and Gould, however, draw what appears to be a conventional downward-sloping demand curve, marked $DD'$ in figure 2. With this they purport to demonstrate that
with free access the flow will be $F_1$ and the cost per traveler will be $C_1$; if a toll is charged to ensure that the cost of travel equals the marginal social cost, then the flow will be $F_2$ and the cost will be $C_3$ (including a toll of $C_2C_3$).

The authors then argue that there are "two further points [that they] have been unable to trace in the literature" (p. 1324). This is not surprising since, as I shall show, they are not only counterintuitive but they are also wrong. First, since the inclusive journey cost (time cost plus toll) is lower after access is controlled (i.e., $C_3 < C_1$), "the efficiency toll may result in all road users' being better off." Second, even if the road is controlled by a monopolist, who maximizes profits by setting a toll such that marginal revenue equals marginal social cost, the cost to road users will still be less than in the free-access case (i.e., $C_5 < C_1$), so that "it is possible for all road users to be better off if the road becomes private property and the owner exerts his monopoly power."

The reason why these two points are wrong can be easily stated. The increase in the flow, and in the speed of flow, after the imposition of the toll is achieved only because the toll deters some road users from embarking on journeys that they would have made if there had been no toll. Those who are deterred from using the road by the toll are clearly worse off than before. So when De Meza and Gould write that all road users would be better off, they are incorrect.
They confuse the flow—the number of journeys completed in a period—and the number of road users at a point in time—the density of traffic on the road. While the flow may increase so that the number of journeys completed in a period may increase, it does not follow that all road users gain.

With the correct diagram this can be clearly shown. In panel A of figure 1, the marginal social cost curve is drawn as a function of density, and its equivalent is derived in panel C as a function of traffic flow. Because it is a curve derived from that in panel A, the marginal social cost curve in panel C shows the social cost of an additional vehicle on the road at different traffic flows, not the social cost of an additional unit of traffic flow at different traffic flows; the latter is the curve that is usually drawn, but it is actually irrelevant and misleading. Only that part of the marginal social cost curve relating to traffic flows below the maximum is shown; that is, it relates to the lower branch of the average cost curve and the upper branch of the derived demand curve. In the absence of a toll, the traffic flow is $F_1$, indicated by the intersection of the lower branch of the derived demand curve with the upper branch of the average cost curve. The cost to users is $C_1$, and as shown in panel A, the density is $D_1$.

The optimal traffic flow is $F_2$, and with a toll equal to $C_2C_3$, marginal social cost will be equal to the price paid by drivers. The optimal density as shown in panel C is $D_2$. Even though the optimal traffic flow is greater than it would be with a toll ($F_2 > F_1$), the number of road users is lower ($D_2 < D_1$) and the cost of travel is higher ($C_3 > C_1$). So it is not true that the introduction of a toll will mean that each traveler pays less; nor are all road users better off with a toll, as De Meza and Gould allege. Some road users are deterred from attempting to travel, and it costs the others more than if there were no toll. It is, moreover, clear that monopolistic pricing by the owner of the road will result in travelers' being worse off.

IV. Conclusions

In this comment, it has been shown that the conventional graphical analysis of the economics of road congestion may be incorrect if a conventional downward-sloping demand curve is drawn relating traffic flow to cost. Since would-be travelers have no control over the traffic flow, only over whether or not they put their vehicles on the road, in any graphical analysis a conventional demand curve should relate density (number of vehicles) to cost.

The analysis is then used to show that De Meza and Gould (1987), who use the incorrect approach, are wrong to conclude that the introduction of a toll on a highly congested road could make all road users
better off. First, some would-be travelers are deterred from traveling by the toll. This eases congestion and so results in faster travel for everybody else, but those who are deterred are obviously worse off. It is only continuing road users who might conceivably be made better off by the introduction of a toll. But, second, in most circumstances the toll raises the cost to travelers by an amount that is greater than the reduction in costs because less time is spent on the journey, so that even continuing road users are not necessarily made better off.

Note that the curve in panel A of figure 1 shows the marginal social cost of an extra vehicle on the road (i.e., an increase in density), and this is also what is shown by the curve in panel C. The latter does not show the marginal social cost of a unit increase in traffic flow, a concept that is meaningless if, for example, one is trying to determine the toll to be charged to a motorist wishing to put a vehicle on the road.

References


