A Taxipooling System with Equity Consideration

Xiaosu Ma, Jinming Ding, Wei Wang, Xuepeng Hua and Yujia Peng

Abstract

In recent years, taxipooling demand emerges in Chinese megacities. Currently taxipooling is usually initiated by taxi drivers which is inefficient and sometimes has safety as well as social equity issues. In this study, we propose a conceptual taxipooling system and associated model formulation. Analytical results are obtained to study the benefit distribution between taxipooling customers and also among customers and taxi drivers under different dynamic fare structures, attempting to find out whether there exists an optimal which is enough to attract both taxi drivers and customers to use taxipooling. The result shows that there is an optimal discount rate combination such that both the taxi driver and customers benefit from taxipooling at the same level if considering equity issue. With the help of current GIS, mobile communication and intelligent transportation technologies, hopefully the proposed system can be easily applied into practice.

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1. Introduction

Taxi is recognized as a more comfortable public transport mode, which provides door-to-door but relatively expensive service. In recent years, taxi demand increases rapidly in Chinese megacities especially in highly dense central areas and transport junctions. According to field surveys in Nanjing, the average waiting time of taxi customers at taxi stations in CBD area during peak hours could reach 45 minutes. The imbalance of taxi supply and demand could not be adjusted simply by increasing the number of taxis. For one, most of the time, the outbreak of taxi demand does not occur all over the city but in limited areas and time periods. For another, vacant taxis are reluctant to go into these areas due to more serious traffic congestion, which normally means more cost in terms of time. Therefore, it is of interest to increase efficiency of the taxi system through demand management approaches.

Taxipooling can be seen as a special variation of carpooling. Carpooling first appeared in Germany and Switzerland in 1940s, where several customers with similar destinations share a car and possible monetary cost. It was extended to taxi system due to its irrefutable potential advantage in saving money and queuing time. And it is encouraged and promoted in countries such as U.S. and Korea as a way to mitigate traffic congestion and emission. However, when there are no laws and regulations, people spontaneously come up with ways to meet their taxi demand. Currently, in Nanjing, China, taxipooling is either initiated by a taxi driver who can obtain much more profit, or privately grouped by impatient waiting customers in a few taxi stations. Apparently, the above ways of sharing a taxi are inefficient and can have safety as well as equity issues. The irony is that this is the reason why many local governments are skeptical about taxi sharing. Therefore it is indispensable to develop an efficient and fair taxipooling system with the interests of all the stakeholders considered.

The theoretical foundation of taxipooling generally stems from the microeconomic theory that is applied in carpooling research (Daganzo, 1982; Lee, 1984; Teal, 1987; Yang and Huang, 1999). Teal (1987) examined the origination, participants, and approaches of carpooling. Yang and Huang (1999) discussed the reasonable carpooling fare structure on High-occupancy-vehicle (HOV) lanes. In recent years, available researches associated with taxipooling mainly include the comparison of various taxipooling modes using either mathematical (Qin and Shi, 2006) or statistical approaches (Liu and Li, 2007), the optimal travel route (Zhang and He, 2008; Zhou et al, 2011; Guo and Wang, 2011), scheduling and algorithms.
(Tao and Chen, 2001; Feng, 2011). Tao and Chen (2001) proposed two heuristic algorithms for dynamic taxipooling problems based on intelligent transportation system technologies, and verified the efficiency of taxipooling in saving resources and energy in Taibei. Qin and Shi (2006) initiated a study on taxipooling route optimization based on “Floyd” algorithm, whereas it was later developed via matrix iteration method (Guo and Wang, 2011). Feng (2011) proposed a dynamic taxipooling scheduling algorithm to improve the success rate of taxipooling. Moreover, GIS, GPS, and smartphone technologies were applied for intelligent taxipooling systems design (Zhang et al., 2013).

On the other hand, there are rich literatures that discuss fare structure and cost distribution among carpoolers under various carpooling circumstances. Fagin and Williams (1983) first defined a notion of “fair share” for carpoolers, i.e. the FW share. Moni (2005) further defined a coalitional game seeking for optimization with equilibrium constraints to study the fairness in the carpool problem. Later on researchers tried to apply those methods in taxipooling fare study with equity consideration (Qin et al., 2008; Zhang, 2009; Zhou et al., 2011). Qin et al. (2008) and Zhang (2009) proposed dynamic taxipooling fare structures where rates change with the number of customers. Zhou et al. (2011) developed a fare optimization model using a genetic algorithm. However, few studies have addressed equity issues in terms of benefit distribution among taxipooling customers and drivers in an analytical framework, which, we believe, is a major factor that determines the success of taxipooling practice.

In this study, we propose a conceptual taxipooling system and associated model formulation for taxi stations in central areas and transport junctions. We study the benefit distribution between taxipooling customers and also among customers and taxi drivers under different dynamic fare structures, attempting to find out whether there exists an optimal which is enough to attract both taxi drivers and customers to use taxipooling. Section 2 describes the basic structure of the taxipooling system and the general model formulation. Analytical results are derived to demonstrate the existence of optimal fare discount rates with equity consideration and the associated conditions. Section 3 describes an experiment taxipooling system. Sensitivity analyses were further conducted to demonstrate the benefit distribution among taxi drivers and customers by varying the individual discount rates and extra travel distances using individual characteristics and preference factors summarized from surveys, and Section 4 some concluding remarks.
2. The Formulation

2.1 The Conceptual System

The basic structure of the conceptual taxipooling system at a taxi station is shown in Fig. 1. Technically, it includes customers’ mobile devices with NFC modules, taxi drivers’ mobile devices with GPS tracking functions, an on-site queuing terminal, taxipooling system server, and necessary wireless communication devices.

The queuing terminal collects customer information, possibly with taxipooling request, through customers’ mobile devices, and sends to taxipooling system server. The server matches potential customers and calculates the corresponding taxipooling rates for each customer in accordance with predetermined principles, e.g. equitable benefit. At the same time, the server collects taxi information through GPS and calculates real-time taxi arrival rate. Eventually, the server sends back those calculation results back to each customer. The customers then make their choices and respond the server through apps on their mobile devices.

- Between customers and the system
  Customer information includes: destination, accept taxipooling or not, his/her personal attributes, such as age, gender, picture, credit, and any special request, etc. Note that personal credit can help to increase the success rate and safety, which is summarized by his/her taxipooling history.

  Queuing and taxipooling information includes: current queuing position, expected waiting time, feasible taxipooling choices and corresponding taxi fare as well as the extra benefit if using taxipooling in terms of time and taxi fare, etc. The feasible taxipooling choices are calculated based on collected customer information, in particular, destinations. Theoretically, the system will match customers with similar destinations, or at least similar directions. The algorithm is not the focus of this study and will be included in our ongoing study.

  As shown in the dotted rectangle, taxipooling request is a specific customer information which is only sent to customer $i$ chosen by interested new customer through the local taxipooling system. Similarly, if customer $i$ accepts this request, he/she will send back the acceptance information.

- Between taxis and the system
  Taxi station information includes: current number of waiting customers, taxipooling request, expected (average) taxi revenue, etc.

  Taxi information is the position and expected arriving time of a vacant taxi who is willing to drive to the station based on current traffic situation.
This information is used to calculate and update the taxi arrival rate which will be later sent to the waiting customer.

Fig. 1 A conceptual taxipooling system

Note that, the proposed taxipooling system is designed not just for potential taxipooling customers, but also act as a part of an intelligent urban public transport system. For a single taxi station, it collects, predicts and provides real-time information for both waiting customers and vacant taxis nearby the station. Since most of the time taxipooling increases the benefit of both customers and taxi drivers, the queue will dissipate more quickly by attracting vacant taxis and facilitating potential taxipooling behaviors. However, the realization of a taxipooling attempt largely depends on customers’ choice.

2.2 Customers’ Choices

When a new customer reaches a taxi station, joins the queue, and exchanges information through the system, his/her decision is whether to share with a potential customer from a given feasible choice set or keep waiting if not interested. According to field survey, fare, travel time, comfort level, safety issues are the four dominant influencing factors. Comfort level and safety issues are normally treated as intangible factors and regarded as random terms in many transport models. Tang et al. (2011) tried to measure in-vehicle comfort level through vehicle speed and number of passengers. Besides, in the proposed taxipooling system, the safety issues can be measured through personal credit by setting it as an ordinal variable. For
example, a five-star rated customer is mostly possible to be selected by another taxipooling customer. Therefore, a multinomial logit model is adopted in the study.

The utility or generalized cost includes travel time, expressed in monetary terms, monetary costs, e.g. fare, travel comfort index and some other intangible factors. As a result, the integral utility of customer $c$, who shares taxi with customer $p$, notated as $U^p_c$, is formulated as:

$$U^p_c = V^p_c + \varepsilon^p_c,$$

where $U^p_c$ is considered as a random variable consisting of the systematic part as expressed in (1) and the random term $\varepsilon^p_c$ follows the identical and independently distributed (IID) Gumbel distribution. $V^p_c$ is expressed as:

$$V^p_c = -t^p_c \cdot vot_c - c^p_c + \delta \cdot W^p_c + \kappa \cdot Y^p_c,$$

where $W^p_c$ is the comfort level; $Y^p_c$ is the credit measure, $Y^p_c = 0$ means it is not a taxipooling choice, other values are the credit of the target customer $p$; $c^p_c$ is monetary cost, i.e. taxi fare, expressed as

$$c^p_c = \tau \cdot l^p_c \cdot \nu^p_c,$$

where $\tau$ is initial fare per kilometer; $l^p_c$ is travel distance; $\nu^p_c$ is the fare discount rate. $t^p_c$ is travel time; $vot_c$ is value of time, expressed as

$$t^p_c = t^p_{c\text{in-veh}} + t^p_{c\text{w}},$$

where $t^p_{c\text{in-veh}}$ is the in-vehicle time, which is a function of travel distance $l^p_c$. $t^p_{c\text{w}}$ is the expected waiting time, which both depends on customers’ queue sequence and taxi arrival rate $\lambda$. As a result, customers’ utility can be written as a function of travel distance, taxi arrival rate, fare discount rate, comfort level, and safety index, others are either customer-specific constants or parameters, expressed as:

$$U^p_c \sim \left( t^p_c \left( l^p_c, \lambda \right), c^p_c \left( \nu^p_c, l^p_c \right), W^p_c, Y^p_c \right).$$

Accordingly, the probability that customer $c$ chooses to share taxi with customer $p$ among other choices is expressed as:

$$Pr^p_c = \frac{\exp \left( \beta^p \cdot V^p_c \right)}{\exp \left( \beta^p \cdot V^p_c \right) + \sum_p \exp \left( \beta^p \cdot V^p_c \right)},$$
2.3 Taxi Drivers’ Choices

Taxi drivers’ choice is whether to drive to the taxi station or keep looking for customers on the road. Similarly, the choice is made based on the utility maximization principle determined by fuel cost, fare revenue, and time. According to logit model, the probability that a vacant taxi driver chooses to drive to taxi station is expressed as

$$Pr_d' = \frac{\exp(\beta_d \cdot V_d')}{\exp(\beta_d \cdot V_d) + \exp(\beta_d \cdot V_d')} = \frac{1}{1 + \exp(\beta_d \cdot (V_d' - V_d))},$$  

(7)

where $V_d', V_d$ are the systematic utility of taxi driver choosing to drive to the station or not to, respectively.

$$V_d' - V_d = E(c'_d) - E(c_d) - (c'_d - c_d) - (t'_d - t_d) \cdot \text{vot}_d,$$  

(8)

where $E(c'_d)$ is the expected fare revenue if driving to the station, $c'_d, t'_d$ are the fuel cost and driving time which are determined by the vacant and occupied driving distance and unit fuel cost $\theta$. $E(c_d), c_d, t_d$ are the corresponding expected fare revenue, fuel cost and driving time if not driving to the station.

2.4 Taxi Fare

The formulation in (1)-(8) models customers’ and taxi drivers’ choices. It is of interest to analyze the benefit distribution among taxipooling customers and taxi drivers under different fare structures. In the following discussion, it is assumed that two customers, $A$ and $B$, have agreed to share a taxi $D$ at the taxi station. It is further assumed that the initial distance of $A$, $l_A$, is equal to or greater than that of $B$, $l_B$, in other words, $B$ is the first to reach his destination, whereas $A$ will travel extra distance, $\Delta l_A$, before reaching his/her destination. In addition, for simplicity, comfort level and safety issues are not considered.

Denote $u_A, u_B$ as the fare discount rate under taxipooling. Denote $v$ as the taxi speed. $v$ is constant, in other words, traffic congestion is not considered. Denote $\Delta U_A, \Delta U_B, \Delta U_D$ as the utility difference for customers and taxi drivers after taxipooling, respectively. Since the customer will not choose taxipooling if he/she will not get extra benefit/utility from it, we define
\[ \Delta U_A, \Delta U_B, \Delta U_D \geq 0, \forall l_A, l_B, \Delta l_A, \nu_A, \nu_B, \]  
\[ 0 \leq \nu_A \leq 1, \]  
\[ 0 \leq \nu_B \leq 1, \]  
\( (9) \)  
\( (10) \)  
\( (11) \)

The other general conditions are defined below:

\( (H_o) \): Customers’ waiting time and taxi drivers’ vacant driving time are not considered.

\( (H_i) \): The initial distance of \( A \) is equal or greater than that of \( B \), expressed as \( l_A \geq l_B \).

\( (H_2) \): Both customers and the taxi driver have same values of time, i.e. \( \nu_A = \nu_B = \nu_D = \nu \).

**Proposition 1** Under conditions \( (H_o) \cdot (H_i) \cdot (H_2) \), there exists one and only one discount rate combination \( \{ \nu_A^*, \nu_B^* \} \) such that the utility differences of \( A \), \( B \) and \( D \) are the same, i.e. utility equilibrium, under the condition that the extra distance of \( A \) is not too much longer than the distance of \( B \).

**Proof** Proposition 1 is equivalent to solve equation (12)-(14), expressed as:

\[ \Delta U_A - \Delta U_B = -\frac{\Delta l}{\nu} \cdot \text{vot} + \tau \cdot \{ l_B \cdot (\nu_B - 1) - [ (l_A + \Delta l_A) \cdot \nu_A - l_A ] \} = 0, \]  
\( (12) \)

\[ \Delta U_A - \Delta U_D = -2 \tau \cdot \nu_A \cdot (l_A + \Delta l_A) - \tau \cdot \nu_B \cdot l_B + 2 \tau \cdot l_A + \theta \cdot \Delta l_A = 0, \]  
\( (13) \)

\[ \Delta U_B - \Delta U_D = \frac{\Delta l}{\nu} \cdot \text{vot} + \tau \cdot l_B \cdot (1 - 2\nu_B) 
\]  
\[ - \tau \cdot \nu_A \cdot (l_A + \Delta l_A) + \tau \cdot l_A + \theta \cdot \Delta l_A = 0, \]  
\( (14) \)

where \( \nu_A \) and \( \nu_B \) are variables, while other factors constants. We seek for the optimal via different fare structures.
Obviously, when (12) and (13) are both satisfied, so is (14). Combining and simplifying (12) and (13), we have:

\[
\tau \cdot l_B \cdot \nu_B - \tau \cdot (l_A + \Delta l_A) \cdot \nu_A = \frac{\Delta l_A}{v} \cdot \nu - \tau \cdot (l_A - l_B), \tag{15}
\]

\[
\tau \cdot l_B \cdot \nu_B + 2\tau \cdot (l_A + \Delta l_A) \cdot \nu_A = 2\tau \cdot l_A + \theta \cdot \Delta l_A, \tag{16}
\]

\[
\tau \cdot l_B > 0, \tag{17}
\]

\[
\tau \cdot (l_A + \Delta l_A) > 0. \tag{18}
\]

Accordingly, we have:

\[
u^e_A = \frac{\tau \cdot (3l_A - l_B) + \theta \cdot \Delta l_A - \frac{\Delta l_A}{v} \cdot \nu}{3\tau \cdot (l_A + \Delta l_A)}, \tag{19}
\]

\[
u^e_B = \frac{2\tau \cdot l_B + \theta \cdot \Delta l_A + \frac{2\Delta l_A}{v} \cdot \nu}{3\tau \cdot l_B}. \tag{20}
\]

Putting (19) and (20) into (12)-(14), we have:

\[
\Delta U^e_A = \Delta U^e_B = \Delta U^e = -\frac{2\Delta l_A}{v} \cdot \nu - \theta \cdot \Delta l_A + \tau \cdot l_B. \tag{21}
\]

To promise positive benefits of \(A\), \(B\) and \(D\), we make (21) fulfill (9), expressed as:

\[
-\frac{2\Delta l_A}{v} \cdot \nu - \theta \cdot \Delta l_A + \tau \cdot l_B \geq 0, \tag{22}
\]

Simplifying (22), we have:

\[
\Delta l_A \leq -\frac{\tau \cdot l_B}{\theta + \frac{2\nu}{v}}. \tag{23}
\]

On the other hand, based on a practical consideration, we make (19) and (20) respectively fulfill (10) and (11), expressed as:

\[
0 \leq \frac{\tau \cdot (3l_A - l_B) + \theta \cdot \Delta l_A - \frac{\Delta l_A}{v} \cdot \nu}{3\tau \cdot (l_A + \Delta l_A)} \leq 1, \tag{24}
\]

\[
0 \leq \frac{2\tau \cdot l_B + \theta \cdot \Delta l_A + \frac{2\Delta l_A}{v} \cdot \nu}{3\tau \cdot l_B} \leq 1. \tag{25}
\]
According to (24)-(25), we can calculate the joint result, expressed as:

$$\Delta I_A \leq \frac{\tau \cdot l_B - \theta}{\theta + \frac{2 \cdot \text{vot}}{\text{v}}}.$$  \hfill (26)

Combine (23) and (26), we obtain the conditions.

**Proposition 2** Under conditions \((H_0) - (H_2)\), the difference of discount rate between A and B at \(\{\nu'_A, \nu'_B\}\) is monotonically decreasing with extra distance of A. There exists an equilibrium point that promises both utility and discount equilibrium with certain constraints, i.e. \(\nu'_A = \nu'_B\).

**Proof** According to (24)-(25), the discount difference at utility equilibrium is expressed as:

$$\nu'_A - \nu'_B = \frac{\tau \cdot (3l_A - l_B) - \left(\theta - \frac{\text{vot}}{\text{v}}\right) \cdot l_A}{3\tau \cdot (l_A + \Delta l_A)} - \frac{\theta + \frac{2 \cdot \text{vot}}{\text{v}}}{3\tau \cdot l_B} \cdot (l_A + \Delta l_A)$$

$$+ \left[\frac{\theta - \frac{\text{vot}}{\text{v}}}{3\tau \cdot l_B} \cdot l_B + \left(\theta + \frac{2 \cdot \text{vot}}{\text{v}}\right) \cdot l_A \right] - \frac{2}{3}$$  \hfill (27)

According to (27), when:

$$\tau \cdot (3l_A - l_B) - \left(\theta - \frac{\text{vot}}{\text{v}}\right) \cdot l_A \geq 0,$$  \hfill (28)

It is straightforward to derive that (27) monotonically decreases with the extra distance of A, \(\Delta l_A\).

When \(\nu'_A = \nu'_B\), we have:

$$a \cdot \Delta l_A^2 + b \cdot \Delta l_A + c = 0.$$  \hfill (29)

where,

$$a = \frac{2 \cdot \text{vot}}{\text{v}} + \theta > 0,$$  \hfill (30)

$$b = \left(\frac{2 \cdot \text{vot}}{\text{v}} + \theta\right) \cdot l_A + \left(2\tau - \theta + \frac{\text{vot}}{\text{v}}\right) \cdot l_B > 0, \forall \tau > \theta,$$  \hfill (31)

$$c = - \tau \cdot l_B \cdot (3l_A - l_B) < 0, \forall l_A > l_B.$$  \hfill (32)
$b > 0$ because practically taxi fare per kilometer is greater than fuel cost per kilometer. $c < 0$ because based on $(H_i)$, the initial distance of $A$ is greater than $B$. As a result, equation (29) should have two feasible solutions. Dropping the negative solution, we have:

$$\Delta l_A^{de} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}. \quad (33)$$

where subscript $de$ represents the discount equilibrium state.

Based on (28) and (33), the relationship of discount difference between $A$ and $B$ is expressed as:

$$\nu_A^e \geq \nu_B^e, \quad \forall \Delta l_A \in [0, \Delta l_A^{de}], \quad (34)$$

$$\nu_A^e \leq \nu_B^e, \quad \forall \Delta l_A \in [\Delta l_A^{de}, +\infty]. \quad (35)$$

In contrast to (28), when:

$$\tau \cdot (3l_A - l_B) - \left( \frac{\nuot}{v} \right) \cdot l_A \leq 0, \quad (36)$$

According to (27) and (36), we have:

$$\nu_A - \nu_B \leq -2 \left\{ \left[ \frac{\left( \theta - \frac{\nuot}{v} \right) \cdot l_A - \tau \cdot (3l_A - l_B)}{9\tau^2 \cdot l_B} \right] \cdot \left( \theta + \frac{2\nuot}{v} \right) \right\}^{1/2}, \quad (37)$$

The maximum can be obtained when:

$$\Delta l_A^{de/\text{max}} = \frac{1}{3\tau} \left\{ \left[ \frac{\left( \theta - \frac{\nuot}{v} \right) \cdot l_A - \tau \cdot (3l_A - l_B)}{l_B} \right] \cdot \left( \theta + \frac{2\nuot}{v} \right) \right\}^{1/2} - l_A. \quad (38)$$

According to (29)-(32), there must be two solutions to the function when it is satisfied for discount equilibrium. Based on (36)-(38), the function is first monotonically increasing with extra distance then decreasing after reaching the maximum point, and the result can be concluded as (34)-(35).
Proposition 2 illustrates the relationship of discount difference between $A$ and $B$ at utility equilibrium for all, which helps to find a more acceptable match from psychological perspective, i.e. Customer $A$ wants more discount due to extra distance and travel time.

3. Case Study

3.1 Survey

A survey in the form of questionnaires and interviews was conducted in both CBD area and railway station in Nanjing on two weekdays in May, 2014. The respondents include both customers with and/or without taxi-pooling experiences. Both Revealed-preference (RP) and Stated-preference (SP) questions were asked concerning attitudes towards taxi-pooling and previous experiences. The results show that, in general, a considerable number of customers felt dissatisfied with current taxi-pooling mode where the discount rate is arbitrarily determined by the driver. In addition, time and fare are the two most important factors affecting the taxi-pooling decision followed by comfort and safety. It is worth mentioning that some customers hesitate to make decisions because they are not sure whether the saved money is enough for the compensation of potential extra distance.

Furthermore, after some crossover data analyses, we found that most people can tolerate 5 to 20 minutes waiting time. More than 80% customers can accept at most 20% extra mileage in contrast to not sharing a taxi. Some of the statistical results are used in the following sensitivity analysis in Section 3.2

3.2 Sensitivity Analyses

The properties derived in the previous section are illustrated by sensitivity analyses here. To be consistent, the analyses here consider that there are only two customers with different destinations, and they decide to share a taxi.

3.2.1 Customers and the Driver’s Benefit Distribution

In Fig. 2, the horizontal axis refers to the discount rate of customer $A$, while the vertical axis refers to the discount rate of customer $B$. This figure is used to illustrate the benefit distribution among customers and the driver, and to prove the existence of an equilibrium point which promises both utility equilibrium and considerable positive benefits for all. Each
discount combination point in the plane refers to one benefit distribution case.

Each point on the thickened blue line represents a discount rate combination so that the benefits of $A$ and $D$ after taxipooling are the same, i.e. $\Delta U_A = \Delta U_D$. Similarly, thickened red line and green line are the utility equilibrium lines between $A, B$, and $B, D$, respectively. The three thickened color lines intersect at one point, $(0.726, 0.7)$, where both $A, B$, and $D$ share the same utility improvement because of taxipooling.

Besides, three thickened color lines divide the figure into six areas. Each area represents a discount rate combination where someone within $A, B$, and $D$ gets the higher benefit than the other two. For example, within Area 2, taxi driver $D$ always get higher benefit than $A, B$.

![Fig. 2 The utility difference among customers and the driver](image)

### 3.2.2 Discount Interactions at Utility Equilibrium for All

Fig. 3 shows the monotonic relationship between extra distance of $A$ and the discount rate difference between $A$ and $B$, i.e. $\nu_A^e - \nu_B^e$. The intersec-
tion point at horizontal axis \((0.66, 0)\) becomes the discount equilibrium point, i.e. \(v_A^e = v_B^e\). In other words, when the extra distance of \(A\) is less than 0.66km, the discount rate of \(A\) is less than that of \(B\), which is interesting.

Fig. 4 is existence conditions for all the cases when both utility and discount equilibrium are achieved among \(A\), \(B\), and \(D\). The horizontal and vertical axis refer to the initial distance of customer \(A\) and \(B\), respectively. This figure illustrates the relationship between initial distances and extra distance at both utility and discount equilibrium for all. For example, \((5, 3)\) is the point where the extra distance of \(A\) is 0.66km, in other words, when the initial distances of \(A\) is 5km, the initial distance of \(B\) is 3km, only when the extra distance of \(A\) is 0.66km according to the suggested taxipooling routes, they will get the same discount rate so as to achieve the utility equilibrium. This observation implies that most of the time customers will not get the same discount rate in order to get the same utility improvement.

![Image](https://example.com/image.png)

**Fig. 3** Discount difference at utility equilibrium
Fig. 4 The distance conditions of both utility and discount equilibrium

4. Conclusion

This paper proposed a conceptual taxipooling system as well as a fare structure for taxi stands. A taxipooling model is developed to study the benefit distribution among customers and taxi drivers. Analytical results are obtained for a representative taxipooling situation. Sensitivity analyses are conducted using field survey data. The dynamic taxipooling fare under the proposed equity scheme is easy to calculate which can realize real-time information communication.

Besides, the proposed taxipooling system is designed not just for potential taxipooling customers, but also act as a part of an intelligent urban public transport system. The conceptual system illustrated in this paper is for a single taxi station. However, the study can extend to multi-station taxipooling system at CBD area which can increase the system performance through coordinated operation.

This is our preliminary study. Therefore many simplified assumptions are made. In our ongoing study we will apply queuing theory to discuss the interaction between taxi demand and supply.
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