PREDICTION OF NON-LINEAR RANDOM WAVE CHARACTERISTICS IN THE SURF ZONE

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MOTIVATION

Net sediment transport in the cross-shore direction is computed as the difference between the on-shore and off-shore transport rates, which are usually of comparable magnitude. For this reason, small errors in the determination of the on-shore and off-shore components may lead to large errors in the net sediment transport rate. Non-linear wave characteristics, such as skewness and asymmetry of the near-bottom wave orbital velocity, may play an important role in determining the on-shore transport component. Therefore, accurate prediction of non-linear wave characteristics, particularly the near-bottom orbital velocity, may be vital to the successful computation of net cross-shore transport.

Tajima (2004) developed a hydrodynamic model whose ultimate purpose was to forecast sediment transport in the surf zone. This hydrodynamic model is based on linear wave theory and therefore involves smaller computational effort than other recent models based on the Boussinesq equations (e.g., Chen et al., 2000). Moreover, Tajima's model is able to compute non-linear wave characteristics by establishing an equivalence between linear and non-linear waves. This makes Tajima's approach useful to predict sediment transport in a computationally inexpensive way.

Tajima's hydrodynamic model describes the propagation of monochromatic or narrow-banded spectral waves into the surf zone. Model predictions have consistently agreed with laboratory experiments. However, real waves in the near-shore region follow more complicated distributions than the simple one assumed by Tajima (2004). The purpose of this work is to investigate how non-linear characteristics of realistic wave distributions differ from Tajima's predictions and, as a result, to develop an improved formulation of Tajima's model.

PROCEDURE

To demonstrate the validity of Tajima's linear wave predictions, we compare his model with existing wave models that are based on linear wave theory (Battjes and Janssen, 1978; Thornton and Guza, 1983). Note that the latter models are purely linear and do not yield non-linear wave characteristics. Hence, they cannot replace Tajima's wave model in sediment transport calculations.

To model incident random waves more realistically than Tajima's (2004) very narrow spectral distribution, defined by H_{rms} and T_{peak}, we represent the incident waves by the joint distribution of wave heights and periods suggested by Longuet-Higgins (1983). This allows us to examine the effects of spectral width, which is assumed zero by Tajima.

Individual wave components, of H and T, are propagated using Tajima's monochromatic wave model. Then, mean wave characteristics are calculated, at various positions in- and outside the surf zone, by averaging the results for all individual wave components. The idea of characterizing the random wave transformation in the surf zone by the propagation of a set of monochromatic waves has been applied in a number of previous studies (e.g., Dally and Dean, 1986). Non-linear wave characteristics can be obtained for the individual waves by making use of the relationships between linear and non-linear waves developed by Tajima (2004). Finally, we compare these results with experimental measurements and obtain a simple non-linear extension of Tajima's spectral model.

RESULTS

In Figure 1 we present a comparison of Tajima's linear wave models with existing models for calculating linear wave propagation (Battjes and Janssen, 1978; Thornton and Guza, 1983) for the experimental case reported by Wang et al. (2002, plunging breaker case). It is emphasized that the “thick solid line” represents predictions by Tajima's spectral model as well as our application of his monochromatic wave-by-wave model for a range of spectral widths 0.1 to 0.6 that includes the experiment's value of 0.22. Thus, predictions of H_{rms} by Tajima's spectral model, which contains no adjustable parameter, are in excellent agreement with measurements and may be safely applied even when the incident wave spectra are not narrow.

With Tajima's model's validity established, we can compare its predictions of non-linear wave characteristics, obtained from the statistical wave-by-wave approach, with measurements. The wave-by-wave approach yields predictions of non-linear wave characteristics but is far more computationally demanding than his spectral model. Therefore, it would be of practical importance if we could obtain information about non-linear wave characteristics from Tajima's spectral model. For this reason, we investigate the relationship between the results of the two approaches, in order to develop a methodology for the extraction of the non-linear wave characteristics pertinent to the computation of cross-shore sediment transport, i.e., skewness and asymmetry of the near-bottom wave orbital velocity, directly from the predictions afforded by Tajima's spectral model. We also compare the model results with experimental results of wave skewness and asymmetry obtained in a large-scale laboratory facility by Hamilton and Ebersole (2001, tests 8A-E).

Since the undertow, in addition to non-linear wave characteristics, contributes to cross-shore sediment transport, we compare (Figure 2) radiation stress gradients predicted by Tajima's spectral model and our application of his monochromatic model in the wave-by-wave approach. Again we conclude that Tajima's spectral model yields predictions that are consistent with the more detailed wave-by-wave approach for a range of spectral widths. The comparison of undertow-related quantities has been extended to examine the effects of surface rollers, including a comparison between Tajima's and existing, e.g., Stive and De Vriend (1994), surface roller models.

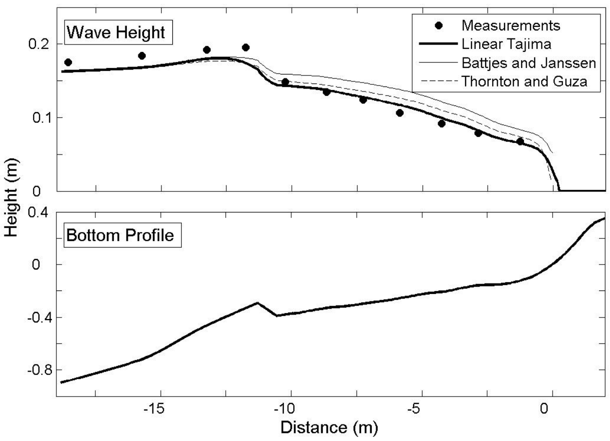

Figure 1: Comparison of measured and predicted wave heights. Measurements (circles, from Wang et al., 2002), Tajima's predicted linear wave heights (thick solid line), Battjes and Janssen's (1978) predictions (thin solid line), Thornton and Guza's (1983) predictions for the recommended parameter values, *B*=0.8 and γ=0.42 (dashed line).

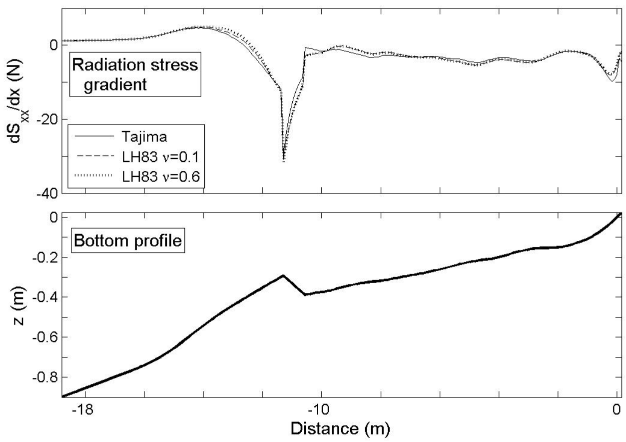

Figure 2: Comparison of radiation stress gradients. Tajima's spectral model predictions (solid line) and wave-by-wave approach results, applying the probability distribution presented by Longuet-Higgins (1983), for spectral width parameter ν=0.1 (dashed line) and ν=0.6 (dotted line).

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