# Modeling Urban-level Impact of a Shared Taxi Market. 

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#### Abstract

Taxi systems are being challenged by alternative, emerging services like Uber, Lyft, and Sidecar, which increasingly offer the option of ride sharing. While the enormous potential of ride sharing has been unveiled in a number of recent papers, it also raised legitimate concerns about the potentially disruptive impact on other transportation modes. In this paper, we introduce a framework for estimating the urban-level impact of ride sharing applied to the current taxicab service. First, we extend a representative economic model of regulated taxi markets to include ride sharing. The model allows predicting the interactions between demand and supply of a shared taxi service based on a few representative parameters, and is rooted on data analytical results. Then, we apply our model to the case study of the New York taxi market. The analysis highlights the dramatic impact of the pricing policy and taxi fleet management on the urban-level, systemic outcomes of a shared taxi system.


[^0]
### 1.1 Introduction

Taxi is a common and comfortable point-to-point transport system, introduced in Europe in the early 17 -th century and playing a major role in urban transportation since then. The services offered to riders evolved following several major technological developments: the diffusion of combustion-based engines at the end of 19-th century; adoption of twoway radios to communicate with dispatch offices in the 1940s; computeroptimized vehicle dispatching in the late 1980s, to cite the major ones. In the last 20 years, GPS technologies and mobile data made it possible to track the position and the availability of each vehicle (and user) in real time, improving service performance and opening up new business opportunities.

### 1.1.1 Many innovations, same business model

Despite the many technological advancements and changes that occurred in the history of this means of transportation, the taxi system business model, the hailing experience, and the fare system have remained the same: the final price of the ride is computed considering an initial amount of money (flag), plus a variable amount of money derived on the basis of time/distance travelled, and extras (such as baggage, tips, night extra fares, highway tolls...), no matter how many people are on-board. Consistently, in traditional taxi systems a cab can be either "vacant" or "occupied", and it can accept trip requests only when in "vacant" state. The result is a massive amount of taxicabs on the streets, about half of which wanders in search of new passengers ${ }^{1}$, which is clearly undesirable for traffic and pollution.

### 1.1.2 The risk of being disrupted by new entrants

The widespread diffusion of Internet-connected devices and the development of new business models leveraging on the emergent socio-economic trend of sharing economy $[12,16]$ are rapidly changing the landscape of individual, point-to-point transportation in urban environments, making the taxi only one of the many alternative modes available to citizens. Carsharing services like car2go [2] or Zipcar [20] are gaining popularity among urban population. People can find an available car with the phone, ride it, pay it per minute/hour, and leave it for someone else when the trip

[^1]is over. Bike and scooter sharing services rely on similar principles. If the user still prefers to be transported as a passenger, a shared van can be cheaper than a taxi while moving groups of people [1]. While these services still represent alternative ways to move in the cities, with a different proposition with respect to taxis, cab companies are increasingly being directly challenged by the so-called TNCs (Transportation Network Companies) like Uber, Lyft, and Sidecar.

TNCs step into the extremely fragmented market of taxicab services by acting as a centralized Internet-based multisided platform that leverages mainly on private citizens' car fleet, re-shapes the user experience, and innovates the pricing mechanism. TNCs take care of the verification of drivers' licenses, provide virtual assistance, additional insurance, driver-rating mechanisms and automatic payment processing systems, replacing the taxicab dispatch central with advanced algorithms. By joining these platforms, the customer is being put at the center of a totally new experience where hailing a car is as simple as pressing a button on the smartphone. Furthermore, all payments are electronically managed, an aspect that is convenient for both parties (passengers avoid to pay by cash, drivers avoid unpaid rides and the risk of robbery). Thanks to their intrinsic scalability, business agility, and a different cost structure, companies like Uber are disrupting the established market, continuously challenging the incumbent taxicab companies by adding new features and lowering fares. The result, as shown in Figure 1.1, is a decreasing demand for traditional taxis, and a consequent drop in the price of taxi medallions after a continuous and uninterrupted growth that lasted for many decades [7].

### 1.1.3 Ride-sharing

The innovation in the value proposition is increasingly including the notion of ride sharing as a key feature to gradually reduce the fare prices, a key strategic move for disruptors [5]. The "UberPool" feature, recently launched by Uber in San Francisco, Paris, New York and Los Angeles, allows a passenger to share the ride with another that is going in the same direction. This service has been described by the company as a new way "to deliver transportation at lower and lower price points" [19], aiming to start a virtuous cycle where demand increases, more cash-flow is generated and money is re-invested in big-data analysis to perfect the server-side dispatch algorithms and maximize driver utilization rates, which in turn enables further price reductions.

The fact that ride sharing is considered a key feature of innovative transportation services should be no surprise to urban planners and policy makers: the evolution of cities in history has been profoundly impacted by
the movement of citizens and goods, and resulting emerging features such as those summarized in the well-known Christaller's central place theory [4] clearly hints to the fact that a large fraction of urban trips should be "shareable". This intuition has been confirmed by recent studies such as [14], which unveiled the immense potential of ride sharing in the city of New York: more than $95 \%$ of taxi trips can be shared, with a minimal impact on passenger discomfort ${ }^{2}$.

The immense potential for ride sharing has raised legitimate concerns regarding the impact of innovative transportation services at urban level. If not wisely implemented, these services might have undesired effects such as reduced job opportunities for taxi drivers, lower demand for public transportation with negative impact on carbon footprint, etc. [8, 13, 15]. Fully addressing these concerns requires performing a comprehensive study of the impact of ride sharing at urban level, and of its integration with other transport modes.

Making a first step in this direction is the goal of this paper. More specifically, we extend current urban economic models of regulated taxi markets to include ride sharing. Ride sharing brings a radical transformation into the market, which becomes an instance of segmented market where the same good (a vacant taxi) can satisfy two classes of customers: those requesting a single trip, and those willing to share their trips. Starting from this model, we build a framework for predicting the interactions between demand and supply of a shared taxi service based on a few representative parameters: the market share $m$ of the ride sharing service, the discount factor $d$ applied to the price of a shared vs. a single ride, and the number $N$ of taxis in the market. The framework is rooted on the data analytical results of [14], which allows accurately predicting the likelihood of sharing a taxi ride as a function of the market share $m$ for the city of New York.

[^2]

Fig. 1.1 San Francisco taxi demand decline: the plot reports the average number of monthly trips per vehicle. (source: S.F. Municipal Transportation Agency).

The application of our framework to the case study of the New York cab market allows for the first time to quantify the dramatic impact of the pricing policy on the urban-level, systemic outcomes of a shared taxi system. Recently raised concerns [8, 13, 15] are legitimate: with an ill-designed pricing policy, ride sharing might actually negatively impact transportation. For instance, the fact that a single taxi can serve multiple customers might lead to a reduction of the number of taxi drivers quantifiable in about 16,600 units in the city of New York. On the other hand, a pricing policy where, defining $P$ as the per-mile price of a ride in the current taxi market: 1) the average per-mile price of a ride in the shared taxi market is $P ; 2$ ) single ride passengers are penalized by paying a per-mile price $P^{\prime}>P$ for the single ride; and 3) shared ride passengers benefit of a reduced price $d \cdot P^{\prime}<P$, leads to a desirable systemic outcome where the total demand of taxi services is unchanged (implying no negative impact on public transport), the total number of miles travelled by taxis is reduced (including both vacant and occupied trips), the number of taxi drivers is unchanged, and the average income of taxi drivers is increased. The analytical framework presented in this paper can help urban planners and policy makers to better understand the transformations brought along by innovative ride sharing services, and to make informed decisions leading to desirable systemic outcomes.

### 1.2. Towards an upgrade of the taxicab operating model

The current taxicab system needs to have many empty cabs on the street to work properly. The perceived quality of the service for the customers relies in the system's response time, i.e., the average waiting time to "find" an empty cab. The search for a vehicle can happen in many different ways: a call to the cab company dispatch, walking to a taxi stand, by using the cab company or a third party smartphone or web app (Uber or the like), or by hailing a car on the street.

In the following, we present a possible model of shared taxi system based on [6]. We start presenting how demand and supply of taxi service are modeled for a traditional, single ride system.

Demand of taxicab service, denoted $Q$, in regulated markets depends on:

- $P$ : the average price (per unit of distance) of the service;
- $V$ : the average number of vacant taxis (per unit of time);
- $X_{D}$ : a variable modeling the effect of exogenous factors such as economic activity (employment data), price of public transportation, summer effect, etc.

Given the above definitions, the demand in a traditional, non-shared taxi system can be expressed as follows:

$$
\begin{equation*}
Q=P^{\beta} \cdot V^{\gamma} \cdot X_{D} \tag{1.1}
\end{equation*}
$$

where $\beta<0$ is the demand elasticity with respect to price, and $\gamma>0$ is the demand elasticity with respect to service availability. Elasticity with respect to service availability is positive since a larger number of vacant taxis typically implies shorter waiting times, hence a better quality of service that, ultimately, results in higher demand. According to equation (1.1), with unchanged exogenous conditions, the demand of taxicabs is therefore expected to increase when either the fare goes down or more vacant taxis are available, since people can easily find a cab when they need one. The offer (market supply of taxicabs) depends on:

- $N$ : the number of taxis in the regulated market, as determined by city regulations;
- $\tau$ : the individual supply of taxicab service (miles driven by each cab per unit of time).

Parameter $\tau$ is intended to model taxi driver's behavior, expressed as an average amount of time he/she spends on the road serving passengers. The total supply $T$ of the taxi cab service can then be computed as

$$
\begin{equation*}
T=N \cdot \tau \tag{1.2}
\end{equation*}
$$

and, by definition, it must satisfy equation $T=Q+V$. I.e., the total supply equals the satisfied demand of the taxi service $(Q)$, plus the amount of vacant taxis (excess supply).

To determine the value of $\tau$, and, hence, the total supply, Flores-Guri assumes a profit maximization driver's strategy, where the profit of the driver depends on price $P$, occupancy ratio $Q / T$, and the operational cost $A>0$. More specifically, $\tau$ can be computed as

$$
\tau^{*}=\operatorname{argmax}_{\tau}\left\{P \cdot \frac{Q}{T} \cdot \tau-A \cdot \tau^{\alpha}\right\}
$$

where $\alpha<1$ accounts for the fact that the operation cost increases more than proportionally with the miles driven (e.g., to include for driver tiredness). The value of $\tau^{*}$ can be derived from first order condition indicating that a profit-maximizing driver chooses $\tau^{*}$ such that the cost of driving one additional distance unit (e.g., a mile) equals the expected revenue for that mile. Hence,

$$
\tau^{*}=\left(\frac{\alpha \cdot P \cdot Q}{A \cdot N}\right)^{\alpha}
$$

which yields

$$
T=N \cdot \tau^{*}=\left(\frac{\alpha}{A}\right)^{\alpha} \cdot P^{\alpha} \cdot Q^{\alpha} \cdot N^{1-\alpha} \cdot X_{S}
$$

where $X_{S}$ has been added to account for exogenous factors such as minimum wage. An alternative approach, more consistent with the scenario in which the taxi is owned by a company and the drivers lease the taxi to operate in shifts, is to simply assume that $\tau$ is a constant, roughly corresponding to the duration of the shift. In the following, we will apply both approaches to study the shared taxi market.

### 1.2.1 Ride sharing models

What happens if taxicabs can be shared with other people going the same way? We first observe that there exist at least two ways of operating a shared taxi system, called static and dynamic ride sharing.

In the static model, the requests for a shared ride are collected by the taxi service operator for a short time interval (say, a few minutes), and only trips in the current pool of collected requests are considered for sharing. If two trips from the pool can be shared, they are matched, and a single taxi is dispatched for accommodating both trip requests. From that time on, and until the time at which the last passenger is dropped, the taxi is considered as occupied and not available for further ride sharing (even if there are still available seats onboard). Thus, similarly to traditional taxi systems in the static scenario the taxi can be in one of two possible states: vacant (no passenger onboard) or occupied (one or more passengers onboard).

In the dynamic model, taxis can instead be in one of three states: vacant, when there is no passenger onboard; shareable, when there is at least one passenger onboard but seats are still available for sharing; and occupied, when all available seats are occupied. In this model, requests for a shared ride are possibly matched not only with currently unserved requests, but also with already ongoing shared trips being served by shareable taxis. In case a new trip is assigned to a shareable taxi, the driver is informed of the new passenger to pick-up, and a re-route is done to pickup the new passenger, possibly before current passengers are dropped off.

Both models have pros and cons. The static model is easier to run and operate, and offers the customer a better travel experience: upon pickup, the customer knows expected travel time to destination (possibly including pickup/drop off of other passengers), and this planned route does not change after departure. On the other hand, the static model is not able to fully exploit potential sharing opportunities offered by partially occupied taxis, as it is instead done by the dynamic model. On the downside, the dynamic model is more complex to run and operate, and undoubtedly offers a lower-quality travel experience to customers, whose arrival time at destination is no longer accurately predictable at pickup time due to possible dynamic re-routing of the taxi.

It is interesting to observe that the two main TNCs, Uber and Lyft, are currently operating ride sharing services adopting different approaches: while Uber is developing its algorithms around a dynamic model [18], Lyft is opting for a static one [10].

In the interest of simplicity and presentation clarity, in the following we present a possible model of a static shared taxi system, based on the assumption that no more than two trips can be combined into a shared trip
(two-trips static sharing). This choice is consistent with one of the sharing scenarios analyzed in [14], and it is also supported by the results reported in [14] showing that, even in this constrained sharing model, more than $95 \%$ of the trips can potentially be shared in the city of New York.

### 1.2.2 Static sharing taxi system model

A shared taxi system shall be analyzed as a market in which the same good (a taxi) is requested by two classes of customers: those requesting an individual ride, and those requesting a shared ride. Hence, we shall consider a scenario in which the market is split between the two classes of customers according to same ratio $0 \leq m \leq 1$, where $m$ models the market share of customers requesting a shared trip. In other words, total demand $Q$ in a shared taxi system shall be intended as the sum of two disjoint demands for individual trips $\left(Q_{I}\right)$ and for shared trips $\left(Q_{S}\right)$, i.e.:

$$
Q_{S M}=Q_{I}+Q_{S}
$$

where

$$
m=\frac{Q_{S}}{Q_{I}+Q_{S}}
$$

Following [6], all quantities reported in equation (1.1) should be considered as average values computed across the whole market. In particular, $P$ in equation (1.1) must be intended as the average price of a ride in the analyzed market. Thus price $P_{S M}$ in the shared taxi market can be computed as follows:

$$
\begin{equation*}
P_{S M}=(1-m) \cdot P_{I}+m \cdot\left[s \cdot P_{S}+(1-s) \cdot P_{I}\right] \tag{1.3}
\end{equation*}
$$

where $s$ denotes the probability that a shared trip request can be matched with another trip to actually form a shared ride, $P_{I}$ denotes the average price of an individual ride, and $P_{S}$ denotes the average price of a shared ride. According to equation (1.3), the average price in the shared taxi market can be computed accounting for the relative market share of individual and shared trip requests (by means of parameter $m$ ); furthermore, in case of a shared trip request, the average price has to take into account that the request for a shared trip can be successfully matched with another request (with probability $s$ ), or cannot be matched, in such case (occurring with probability $(1-s)$ ) being served as an individual trip request.

Equation (1.3) can be simplified under the assumption that the price for a shared ride is computed as a discounted fare with respect to an individual
ride, i.e., $P_{S}=d \cdot P_{I}$, where $0<d<1$ is the discount factor for the shared ride service. Under this assumption, the average ride price can be rewritten as:

$$
\begin{equation*}
P_{S M}=P_{S M}(m, s, d)=P_{I} \cdot[1-s \cdot m \cdot(1-d)] \tag{1.4}
\end{equation*}
$$

A further simplification is possible observing that $s$ - the probability of successfully sharing a trip - is positively correlated with $m$ - the market share of shared trips: total demand $Q_{S M}=Q_{I}+Q_{S}$ being equal, a higher value of $m$ implies a larger demand $Q_{S}$ of shared trips; hence, a larger pool of potentially shareable trips and, ultimately, a larger value of $s$. By elaborating on the findings of [14], it is possible to describe the relationship between $s$ and $m$ by means of the Hill's equation, as reported below:

$$
\begin{equation*}
s=f(m) \approx \frac{K \cdot m^{n}}{1+K \cdot m^{n}} \tag{1.5}
\end{equation*}
$$

where $K=11,462.1$ and $n=1.77 .{ }^{3}$ The curve $f(m)$ describing the relationship between $s$ and $m$ is reported in Figure 1.2.


Fig. 1.2 Curve $f(m)$ describing the relationship between $s$ and $m$ in the city of New York, derived from the over 150 million taxi trips analyzed in [14].

[^3]Substituting (1.5) into (1.4) yields:

$$
P_{S M}=P_{S M}(m, d)=P_{I} \cdot\left[1-\frac{K \cdot m^{n+1}}{1+K \cdot m^{n}} \cdot(1-d)\right]
$$

We are now equipped with all necessary definitions to define the total demand with static taxi ride sharing, which can be expressed as follows:

$$
\begin{equation*}
Q_{S M}=P_{S M}^{\beta} \cdot V_{S M}^{\gamma} \cdot X_{D} \tag{1.6}
\end{equation*}
$$

where $V_{S M}$ denotes the fact that, as we'll see next, all other parameters being equal, the supply in case of a shared taxi system is in general different from that of a traditional system, due to the fact that a single taxi can satisfy multiple (shared) trip requests simultaneously.

We now turn our attention to the supply side of the market. As observed above, the fundamental difference between the traditional and the shared taxi market is that in the shared taxi market a single taxi ride can serve multiple customers (up to two in the model at hand). Thus, the first step in our analysis is computing the expected number of customers served in a typical ride. This can be computed as follows:

$$
\begin{align*}
n_{c}=n_{c}(m)= & 1 \cdot(1-m)+1 \cdot m \cdot(1-s)+2 \cdot m \cdot s  \tag{1.7}\\
& =1+\frac{K \cdot m^{n+1}}{1+K \cdot m^{n}}
\end{align*}
$$

accounting for the fact that a taxi serves a single passenger in case of a single ride request (occurring with probability $(1-m)$ ), or in case of an unsuccessful shared ride request (occurring with probability $m \cdot(1-s)$ ), and serves two passengers otherwise. Notice that the last equality in (1.7) follows from equation (1.5) and simple algebraic manipulation. Notice also that, as expected, $1 \leq n_{c} \leq 2$, and $n_{c} \rightarrow 2$ as $m \rightarrow 1$.

The total supply $T_{S M}$ in the shared taxi market can then be defined by generalizing (1.2) as follows:

$$
\begin{equation*}
T_{S M}=n_{c} \cdot N \cdot \tau_{S M} \tag{1.8}
\end{equation*}
$$

Following the profit maximizing driver's strategy of [6], we now proceed to defining the expected driver's profit, which can be expressed as follows:

$$
\begin{equation*}
\tau_{S M}^{*}=\operatorname{argmax}_{\tau}\left\{n_{c} \cdot P_{S M} \cdot \frac{Q_{S M}}{T_{S M}} \cdot \tau_{S M}-A \cdot \tau^{\alpha}\right\} \tag{1.9}
\end{equation*}
$$

accounting for the fact that the driver serves, on average, $n_{c} \geq 1$ customers per trip. Substituting (1.8) into (1.9) yields

$$
\tau_{S M}^{*}=\operatorname{argmax}_{\tau}\left\{P_{S M} \cdot \frac{Q_{S M}}{N \cdot \tau} \cdot \tau_{S M}-A \cdot \tau_{S M}^{\alpha}\right\}
$$

implying that, by the same first order analysis as in [6], we have:

$$
\tau_{S M}^{*}=\left(\frac{\propto \cdot P_{S M} \cdot Q_{S M}}{A \cdot N}\right)^{\propto}
$$

It is insightful to observe that $\tau_{S M}^{*}$ and $\tau^{*}$ are defined similarly, the only difference being the values of average price and demand. This is because, on one hand, sharing rides implies a value of $n_{c}$ greater than 1 , hence, a potential higher revenue for the driver. On the other hand, the expected occupancy ratio $Q_{S M} / T_{S M}$ is reduced of a corresponding factor $n_{c}$, reflecting the fact that, all other parameters being the same, a single taxi serves $n_{c}$ customers: this means that less passengers are available for pickup, negatively impacting the occupancy ratio. Alternatively, $\tau_{S M}$ can be defined to be a constant independent of demand, largely determined by the duration of the shift.

In the next section, we show how the notions of demand and supply derived herein can be applied to achieve different desired system outcomes by acting on the fare price and share discount, which should be thought as control parameters of the shared taxi system.

### 1.3 Case studies

We now show different applications of the model derived in the previous section. The analysis is referred to the taxi market of New York City. For this market, we have the following parameters, taken from [11] and [6]: $P=5.15 \$ /$ mile $, N=10,500, \beta=-0.937$ and $\gamma=0.102$.

### 1.3.1 Constant demand

The first case study considers a situation in which the regulator is interested in keeping the total demand of taxi service unchanged in the transition from traditional to shared taxi system. This scenario finds its motivation in the fact that increasing the demand of taxi as a result of ride sharing might be considered detrimental by city authorities, since this additional demand might come at the expense of a reduced demand for public transportation
services - which should be preferred due to the reduced impact on traffic and pollution. Concerns about the fact that taxi sharing services might reduce the demand for public transportation have been recently expressed in the literature $[8,15]$.

Based on the above, we impose the condition:

$$
Q=Q_{S M}
$$

yielding

$$
P^{\beta} \cdot V^{\gamma}=P_{S M}^{\beta} \cdot V_{S M}^{\gamma}
$$

under the assumption that exogenous factors have a similar effect on both markets. This promptly yields the following expression for the average price of a ride in the shared taxi system:

$$
P_{S M}=\left(P^{\beta} \cdot \frac{V^{\gamma}}{V_{S M}^{\gamma}}\right)^{\frac{1}{\beta}}=P \cdot\left(\frac{V}{V_{S M}}\right)^{\frac{\gamma}{\beta}}
$$

The value of $P_{S M}$ as a function of the ratio $\frac{V}{V_{S M}}$ is reported in Figure 1.3.


Fig. 1.3 Value of the average ride price in a shared taxi system, as a function of the ratio $\frac{\mathrm{V}}{\mathrm{V}_{\mathrm{SM}}}$. The plot also reports the average price for the traditional taxi system (red line), taken from [11].

According to Figure 1.3 and to intuition, if the number of vacant taxis does not change (i.e. $\frac{V}{V_{S M}}=1$ ), then the only way of keeping the demand constant is to set $P_{S M}=P$, i.e., the average price should not change as well. Notice that, in a shared taxi market, the average price is determined by the price of the single ride $P_{I}$, as well as by parameters $m$ and $d$, implying that the regulator can act on the combination of these parameters to achieve the desired output. By fixing any two of these parameters, it is possible to compute the third in such a way that the resultant value of $P_{S M}$ is as desired. Indeed, while two of the three mentioned parameters $-P_{I}$ and $d$ - are under full control of the regulator, the third parameter - the split $m$ between single and shared ride passengers - cannot be directly controlled. One way of dealing with this is to consider $m$ as an input parameter given by market conditions, and to act only on $P_{I}$ and $d$ to achieve the desired average price $P_{S M}$ : this is the approach undertaken in this paper. In case the current market split $m$ is not desirable - e.g., because the number of shared rides is very small -, the regulator might apply pro-active policies such as increasing the discount factor $d$ to impact the value of $m$ in the desired direction. In a more general sense, then, parameters $m$ and $\left\{P_{I}, d\right\}$ should be considered as mutually dependent. Quantifying this inter-independence requires having access to shared mobility data, and is left outside the scope of the present paper.

The value of $P_{I}$ resulting in $P_{S M}=P$ for different market shares $m$ (keeping $d$ fixed to 0.7 ) is reported in Figure 1.4 , while Figure 1.5 reports the same value of $P_{I}$ as a function of $d$ when $m$ is 0.5 . As seen from the plots, the only way of achieving $P_{S M}=P$ is to increase the price for single ride passengers above $P$. This increase gives some slack to reduce price for shared ride passengers below $P$, and keep the average price $P_{S M}$ the same as in the traditional market. Thus, at least a minimal number of single ride passengers is beneficial for shared ride passengers: when $m<1$, the price of a shared ride is strictly lower than $P$, but it becomes equal to $P$ as $m \rightarrow 1$.


Fig. 1.4 Price of the single ride (blue curve) in the shared taxi market, for different values of $\boldsymbol{m}$. Parameter $\boldsymbol{d}$ is set to 0.7 . The plot reports also the price of the shared ride (orange), and of the single ride in the traditional taxi system (red).

It is interesting to analyze also the supply side of the market, keeping the working assumptions of $Q_{S M}=Q$ (unchanged demand) and $V_{S M}=V$ (unchanged number of vacant taxis), which implies $P_{S M}=P$ as we have seen. Given that $T=Q+V=Q_{S M}+V_{S M}=T_{S M}$ we have:

$$
\begin{equation*}
T=N \cdot \tau=n_{c} \cdot N_{S M} \cdot \tau_{S M}=T_{S M} \tag{1.10}
\end{equation*}
$$



Fig. 1.5 Price of the single ride (blue curve) in the shared taxi market, for different values of $\boldsymbol{d}$. Parameter $\boldsymbol{m}$ is set to 0.5 . The plot reports also the price of the shared ride (orange), and of the single ride in the traditional taxi system (red).


Fig. 1.6 Percentage reduction in number of taxis as a function of the market split $m$.

Constant individual supply. Let us consider the constant individual supply model, i.e., $\tau_{S M}$ is constant and does not depend on demand. Under the assumption that $\tau_{S M}=\tau$, i.e., that the hours spent on road by a taxi are the same as in the traditional taxi market, equation (1.10) yields

$$
N_{S M}=\frac{N}{n_{c}}
$$

This implies that the number of taxis in the shared market should be reduced with respect to the case of traditional taxi market to keep demand unchanged. The amount of this reduction as a function of the market split $m$ is reported in Figure 1.6: as the market share of shared rides increases, the number of taxis needed in the market reduces, up to close to $50 \%$ when $m$ approaches 1 . Notice that reducing the number of taxis might be desirable for reducing traffic; however, reducing of a similar amount the number of taxi drivers might be undesirable from the societal perspective. However, the fact that the revenue generated by a single taxi would increase of a factor $n_{c}$, suggests a possible way of reducing the negative impact on workforce: reducing the duration of the shift.

Let us clarify this point with an example. According to [11], the average duration of a shift in New York is 9.5 hours. Assuming a two-shift taxi utilization scheme, we have that $\tau=\tau_{S M}=19 \mathrm{~h}$. Assume for the sake of this example that $m=0.5$, which implies $n_{c}=1.499$. If a single driver operates the taxi during a shift, his/her profit in the shared taxi market would be about $50 \%$ higher than in the traditional market. This would make the drivers that remain in the market very happy, but would have the undesirable consequence of leaving about $33 \%$ of the taxi drivers out of work, which corresponds to about 16,600 people in the city of New York [11].

Alternatively, one can think of designing a more flexible shift structure, in which taxis and drivers are pooled together. A driver can, say, have a shift composed of two mini-shifts: one on taxi $A$ for, say, 4 hours, and one on taxi $B$ for, say, 4 more hours. The notion of mini-shift allows a single taxi to be operated for 19 hours daily (i.e., $\tau_{S M}=\tau$ ), but this occurs being driven by $2 \cdot n_{c}$ drivers on the average, instead of 2 as in the traditional taxi market. Let us elaborate more the previous example under this scenario. Suppose the goal of regulators is keeping the overall driver's profit unchanged. In this case, thanks to the notion of mini-shift it is possible to keep the workforce unchanged, with a total duration of the per-driver shift of about 6.5 hours.


Fig. 1.7 Average number of passengers per ride, as a function of the market split $m$.

Profit maximizing driver strategy. Let us consider the profit maximizing driver strategy model. Assume $N=N_{S M}$, which implies $\tau_{A}^{*}=\frac{\tau^{*}}{n_{c}}$. In this case, the only way of keeping the total supply unchanged, thus satisfying equation (1.10), is to have taxis spending shorter times on the road with respect to the traditional taxi market. The ratio between $\tau_{S M}^{*}$ and $\tau^{*}$ equals $\frac{1}{n_{c}}$, which, in turns, depends on the market share $m$ of shared rides: the higher $m$, the higher the expected number of passengers on a ride, with corresponding reduction in number of hours driven - see Figure 1.7. This is exactly the same reduction in driving time as in the case of mini-shift described above. The difference is that in this case a single taxi is operated for a time shorter than the 19 hours of the traditional taxi market, and taxi drivers do not need to operate on mini-shifts and rotate taxis.

Discussion. When collectively considered, the analyses reported in this section show that, independently of whether drivers follow a profit maximizing or a fixed driving time strategy, it is possible to envision scenarios in which the total workforce is not reduced, and drivers actually obtain better conditions receiver higher hourly wages. In both analyzed models, the total miles driven by the taxi fleet would be reduced of a factor $n_{c}$ with respect to the case of traditional taxi market (e.g., of about $33 \%$ when $m=.5$ ), with corresponding reductions in traffic and pollution generated by the taxi fleet.

### 1.3.2 Increasing demand

We now consider a scenario in which the demand of taxi service is increased of, say, $20 \%$; i.e., $Q_{S M}=1.2 \cdot Q$. In turn, this implies

$$
\begin{equation*}
P_{S M}=P \cdot 1.2^{\frac{1}{\beta}} \cdot\left(\frac{V}{V_{S M}}\right)^{\frac{\gamma}{\beta}} \tag{1.11}
\end{equation*}
$$

Also, in order to avoid draining demand for public transportation, we assume that the average price of a shared taxi trip is at least twice as expensive as the single ride mass transport ticket price. Considering that in New York the price of mass transport ticket is $2.50 \$$, and that the average length of a taxi trip is 2.6 miles [11], we have

$$
2.6 \cdot d \cdot P_{I} \geq 2 \cdot 2.5
$$

i.e., $d \cdot P_{I} \geq 1.92$. This equation gives a lower bound on the price for a shared ride, which is considered in the scenario at hand. In the following, we consider a scenario where $P_{I}=P$, i.e., where the single ride passenger is not penalized nor favored in the transition to the shared taxi market. This yiels $d \geq 0.37$. Assuming an aggressive discount policy for shared rides, we set $d=0.4$ in the following. By equation (1.11), and assuming $V_{S M}=1.2 \cdot V$ so that the total supply is also increased of $20 \%$ with respect to the traditional taxi market, we obtain that the above setting of the parameters is possible when the market split $m$ equals .3 , i.e., only $30 \%$ of the requested rides are shareable. This mimics an early stage of the shared taxi market, where most of the customers still use single rides, and the price policy is designed to facilitate transition to a more "shared" market while not sacrificing single ride passengers.

What are the implications of the above pricing policy and demand shaping on the supply side? For the sake of simplicity, let us assume constant individual supply, i.e., that $\tau_{S M}$ is constant and does not depend on demand; furthermore, assume $\tau_{S M}=\tau$, i.e., that the hours spent on road by a taxi are the same as in the traditional taxi market. Observing that $T_{S M}=1.2 \cdot T$, we obtain

$$
N_{S M}=1.2 \cdot \frac{N}{n_{c}}
$$

which, given that $n_{c}=1.3$ with the parameters at hand, implies a marginal reduction of about $8 \%$ in the total number of taxis needed to accommodate the demand of the shared taxi system. As explained in the previous section, this reduction does not necessarily imply a corresponding reduction in number of taxi drivers, since the mini-shift approach can be used to distribute the relatively higher welfare generated by the shared taxi market to a larger number of drivers by reducing the per-driver shift.

Notice that that the $8 \%$ reduction in number of taxis corresponds to an equal reduction in total traveled miles in the scenario at hand. Summarizing, in a scenario where the total demand of taxi service is increased by $20 \%$, but the price structure is designed to prevent draining demand for public transport, we can still observe benefits of a shared taxi system, which is able to satisfy more passengers than the traditional system while decreasing of about $8 \%$ taxi related traffic. Notice that compounding traffic benefits can be achieved if the higher demand satisfied by the shared taxi system comes from private vehicle traffic.

### 1.4 Conclusion

In this paper, we have extended an existing economic model of demand and supply of taxicabs considering the implementation of a static ridesharing model, basing our analysis on real data from the city of New York. Our analysis clearly points out how the introduction of taxicab ride-sharing services can produce contrasting effects. This opens for a discussion at the policy making level, where an operational systemic cooperation among medallion owners (being them companies or private drivers) has to be promoted, and both existing pricing strategies and fleets operations need to evolve. The goal of this study is stimulating further analysis and discussion on this topic, highlighting the main variables and interdependences that have to be considered while envisioning the implementation of a taxicab ride sharing service, considering both the demand and supply side dynam-
ics. New Transportation Network Companies, such as Uber, Lyft or Sidecar, are challenging the incumbents by leveraging on big-data intelligence. We believe that the adoption of similar data-intensive systems, complemented by new bold policy decisions, can help traditional yellow cabs to run for many miles more.

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## References

[1] Bridj. http://www.bridj.com. Accessed 15 January 2015.
[2] Car2go. http://www.car2go.com. Accessed 18 January 2015.
[3] Charitou, C. \& Markides, C. (2012). Responses to Disruptive Strategic Innovations. MIT Sloan Management Review, 44(2), 55-63.
[4] Christaller, W. (1933). Die Zentralen Orte in Suddeutschland, Jena: Fischer Verlag.
[5] Christensen, C., Scott, A. \& Roth, E.A. (2013) Seeing what's next: Using the theories of innovation to predict industry change. Boston, MA: Harvard Business School Press.
[6] Flores-Guri, D. (2003). An Economic Analysis of Regulated Taxicab Markets. Review of Industrial Organization, 23(3-4), 255-266.
[7] Kwong, J. (2014). Report says SF taxis suffering greatly. The Examiner. http://www.sfexaminer.com/sanfrancisco/report-says-sf-taxis-sufferinggreatly/Content?oid=2899618. Accessed 10 January 2015.
[8] Lopez, L., Domingos, T., Cadarso, M.A., Zafrilla, J.E. (2014). The Smarter, the Cleaner? Collaborative Footprint: A Further Look to Taxi Sharing. Proc. National Academy of Science, 111(51), E5488 (2014). doi:10.1073/pnas. 1420242112 .
[9] Lyft. http://www.lyft.com. Accessed 25 November 2014.
[10] Lyft. Lyft Line. https://www.lyft.com/drive/help/article/1650772. Accessed 28 February 2015.
[11] NYC Taxi and Limousine Commission (2014). 2014 Taxicab Factbook. New York Taxi and Limousine Commission. http://www.nyc.gov/html/tlc/downloads/pdf/2014_taxicab_fact_book.pdf. Accessed 1 December 2014.
[12] Rosenberg, T. (2013). It's Not Just Nice to Share, It's the Future. The New York Times. http://opinionator.blogs.nytimes.com/2013/06/05/its-not-just-nice-to-share-its-the-future/?_r=0. Accessed 4 December 2014.
[13] Rubinstein, D. (2014). Economist: Big taxi-sharing study gets it wrong. Capital. http://www.capitalnewyork.com/article/city-hall/2014/10/8553625/economist-big-taxi-sharing-study-gets-it-wrong. Accessed 10 January 2015.
[14] Santi, P., Resta, G., Szell, M., Sobolevsky, S., Strogatz, S.H., Ratti, C. (2014). Quantifying the Benefits of Vehicle Pooling with Shareability Networks. Proc. National Academy of Science, 111(37), 13290-13294 (2014). doi:10.1073/pnas.1403657111.
[15] Santi, P., Resta, G., Szell, M., Sobolevsky, S., Strogatz, S.H., Ratti, C. (2014). Reply to Lopez et al.: Sustainable Implementation of Taxi Sharing Requires Understanding Systemic Effects. Proc. National Academy of Science, 111(51), E5489 (2014). doi:10.1073/pnas. 1421300112
[16] Sundararajan, A. (2013). From Zipcar to the Sharing Economy. Harvard Business Review. https://hbr.org/2013/01/from-zipcar-to-the-sharing-eco/. Accessed 15 November 2013.
[17] Uber. http://www.uber.com. Accessed 2 March 2015.
[18] Rinkus, S. (2014). How Uberpool works. Uberblog. http://blog.uber.com/UberPoolDriver. Accessed 2 March 2015.
[19] Rinkus, S. (2015). Uber's new BHAG: Uberpool. Uberblog. http://blog.uber.com/uberPOOL-2015. Accessed 2 March 2015.
[20] Zipcar. http://www.zipcar.com. Accessed 10 January 2015.


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[^1]:    ${ }^{1}$ According to [11], the average occupancy rate of taxicabs in New York City is about 50\%.

[^2]:    ${ }^{2}$ Passenger discomfort is measured in terms of delay in reaching the destination vs. the case of a single ride.

[^3]:    ${ }^{3}$ The parameters of the $f(m)$ curve have been obtained from those reported in [14], by considering the fact that the average number of daily taxi trips in New York is about $4.5 \cdot 10^{5}$

