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Modeling epistemic subsurface reservoir uncertainty using a reverse Wiener jump–diffusion process

Jijun Lin [a,*], Olivier de Weck [b], David MacGowan [c]

[a] BP America Inc., 501 Westlake Park Blvd., Houston, TX 77079, USA
[b] Engineering Systems Division, Massachusetts Institute of Technology, 77 Mass Ave., Cambridge, MA 02139, USA
[c] BP Exploration Operating Company Ltd., Chertsey Road, Sunbury on Thames, Middlesex, TW16 7BP, UK

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ABSTRACT

This paper develops a mathematical model of how subsurface reservoir uncertainty, in particular estimated ultimate recovery (EUR), can evolve over time. The model assumes that epistemic uncertainty reduces over time as information from seismic surveys, appraisal wells and production logs is used to improve EUR estimates. A reverse Wiener diffusion process with superimposed jumps is developed to capture the exponential decrease in estimate volatility due to learning but also the existence of sudden jumps in estimates due to unexpected discoveries such as reservoir fault lines or aquifer support. The model can be applied to quantify the evolution of reservoir uncertainty over time during appraisal and planning of new oil and gas development projects. Appreciation Factor data from 34 North Sea fields is used to calibrate and validate the model showing that the evolution of EUR estimates is predicted with 82.4% of validation data points within the simulated P10 and P90 uncertainty envelope, which should theoretically cover 80% of data points if there is no model error. The key parameters in the model are the initial EUR distribution, as well as the exponential decay rates for EUR volatility and the likelihood of occurrence of discrete jumps.

1. Introduction

In the Exploration and Production (E&P) of oil and gas fields, subsurface reservoir uncertainty is one of the major uncertainties with preeminent influence on project economic viability. Specifically, the recoverable volume known as, estimated ultimate recovery (EUR), defines the estimated amount of hydrocarbons that can ultimately be produced from reservoirs. It is convenient to use EUR as it can be applied at all stages of development planning whereas alternative terms such as resource estimates and reserves are more appropriate to prospects and from the time of project sanction respectively. EUR is an important source of uncertainty that influences the strategic planning of field development. To some extent, the unfolding of subsurface uncertainty over time (e.g., EUR) drives the direction and progression of E&P projects. Major energy companies spend a significant amount of resources on quantifying and managing subsurface uncertainty. A reservoir is a complex geological system and many factors (e.g., geological structures, rock properties, fluid characteristics, reservoir drive mechanism, and reservoir connectivity) affect EUR.

In E&P, a traditional approach normally describes two periods related to estimating recoverable volumes for a hydrocarbon accumulation during the appraisal stage. Fig. 1 illustrates the typical development phases for E&P projects and their associated range of subsurface uncertainty. There are two consecutive stages for the appraisal phase.

(1) Scale appraisal: a period marked by large jumps in the estimation of resources. The purpose of scale appraisal is to understand the potential size of reservoirs and associated characteristics post discovery of a field. Normally, seismic surveys, exploration or appraisal well drilling, are utilized to collect data and hopefully reduce the uncertainty. Sometimes scale appraisal is also called primary appraisal or resource appraisal. Scale appraisal activities usually reside in the exploration division of an E&P company.
(2) Confidence appraisal: a period during which gradual narrowing of the EUR distribution occurs although sometimes it may not lead to desired continuous reduction of uncertainty. Confidence appraisal is also called secondary appraisal which leads up to project sanction, which means the development of a field is technically feasible and economically viable. Confidence appraisal usually resides in the projects or developments division of an E&P company.

Thunnissen (2003) gives a literature survey of uncertainty definitions and classifications from various fields ranging from social sciences, to natural sciences, to engineering. Thunnissen proposes an uncertainty classification framework for design and development of

* Corresponding author. Tel.: +1 281 366 0592.
E-mail address: Jijun.lin@bp.com (J. Lin).

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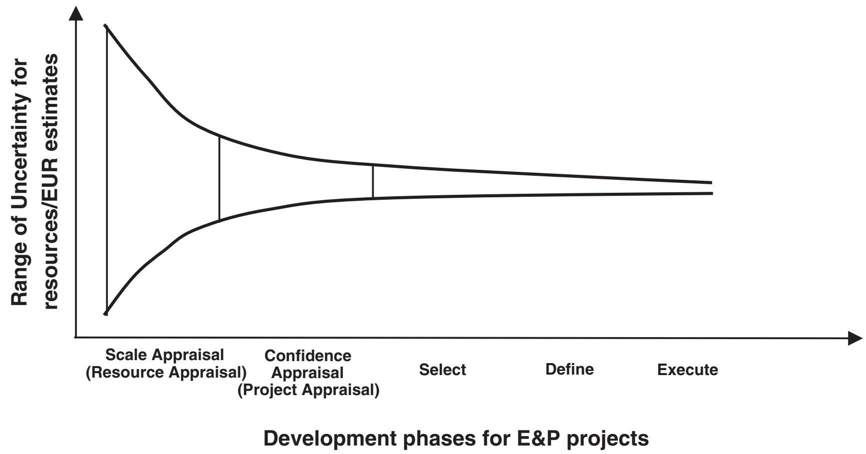

Fig. 1. Development phases for E&P Projects.

complex systems. In this framework, there are four types of uncertainty: ambiguity, aleatoric, epistemic, and interaction uncertainty:

(1) *Ambiguity uncertainty* is imprecision due to vague definitions and communication.
(2) *Aleatoric uncertainty* is inherent variation associated with a physical system or environment under consideration. Aleatoric uncertainty has several other names: irreducible uncertainty, inherent uncertainty, noise, type A uncertainty.
(3) *Epistemic uncertainty* is due to lack of knowledge or information in any phase or activity of the modeling process. In the literature, epistemic uncertainty also goes by the names: reducible uncertainty, subjective uncertainty, model form uncertainty, state of knowledge, and type B uncertainty.
(4) *Interaction uncertainty* arises from unanticipated interaction of many events and/or disciplines, each of which might, in principle, be foreseeable.

Under Thunnissen's uncertainty framework, market uncertainty (such as prices for crude oil and gas products) is aleatoric uncertainty, or irreducible uncertainty, as the market prices of crude oil are continuously evolving into the future and individual firms have little influence on future crude prices. Much research has been done and models have been developed to simulate the propagation of this type of aleatoric uncertainty in financial options and real options theory (Trigeorgis, 2002; Copeland and Antikarov, 2003). However, reservoir subsurface uncertainty can be characterized as epistemic uncertainty, or reducible uncertainty, because the uncertainty stems from limited human knowledge about the reservoirs underground during a project's planning and operation stages. The fundamental characteristics of oil and gas reservoirs have evolved over millions of years; therefore they are essentially static (e.g., original oil in place) on the time-scale of their development and exploitation. To the authors' knowledge, very limited research has been devoted to model and simulate the stochastic process of human learning on epistemic uncertainty, particularly the evolution of reservoir subsurface uncertainty. The rest of this section further motivates the need for such a model and describes the learning behavior of epistemic uncertainty based on actual reservoir uncertainty examples.

For reservoir subsurface uncertainty, there is an important distinction between the unknown "true" state of a reservoir and human perception of this state (i.e., estimates of the subsurface parameters). A reservoir's geological structures, fluid properties, and quantity and quality of hydrocarbons are in fact "deterministic" as they are physical entities and have evolved over millions of years to a quasi steady-state before any human intervention, such as exploration and appraisal, and production well drilling occurs. However, the human perception (or estimation) of reservoirs' physical conditions is evolving over time as new information is acquired through field exploration and production. The decisions about field development are made based on human perception of reservoir conditions rather than the underlying "true" values, which are unknown.

The true quantity of original hydrocarbons in place for a specific field is a deterministic number. It is unlikely to change in a short period of time and neither is the recoverable quantity based on a given recovery mechanism. However, the estimates of recoverable hydrocarbons even from a fixed recovery mechanism are uncertain. Through investment in the exploration and production, such as seismic surveys, appraisal or production well drilling, and well logging, more information is gathered which hopefully reduces the uncertainty of EUR over time. Fig. 2 illustrates the concepts of evolution of subsurface uncertainty. It assumes that at any given point of time, the estimate of an uncertain variable (i.e., EUR) follows a distribution characterized by a mean and standard deviation. The solid line represents one possible trajectory for the estimated mean. It is assumed that the estimated mean progressively approaches the "true" underlying value. This trajectory may not be monotonic. It is possible that the estimate initially approaches a "false" underlying value, and then suddenly the estimate changes significantly as discrete pieces of new information are discovered about the reservoirs. For example, as shown in Fig. 2, the estimate of existing EUR decreases significantly at t_2 which may be due to discovery of a new fault structure in a reservoir, which reduces flow connectivity among different compartments of the reservoir. As a result, the EUR reduces if following the same development plan (i.e., the same number of production wells). In theory as well as in practice, it is also possible that the true underlying value is greater than the initial estimate. For example, the actual hydrocarbon-bearing area and play or porosity could turn out to be much higher than the initial estimates may have suggested. In Fig. 2, $\Delta\mu$ represents the initial estimation error for the mean. Fig. 2 simply represents one trajectory for the mean estimate among many possible evolutionary trajectories for these estimates. However, for any past project, there is only one "realized" estimation trajectory. Therefore, an important question is: *How does one simulate the evolutionary trajectory of a variable subject to epistemic uncertainty and human learning a-priori before any such learning has occurred?*

In this paper, we develop a stochastic model to simulate epistemic uncertainty for subsurface EUR over time. This model is based on a reverse Wiener stochastic process ("random walk") with superimposed discrete jumps. The classical Wiener process is continuous but we found that jumps are necessary to represent "surprise" discoveries of new information over time. This model is applicable to the evolution of EUR uncertainty either pre- or post-discovery of a field.

This epistemic uncertainty model is motivated by real world examples. Fig. 3 shows the distribution of two projects' EUR estimates

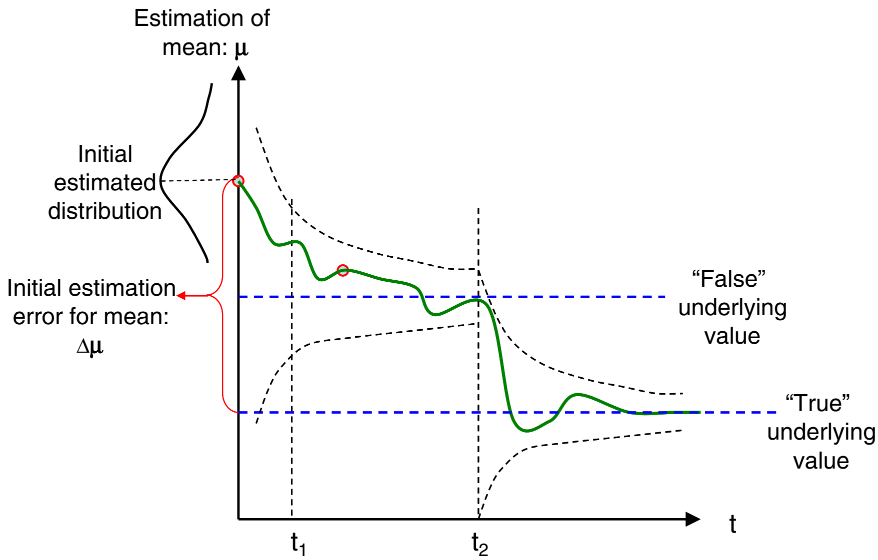

Fig. 2. Evolution of epistemic subsurface uncertainty over time.

from early exploration discovery to the end of appraisal. We can extract several key observations from these two examples:

(1) First, the range of uncertainty can increase or decreases over time. For the first example in Fig. 3, the range between the 10th percentile (P10) and the 90th percentile (P90) narrows from exploration discovery to end of appraisal, although it is not a monotonic trend. For the second example in Fig. 3, the range of uncertainty actually increases for a certain period of time.
(2) Secondly, there are some discontinuous "jumps" in median (P50) estimates. When these jumps occur, the range of uncertainty increases simultaneously.

The fact that the range of uncertainty for EUR does not necessarily diminish has been previously reported in the literature. Demirmen (2007) shows the non-diminishing uncertainty for a field in the North Sea and a producing well in Colorado. Watkins (2000) conducted a study on reserve Appreciation Factors (*AF*) for 126 fields in the North Sea. Fig. 4 shows three actual evolutionary trajectories for reserve *AF* from the Watkins study. The *AF* is defined as the ratio between the EUR at time *t* and the initial value at time t_0. There are three distinct evolutionary behaviors for these three oil fields. For Valhall, the *AF* increases four fold during the first 15 years. For Beryl, there are two discrete jumps, and as a result, *AF* increases nearly 200%. For Tartan, on the other hand there is a rapid reduction of *AF* in the first 2 years and then *AF* remains relatively stable for the remaining years. These examples show just three evolutionary trajectories. In reality there are many potential evolutionary trajectories for *AF* (or EUR) that could have occurred for each of these fields depending on the arrival of specific subsurface information and the interpretation of such information. The question is how we can conceptually model the human learning process and simulate the underlying epistemic stochastic process starting from t_0 where only the initial EUR distribution may exist.

There are many reasons that could change EUR distributions over time. First, the inherent reason for too high or too low initial EUR is geological complexity of fields. These inherent geological factors and

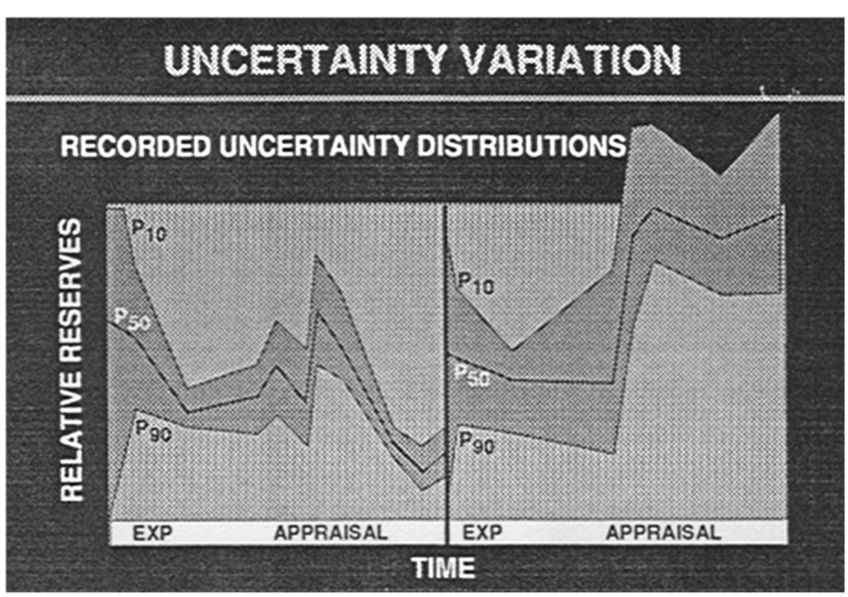

Fig. 3. Reserve estimates from exploration discovery to end of appraisal for two unnamed fields.
Source: BP Exploration and Production.

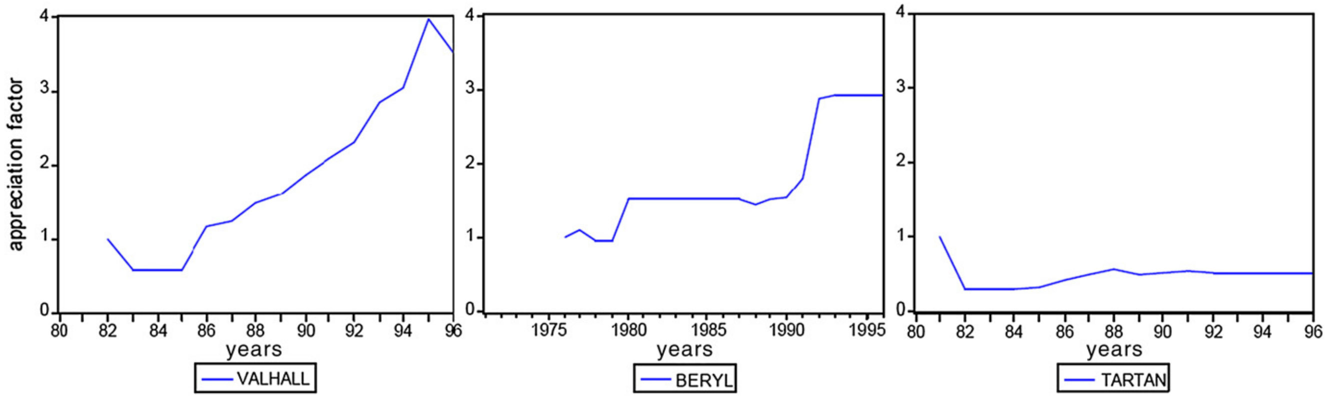

Fig. 4. Evolution of reserve appreciation factors for three oil fields in the North Sea.
Figures are adapted from Watkins (2000).

reservoir characteristics (such as gross rock volume, reservoir drive mechanism, reservoir connectivity and fluid and petrophysical parameters) are not well understood at the beginning. These factors include

(1) Trap geometry and reservoir distribution/thickness, which impact the gross volume estimate.
(2) Reservoir drive mechanism: such as the presence of aquifer support or a gas gap, which affect the reservoirs' recovery factor.
(3) Petrophysical parameters: such as rock porosity, oil saturation, permeability.
(4) Reservoir fluid characteristics: such as viscosity, pressure, volume, temperature (PVT), and volumetric factor.

As field development progresses from appraisal to execute, knowledge about the subsurface accumulates over time through seismic surveys and analysis and appraisal well drilling. Ideally, the range of uncertainty in EUR reduces monotonically and eventually approaches the underlying true values. In many cases, not until oil production starts to decline, can true reservoir dynamics be fully observed and understood. Therefore, uncertainty reduction in EUR is the result of active scientific and engineering activities, which cannot be obtained without capital investment in E&P (usually millions or billions of dollars) and time (around 10 years from exploration to production). Secondly, it is possible that there are "sudden jumps" in EUR estimates with time. These discontinuities may be due to the discovery of static reservoir structures (e.g., faults, compartment size[1]) and dynamic behaviors (e.g., aquifer support). These new discoveries will potentially cause a revision of previous estimates and this can happen at any stage of the field life cycle. In general, the probability of having such discrete jumps is higher in the early stages as there is limited knowledge about reservoirs. Thirdly, EUR is not independent of field development plans. The development decisions, such as well spacing, water and gas injection, wet or dry trees, will all impact the EUR. Alongside the technical challenges of obtaining accurate EUR, human, economic, and political factors are also responsible for estimating bias, because public knowledge of EUR may affect an E&P Company's worth or stock price in the market (Mackay, 2004). Therefore, understanding and estimating the underlying true EUR is very critical during the field exploration and appraisal stages. Such estimates determine key decisions about field development, facility concepts, field architectures, and designed processing capacities.

[1] Although an E&P Company could possibly maintain the same EUR by adapting field development plans (such as drilling more wells with discovery of smaller compartment size), which just means more capital investment would be needed to recover the same amount of reserve.

2. Literature review

In the petroleum engineering literature, subsurface volumetric uncertainty, such as Stock Tank Original Oil In Place (STOOIP) and Ultimate Recovery (UR), has been studied and described in terms of Probability Distribution Function (PDF) or Expectation Curve approaches, and decline curve approaches (Arps, 1945; Arps, 1956) etc.

The standard textbook formula (Jahn et al., 1998) for calculating the STOOIP and UR is shown as follows:

$$STOOIP = GRV \cdot \frac{N}{G} \cdot \varphi \cdot S_o \cdot \frac{1}{B_o} \quad (1)$$

$$UR = STOOIP \cdot RF \quad (2)$$

where

STOOIP stands for stock tank original oil in place. It normalizes volume of oil present in subsurface conditions to the standard surface conditions.
GRV is Gross Rock Volume of the hydrocarbon-bearing interval. It can be estimated based on the area containing hydrocarbons and the interval thickness.
N/G is Net to Gross Ratio (N/G). It is the ratio between the thicknesses of productive reservoir rock within the total (gross) reservoir thickness.
φ is the porosity of the productive reservoir rock. It is the percentage of volume for bearing fluids within the reservoir rock.
S_o is the oil saturation. It is the percentage of pore space which contains oil.
B_o is the oil formation volume factor, which transforms volume at reservoir conditions to standard surface conditions.
UR Ultimate Recovery. It is linked to volumes initially in place by the recovery factor. They are a fraction of the initial volume. It is the quantity of oil that is ultimately produced.
RF Recovery Factor. It depends on reservoir drive mechanisms (primary, secondary, and tertiary recovery) and field production schemes and is the fraction of oil in place that is ultimately recovered.

All the parameters for calculating STOOIP are uncertain. They are estimated using various techniques. For example, seismic surveys and exploration well drilling can be used to estimate reservoir location and area, and core samples can be used to estimate the net gross ratio, formation volume factor, and porosity. Given the heterogeneous nature of an oilfield, the values for these parameters may vary across fields. For offshore petroleum fields, it is very expensive (tens or even hundreds of millions of dollars) to get log samples by drilling an appraisal well. Thus, the estimates of these parameters are generally uncertain given limited samples. Therefore, the estimates of STOOIP and EUR are uncertain. Quantifying the uncertainty of STOOIP and the Estimate of Ultimate Recovery (EUR) is one of the major tasks in the early phases of field development. In practice, these uncertainties are expressed in terms of probability distributions or cumulative probability distributions (called expectation curves in petroleum engineering). In the petroleum engineering literature, there are two general approaches for estimating the distributions of STOOIP and EUR:

(1) *Monte Carlo simulation*: Input parameters are sampled from their assumed distributions. Each sample combines these uncertain inputs and obtains one instance of STOOIP. After obtaining a large number of samples, a frequency histogram can be obtained to approximate the probability distributions of STOOIP or EUR. Table 1 shows the assumed distributions for input parameters. For simplicity, all input parameters are assumed to have a normal distribution. Fig. 5 shows a histogram and expectation curve for EUR based on the inputs assumption in Table 1. The Monte Carlo sampling results are based on 5000 samples. The estimated distributions for EUR appear to be lognormal. This is not a surprise since probability theory (as an extension of the central limit theorem, Rice, 1995) says that the product of multiple independent normally (or any) distributed variables has a lognormal distribution. The expectation curve shown in Fig. 5 is essentially equivalent to one minus cumulative distribution function. For example, the point (200, 0.2) on the curve means that there is a 20% chance that the EUR is greater than 200 mmbbls. For an undrilled prospect, there is a finite probability to have zero recoverable hydrocarbons. Therefore, the expectation curve will not reach one.

(2) *Parametric method*: This is an established statistical technique used for combining variables containing uncertainties. This method is based on basic statistical rules to add or multiply uncertain variables if each variable can be characterized by a distribution with its own mean and standard deviation. The parametric method provides a convenient way to estimate the relative contribution of each input parameter's uncertainty. If we assume that the inputs are independent of each other, the relative contribution of each input parameter to the overall uncertainty is $(1+K_i^2)$, where $K_i=\sigma_i/\mu_i$ for each input parameter. Applying this formula to the previous example, Fig. 6 shows the relative impact of input parameters on uncertainty of STOOIP and EUR. Typically, gross rock volume is the most uncertain, followed by recovery factor, geological model factors (i.e., net-to-gross), followed by petrophysical parameters (i.e., porosity and oil saturation) and finally formation volume factor as the least uncertain parameter. However, the actual ranking of these factors for a specific field will depend on the assumptions on the input data shown in Table 1.

Table 1
Input parameters for STOOIP and EUR.

Input parameters	Assumed distribution	Definition of distribution
GRV	Normal distribution	Mean = 10^{10} barrels, std = 3×10^9
N/G	Normal distribution	Mean = 0.4, std = 0.1
φ	Normal distribution	Mean = 0.5, std = 0.1
S_o	Normal distribution	Mean = 0.2, std = 0.03
B_o	Normal distribution	Mean = 1.2, std = 0.1
RF	Normal distribution	Mean = 0.4, std = 0.1

Both Monte Carlo sampling and the parametric method for subsurface uncertainty assessment have been widely applied in practice. However, the classical reservoir uncertainty modeling approaches are still static in the sense that they only provide a snapshot of uncertainty at a given point in time. These approaches do not capture the epistemic aspect of uncertainty, in other words, how our understanding of uncertain variables could possibly evolve over time.

Recently, an extensive body of research has emerged with focus on quantifying and modeling market (i.e. oil price) and subsurface (i.e. EUR, production rates) uncertainty and their impact on field development and production strategies. A special issue (Suslick and Schiozer, 2004) in the Journal of Petroleum Science and Engineering was devoted to risk analysis and its application to petroleum exploration and production. There are two streams of literature in this research area:

(1) *Market uncertainty*: Dias (2004) gives an overview of classical real option models for evaluating E&P assets, and presents various stochastic models (i.e., Geometric Brownian Motion (GBM), Mean–Reversion Model (MRM), and two and three factor models) for oil price uncertainty. However, subsurface uncertainty remains unaddressed in the classical real options models. Lima and Suslick (2006) estimate the volatility of 12 deep-water offshore oil projects considering that oil price will evolve according to either GBM or MRM. They assume that volatility only stems from the uncertainty in oil price. A fairly recent paper by Abid and Kaffel (2009) shows a methodology to evaluate an option to defer an oilfield development. The oil price is modeled as GBM with discrete jumps.

(2) *Subsurface uncertainty*: Chang and Lin (1999) develop a stochastic method based on decline curve analysis to predict the future production rates and EUR probabilistically. Armstrong et al. (2004) incorporate technical uncertainty using Bayesian updating based on Archimedean copulas and evaluate the option to acquire more information through a production logging tool. Subbey et al. (2004) present an approach for generating uncertain reservoir performance predictions using a Bayesian framework and an adaptive sampling technique. Caumon et al. (2004) proposed a workflow based on Bayesian analysis to assess the uncertainty about a global reservoir parameter such as net-to-gross given multiple geological scenarios during early exploration. Zabalza-Mezghani et al. (2004) developed a proxy model for reservoirs based on a Design of Experiments approach to quantify the impact of uncertainty on production forecasts. A recent paper by Maschio et al. (2010) presents a methodology to reduce uncertainties in reservoir simulation models using observed data and the Latin hypercube sampling technique.

Although uncertainty has been addressed from two fronts separately (oil price and subsurface), to authors' knowledge there has not been any research on modeling how epistemic subsurface uncertainty evolves over time. For the first stream of research cited above, stochastic models, such as GBM with jumps (Dias and Rocha, 1999; Abid and Kaffel, 2009), have been developed to model oil price uncertainty. In the Dias and Rocha model, jumps are used to model sudden changes of oil prices due to abnormal events, such as wars and market crashes. This model assumes that the probability of jumps occurring is constant over time. However, this underlying assumption does not fit the human learning process (i.e., progressive learning) regarding subsurface uncertainty. In general, fewer surprises would be expected as we learn more about reservoirs through field exploration and production. For the second body of literature on subsurface uncertainty, various statistical or probabilistic approaches have been developed and applied, such as the Bayesian approach, decline curve analysis, and the proxy model. However, these approaches generally require some level of field data (i.e., production rates) and they are more suitable for performance

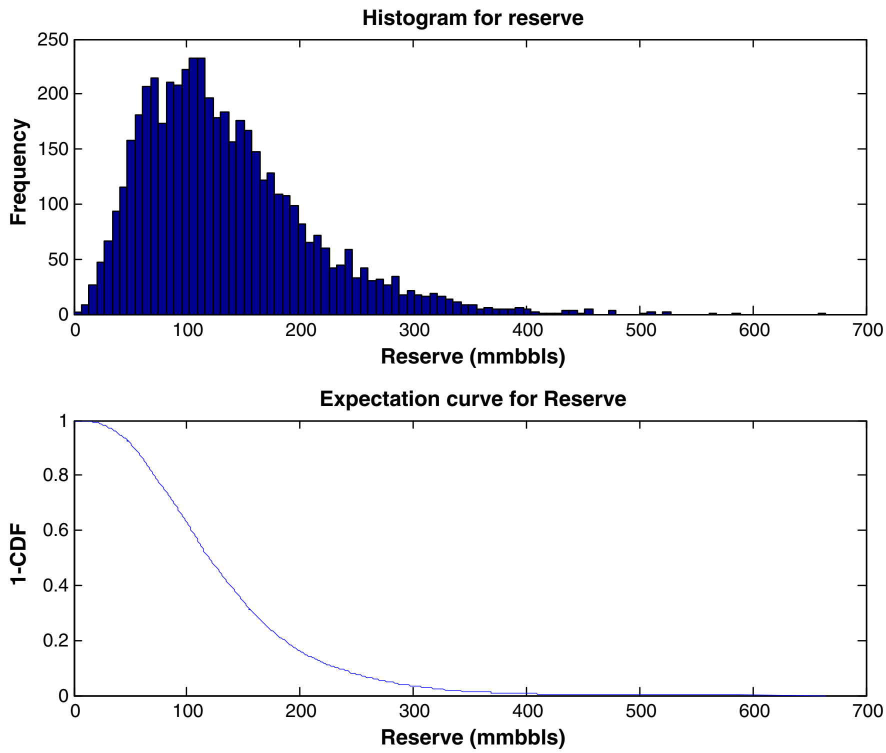

Fig. 5. Histogram and Expectation Curve for EUR (5000 samples).

prediction when a field is in the production stage. Moreover, classical reserve estimation approaches (Demirmen, 2007), such as deterministic or stochastic volumetric calculations are all bottom–up. They first assume certain probability distributions for input parameters and then combine these through analytic formulae or Monte Carlo simulation to yield a probability distribution for EUR. While valid these methods only give a snap shot of the EUR distribution at one particular point in time but leave the question of how EUR evolves over time unanswered.

This paper proposes a top–down stochastic model to simulate the evolution of EUR considering both progressive learning and "surprise" changes. Knowledge of possible EUR distribution trajectories is important because irreversible investment decisions are based on human perception of EUR and not on the underlying "unknown" true ultimate recovery. This important distinction is glossed over in much of the petroleum-related real options literature. Therefore, the purpose of this model is not to accurately predict reservoir performance per se, but to give decision makers a view of how their current EUR could evolve over time. Given the nature of this problem, generally there are two types of modeling approaches:

(1) *Data-driven approach*: requires samples of petroleum projects with historical data on EUR, and then applying statistical techniques (i.e., regression, response surface method) to fit empirical models to the distribution of EUR and its evolutionary behavior.
(2) *Analytical approach*: assumes an initial distribution for uncertain variables and uses several parameters (i.e., mean, standard deviation) to describe the distribution and the evolutionary behavior

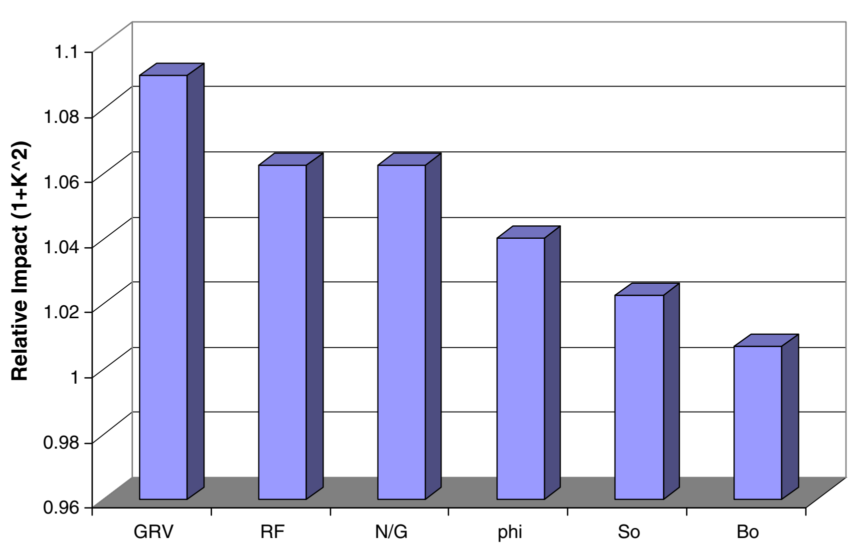

Fig. 6. Relative impact of input parameters on uncertainties in STOOIP and EUR.

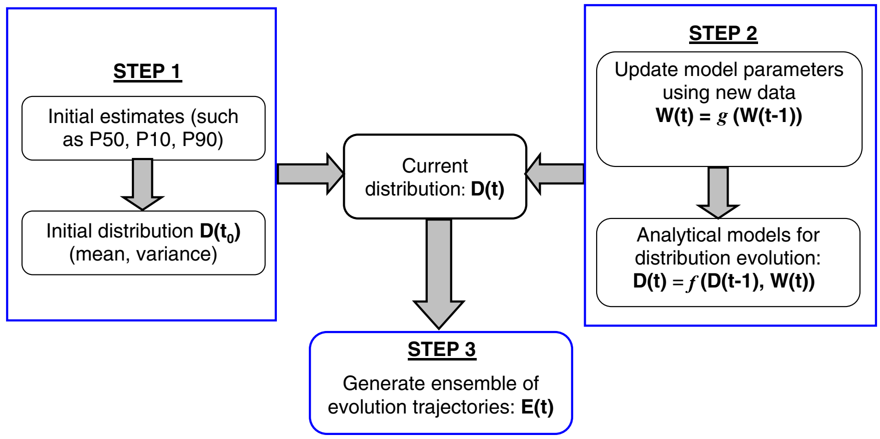

Fig. 7. Modeling steps for epistemic uncertainty (i.e., evolution of EUR).

(i.e., speed of convergence, probability of sudden jumps) of EUR distributions over time. Once an analytical model is in place, it can be calibrated against historical data. Recently, the Bayesian approach (Armstrong et al., 2004; Caumon et al., 2004; Subbey et al., 2004; Sarma, 2006) has been proposed in the literature to update posterior probability distribution of uncertainty estimates by incorporating new information.

For the first approach, there are several limitations: First, it requires a large number of samples in order to obtain statistically significant estimates of model parameters. In practice, it could very challenging to get access to a reasonable number of samples for reservoirs' historical data. Second, since each reservoir is unique, the estimated parameters tend to mix reservoirs with distinct geological characteristics. As a result, the estimate may not be very relevant to any particular reservoir. This paper develops a stochastic reservoir uncertainty model based on the second analytical approach.

3. Model development

First, this model makes several initial assumptions about EUR (i.e., type of distribution, speed of convergence), this approach avoids requiring large samples of historical data initially and it allows updating the model parameters when more information becomes available later. Therefore, the model can be tuned to a specific reservoir by acquiring more information from the reservoir or benchmarking to other similar reservoirs. The rest of this section presents a stochastic reservoir uncertainty model based on a reverse Wiener jump–diffusion process.

The theoretical foundation for this model is Geometric Brownian Motion (GBM) or Wiener processes. GBM and Wiener processes appeared in the finance literature to model commodity price uncertainty (Black and Scholes, 1973; Merton, 1973) and have been applied to model oil price uncertainty (Paddock et al., 1988; Dias and Rocha, 1999) in petroleum E&P projects. If we assume P as a stochastic variable (i.e., oil price) and it follows a GBM, the following stochastic differential equation describes the discrete dynamics of P:

$$\frac{dP}{P} = \alpha_P dt + \sigma_P dz \tag{4}$$

where α_P is the drift rate, σ_P is the instantaneous volatility and dt is the time increment of the Wiener process. dz is a random sample from a standard normal distribution $dz \sim N(0,1)$. Essentially, a GBM is a continuous or discrete "random walk" from an initial position to future positions, whereby the stochasticity comes from dz.

In this paper, we adapt the Wiener process by adding stochastic behavior that is more appropriate to model the evolution of epistemic uncertainty:

(1) *"Reverse" Wiener process*: The proposed model retains the random walk behavior but exponentially decreases the volatility due to random walk over time. In a classical random walk the initial starting position is known but subsequent positions that are further and further away from the start time are increasingly uncertain. In modeling epistemic uncertainty, the difference between the initial value and the true value could be large initially and then tends to decrease over time in general. Adding a "damping" factor (the exponentially declining rate for the volatility) to the Wiener process, diminishes the contribution of the random walk over time to allow gradual convergence to a steady value for each sample. However, for an ensemble of samples starting from the same initial value, the final values of the samples still have a distribution of outcome, thus, this modified process retains the diffusion characteristics of the standard Wiener process. Because each sample converges – rather than diverges – to a steady value over time, we add the word "reverse" to the original Wiener process. The exponential decline rate reflects the speed of human learning.

(2) *Jump diffusion process*: This model adds with a certain probability discrete jumps in the stochastic process, and this probability also exponentially decreases over time. Whenever a discrete jump occurs, it resets the current volatility to a fraction of the initial volatility (e.g., applying a reset factor between 0.5 and 1.5) and therefore partially restarts the random walk process. Jump diffusion reflects the fact that human learning is not perfect.

By adding these two stochastic characteristics to the Wiener process, we are able to qualitatively and quantitatively model the human learning process regarding epistemic uncertainty. These two model enhancements and their application to modeling EUR uncertainty in E&P projects are the main contribution of this paper. The rest of this section will show the detailed mathematical development of the reverse Wiener jump–diffusion process.

The overall modeling steps are illustrated in Fig. 7.

(3) The first step is to define an initial probability distribution for EUR given a snapshot at t_0. The vector $\boldsymbol{D}(t_0)$ contains the moments of the distribution, such as the mean and standard deviation for a lognormal distribution. In the previous section, we have shown that EUR follows a lognormal distribution if each individual factor is assumed to have an independent normal distribution. Other distributions (such as beta distribution) can be used for EUR as long as the moments of the chosen distribution fit the moments of

Table 2
An example of initial EUR. (Percentile estimates are normalized based on the initial P50 estimate).

	P100	P90	P80	P70	P60	P50	P40	P30	P20	P10	P0
Initial inputs	0	0	0	0.57	0.82	1.00	1.18	1.36	1.57	1.89	3.82
$D(t_o)$ model	0	0	0	0.71	0.88	1.02	1.18	1.36	1.59	1.96	

the statistical data for a field under study. The selection of lognormal distributions in this paper is not critical as our approach does not rely on the underlying distribution assumption and it is applicable to other distributions.

(4) The second step is to update the distribution vector ***D*** from the previous time step $t-1$ to the current time step at t. In this process, the model parameter vector, $\boldsymbol{W}(t)$, needs to be updated once an actual EUR becomes available. If the information is not available, $\boldsymbol{W}(t)$ remains the same as $\boldsymbol{W}(t-1)$.

(5) The third step consists of generating an ensemble of EUR trajectories $\boldsymbol{E}(t)$ given the model. These trajectories give petroleum engineers and decision makers a view of how the initial EUR could potentially evolve over time. Modelers can use these evolutionary trajectories as inputs for Monte Carlo simulations to test how well various field development plans respond to this type of epistemic uncertainty.

In this model, there are two functions, the first one is the update function $\boldsymbol{W}(t)=g(\boldsymbol{W}(t-1))$, which updates model parameters using available data (such as historical data of EUR). One type of update function is based on Bayesian theory, however, as new information is not usually available in the early exploration and appraisal phases of a field, we assume that $W(t)=W(t-1)$. The other one is the distribution evolution function $\boldsymbol{D}(t)=f(\boldsymbol{D}(t-1), \boldsymbol{W}(t))$, which updates the mean and variance of actual EUR. The rest of this section will describe the details of the three modeling steps shown in Fig. 7.

3.1. Step 1: generate an initial distribution vector $D(t_0)$

Inputs for the first step include quantities defining the initial distribution of EURs, such as P0–P100, which are generally available for prospect, discovery and appraisal fields.

Table 2 gives the initial EUR for a hypothetical prospect. At any given point in time, we assume that EUR follows a lognormal distribution, which is characterized by its mean[2] (μ) and standard deviation (σ) of $log(x)$, where x is EUR. These two parameters are estimated using the least squares method such that the mean and standard deviation minimize the sum of squares of residuals $\sum_{i=1}^{k} r_i^2$, where r are the residuals between input data and estimations of the fitted model for P10–P90. Because the theoretical P0 for a lognormal distribution is infinity, we ignore the P0 data point for least squares curve fitting. For a prospect[3] field, there is a finite probability of having zero recoverable hydrocarbons. The example shown in Table 2 has 80% chance of success for the prospect field. The probability distribution of EUR can be approximated by a delta function (finite probability with zero EUR) plus a "scaled" lognormal distribution. For the specific case shown in Table 2, we have seven data points ($k=7$) for least squares fitting.

Fig. 8 shows the resulting expectation curve obtained by least squares fitting based on the data in Table 2. The fitted initial

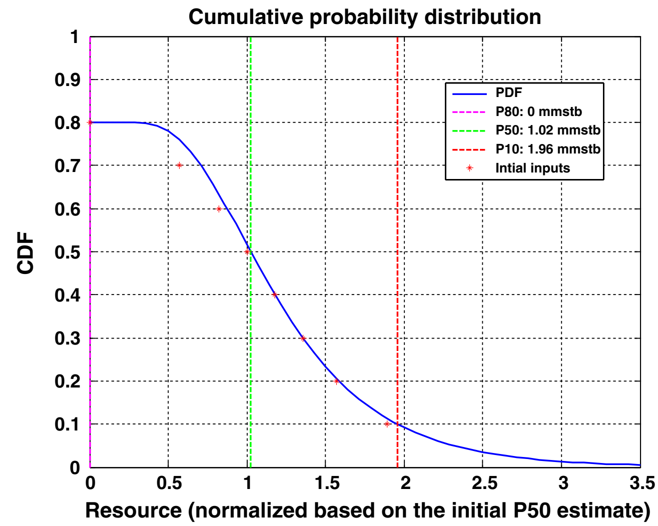

Fig. 8. A fitted expectation curve based on the $\boldsymbol{D}(t_o)$ initial EUR.

distribution is a scaled lognormal distribution (P10–P80) plus a delta function (P80–P100). Fig. 9 shows the simulated initial probability distribution for EUR estimated based on the fitted model. In this example, the distribution vector $\boldsymbol{D}(t_0)$ contains three elements:

$$D(t_0) = \begin{bmatrix} \mu(t_0) \\ \sigma(t_0) \\ \lambda(t_0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0.13 \\ 0.2 \end{bmatrix} \quad (5)$$

$\mu(t_0)$ and $\sigma(t_0)$ define the scaled lognormal distribution between P10 to P80. $\lambda(t_0)$ defines the delta function for cumulative probability from $(1-\lambda(t_0))$ to 1. The value of $\sigma(t_0)$ is normalized against $\mu(t_0)$. Therefore, $D(t_0)$ defines the initial distribution of uncertainty variable x. In this particular application, x represents the EUR normalized to its initial P50 estimate.

3.2. Step 2: update the model parameter $\boldsymbol{W}(t)$ and distribution vectors $\boldsymbol{D}(t)$ at time t

The second step is to update the distribution vector ***D*** from time $t-1$ to t. A set of parameters and functions will define how each element of $\boldsymbol{D}(t)$ evolves over time.

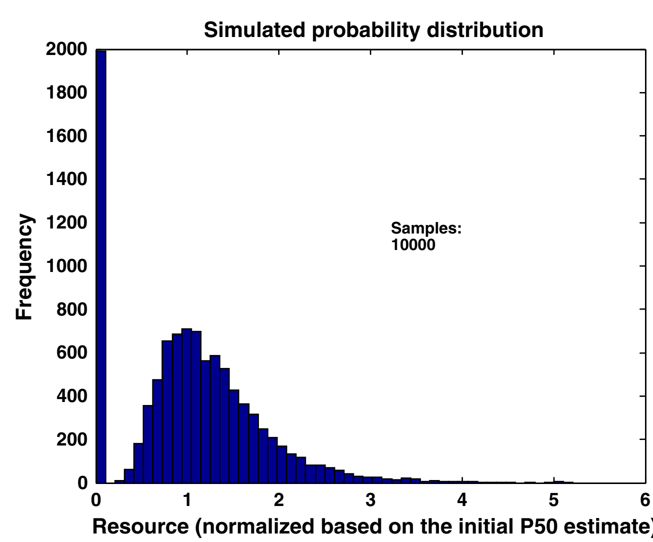

Fig. 9. Simulated probability distribution for EUR at t_0.

[2] Mean and standard deviation define the random variable $log(x)$, where x follows a lognormal distribution. According to statistical theory, $log(x)$ follows a normal distribution.

[3] For a prospect field, there is a finite probability of having zero EUR underground; for a discovery field, it is certain that the EUR are positive.

- $\mu(t)$: There are two assumptions governing the evolution of $\mu(t)$. First, we assume that the mean of $\log(x)$ starts from the initial estimate and follows a random walk at each time step. Secondly, the volatility of the random walk is assumed to decrease exponentially over time to reflect the effect of learning. Ideally, this decrease rate should be estimated from historical data (if available) in a similar geographical region. It also assumes that the probability of a discrete jump decreases exponentially. Hence, the following equations can be used to define the evolution of the mean EUR $\mu(t)$:

$$\mu(t) = \mu(t-\Delta t) + \Delta_t \tag{6}$$

$$\begin{cases} \Delta_t = b\Sigma_0 e^{-\beta t} & \text{if} \quad a \geq p(t) \\ \Delta_t = c\Sigma_0 & \text{if} \quad a < p(t) \end{cases} \tag{7}$$

$$p(t) = p_0 e^{-\gamma t} \tag{8}$$

where

$p(t)$	the probability of a sudden jump at time t
β	exponential decline rate for the variation of random walk
γ	exponential decline rate for the probability of a sudden jump
a	a random number drawn from a uniform distribution between 0–1
b	a random number drawn from a standard normal distribution
c	a random number drawn from a uniform distribution either between 0.5 and 1.5 or between −1.5 and −0.5 with equal probability
Σ_0	the initial standard deviation for random walk of $log(x)$, default [0.2]
p_0	the initial probability of a sudden jump, default [0.05]
Δt	the discrete time increment, default [3 months].

In each incremental time step, there is $p(t)$ probability to have a discrete jump, where the probability $p(t)$ exponentially decreases over time governed by the parameter γ. There is an equal probability to jump up or down. The jump size is the multiplication of the initial volatility and a random sample from a uniform distribution between [0.5 1.5] (jump up) or [−1.5 −0.5] (jump down). In each incremental time step, there is a $(1-p(t))$ probability to continue a random walk with exponentially decreasing volatility governed by the parameter β.

(1) $\sigma(t)$: The standard deviation of the uncertain variable (i.e., log (x)) starts from an initial value and decreases exponentially. Whenever the mean $\mu(t)$ has a sudden jump, the standard deviation increases simultaneously.

$$\begin{cases} \sigma(t) = \sigma(t-\Delta t)e^{-\alpha} & \text{if} \quad a \geq p(t) \\ \sigma(t) = \max\left(\sigma(t-\Delta t)e^{-\alpha}, \quad d\sigma_0\right) & \text{if} \quad a < p(t) \end{cases} \tag{9}$$

where

α	exponential decline rate for the standard deviation of $\log(x)$
d	a random number drawn from a uniform distribution between 0.5–1
σ_0	the initial standard deviation of the uncertain variable, log (x), default [0.3].

The random sample d partially resets the volatility to the initial value whenever a jump occurs. The *max* function is to ensure that the resulting volatility following a jump is no less than the volatility if the underlying random walk had continued.

(2) $\lambda(t)$: This parameter is non-zero for a prospect field, which has a finite probability of having a zero EUR. For discovered fields, $\lambda(t) = 0$. If no future information is available, the model assumes that this parameter remains at the initial estimate. If future information is available, it is possible to estimate the evolutionary trend for $\lambda(t)$. For simplicity, we assume:

$$\lambda(t) = \lambda(t_0). \tag{10}$$

Eqs. (6) through (10) define the function $\boldsymbol{g}$ for each element of $\boldsymbol{D}(t)$. Function $\boldsymbol{g}(\boldsymbol{D}(t) = \boldsymbol{g}(\boldsymbol{D}(t-1), \boldsymbol{W}(t)))$ updates $\boldsymbol{D}(t)$ from time step $t-1$ to t. The model parameter vector $\boldsymbol{W}(t)$ can also be a function of time. $\boldsymbol{W}(t)$ includes parameters, such as $\alpha(t)$, $\beta(t)$, and $\gamma(t)$, which define the various exponential decline rates. An extended version of the $\boldsymbol{W}(t)$ vector would also need to include the parameters a, b, c, and d. We also need to define the initial parameters of the model, such as σ_0, Σ_0, and p_0. If improved EUR distributions become available over time, the parameter vector should be updated accordingly (e.g., Bayesian approach). If only a snap shot of EUR is available, we assume that the model parameter vector $\boldsymbol{W}(t)$ is constant.

With this model, we can generate an ensemble of possible evolutionary trajectories for EUR given the best knowledge of reservoirs today. However, the actual evolution history for EUR is only one realization among many possibilities. This is the main difference between human perception of EUR evolution (many possibilities) and the actual evolution history (only one evolutionary trajectory). The ensemble of EUR evolution trajectories allows decision makers to experiment with various field development strategies in view of possible EUR evolutions. In this type of application, since the future has not yet unfolded, we can assume $\boldsymbol{W}(t) = \boldsymbol{W}(t_0)$ as a constant vector. The next section of this paper shows a set of numerical experiments simulated by this model.

3.3. Step 3: generating an ensemble of future scenarios

The step 3 is to generate an ensemble of future scenarios $\boldsymbol{E}(t)$ given the model parameters as defined in steps 1 and 2. We will illustrate the evolutionary behavior of scenarios using numerical experiments.

With the defined stochastic processes of the model, an ensemble of future scenarios $\boldsymbol{E}(t)$ can be generated using Monte Carlo simulations. This is step 3 as shown in Fig. 7. The simulation is discretized in time steps Δt (typically 3 months over a 25 year lifecycle). Within each simulation time step, samples are drawn from given distributions, and the evolution of EUR is simulated according the procedures in steps 1 and 2. Fig. 10 shows two sample evolution trajectories for EUR with two different resulting underlying values. These two trajectories start from the same initial estimate and diverge in different directions. In year 12, one of the trajectories has a sudden jump in its EUR. The other trajectory follows a random walk and the uncertainty reduces monotonically. Eventually the estimation trajectories approach their underlying values. As a result, the range of uncertainty decreases simultaneously. The behavior of the stochastic model can be tuned by changing the model parameter vector $\boldsymbol{W}(t)$. Fig. 10 shows only two possible evolution trajectories for EUR.

One practical way to quantify the range of EUR uncertainty is to define an uncertainty spread metric as the ratio of (P10–P90)/P50. Fig. 11 shows the uncertainty spread for the same two realizations as in Fig. 10. Both realizations follow the same uncertainty curve up to year 11. In year 12, the first realization has a jump which instantaneously increases its uncertainty spread and then declines. Because this model assumes exponential decline of variation, governed by the parameter β, the uncertainty spread curves also follow similar exponential decline functions. However, the decline rate of uncertainty spread is less than β because uncertainty spread only computes part (between P90 and P10) of the full uncertainty range.

Fig. 12 shows 100 realizations of the P50 EUR over 24 years. All 100 realizations start from the same P50 value (800 mmstb) and evolve to

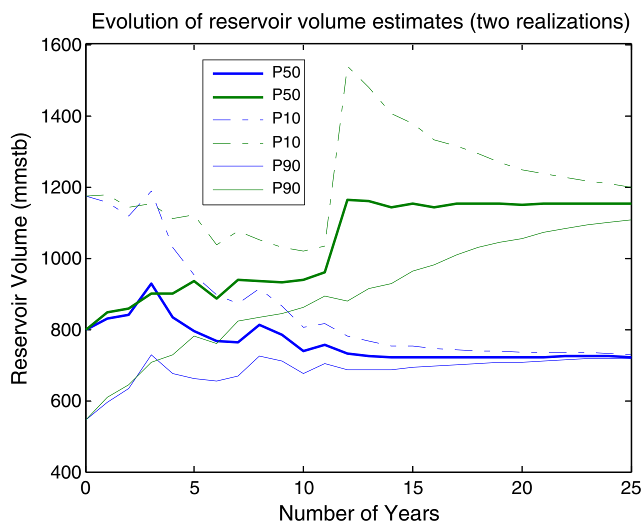

Fig. 10. Two evolution trajectories of EUR. (Model parameters: $\alpha=0.15$, $\beta=0.2$, $\gamma=0.15$, $\sigma_0=0.3$,$\Sigma_0=0.2$, $p_0=0.05$).

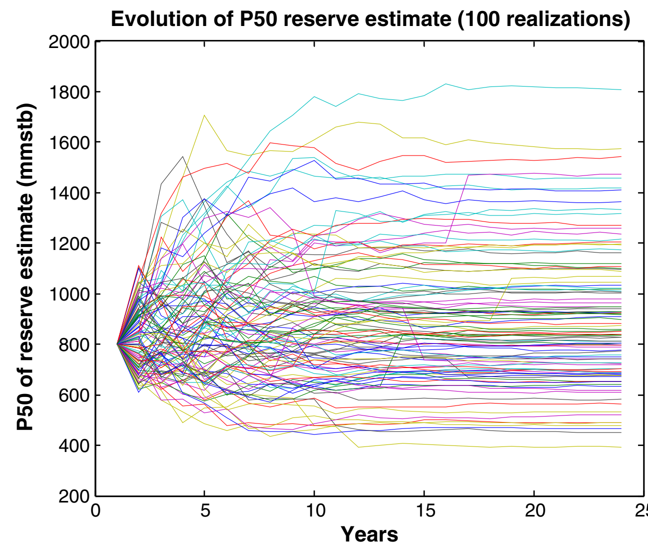

Fig. 12. Evolution trajectories for P50 EUR (100 realizations).

different final values, which are initially unknown. As shown in Fig. 12, some realizations have increasing trends of EUR while others have decreasing trends over time. Occasionally, there are some jumps in the evolution trajectories. Since the underlying value is unknown initially, the underlying value predicted by this model shows a distribution of outcomes at the end. It is important to retain a distribution of outcomes for the underlying EUR because at the time the model is developed or used the true underlying EUR is unknown. Each trajectory has a certain probability to become the "realized" trajectory for the actual EUR. As more information becomes available over time, the model parameters can be adjusted so that the range of epistemic uncertainty narrows and EUR approaches to the true underlying value. This model is used to generate possible evolutionary trajectories given the best knowledge of reservoirs at any given point of time, but not to predict the underlying "unknown" true value.

4. Model validation

The model was developed to reproduce qualitatively the behavior seen in several historical data sets and it has a number of parameters

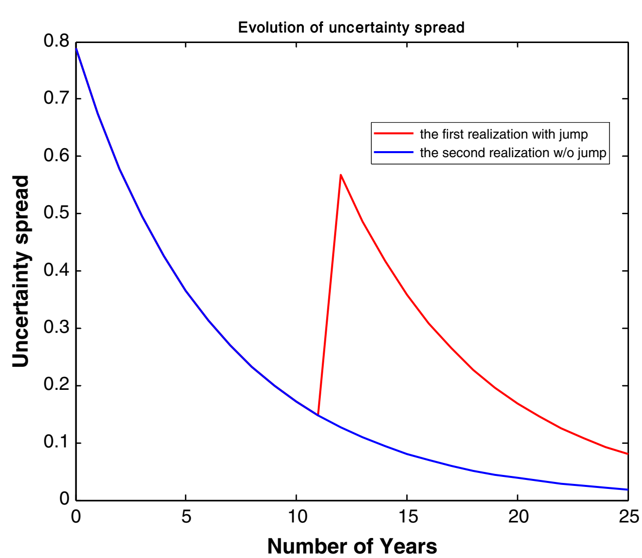

Fig. 11. Evolution of uncertainty spread (corresponding to the two realizations shown in Fig. 10).

to tune the evolutionary behavior of EUR. We use the data presented by Watkins (2000) to perform model validation. The most important aspect which the model must reproduce is the appreciation factor (the ratio of the EUR at times t and $t_o=0$). To validate the predicted outcome for this quantity, a comparison between the model results and the data from Watkins (2000) was made. The validation procedure has the following three steps:

(1) Firstly, split the original data set randomly into two groups with equal number of field data (17 fields each). The purpose of random allocation of data into two data sets is to maintain similar statistics. The first data set is used to calibrate the model parameters while the second data set is used for model validation.
(2) Modify model parameter Σ_0 until that the model predicted P10 to P90 uncertainty envelope covers 80% of the data points in the calibration data set. In this case, we only change one model parameter. This model tuning process is based on the first half of the data.
(3) Plot the second data set on the same chart and verify whether 80% of data points fall within the P10 to P90 uncertainty envelope predicted by the model. This model validation process is based on the second half of the data.

Fig. 13 overlays the actual data ("+" symbols for calibration data set, "o" symbols for validation data set) and the envelope curves predicted by the model. The P10, P50, and P90 envelope curves are based on the results of 200 simulated trajectories and all the model parameters were set at the default values except for $\Sigma_0=0.16$ where the default value was originally 0.2. By tuning the model parameter Σ_0 to 0.16, exactly 80% of the data points fall within the predicted P10 to P90 uncertainty envelope. Then we plot the validation data set on the same graph and count the number of data points falling within the P10 to P90 uncertainty envelope. As shown in Fig. 13, 82.4% of validation data points fall within the same envelope predicted by the reverse Wiener jump–diffusion model. Although 82.4% is not an exact match of 80% that the P10 to P90 uncertainty envelope should theoretically cover, the result validates the overall model behavior well given given the limited data available for calibrating and validating the model parameters. As shown in Fig. 13, there are a couple of outliers, and in one instance AF goes up to 4 in year 12. They are clearly outside the P10 and P90 envelope. This could be inherent variability in the dataset. A sensitivity study of including or excluding certain data points could help understand the robustness of the model. We reserve this as future work. This process demonstrates that, given the historical data, this model can be

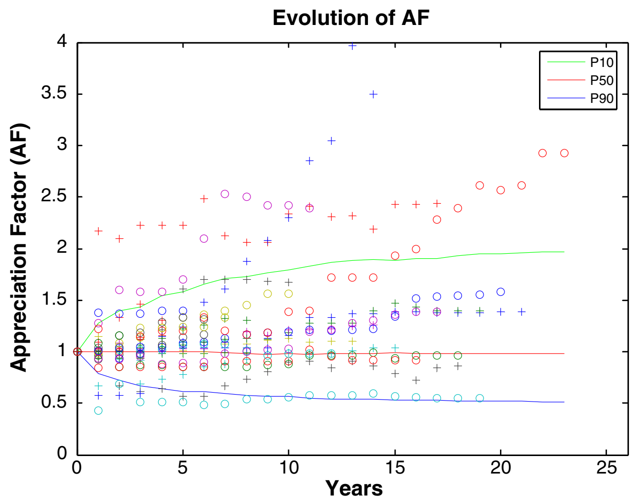

Fig. 13. Comparison of Appreciation Factors: P10, P50, and P90 are based on the model; points ("+" or "o") are actual data from 34 North Sea fields from 1973 to 1996 (with various durations). Points "+" are from the calibration data set and points "o" are from the validation data set.

calibrated by tuning the parameters, such as the initial volatility and other parameters governing the jump–diffusion process, to reproduce the evolutionary behavior of epistemic uncertainty.

5. Conclusions

This paper develops a reverse Wiener jump–diffusion process model to simulate the propagation of epistemic uncertainty. This model is developed for the purpose of simulating the evolution of subsurface uncertainty (e.g., EUR). Based on the comparison between the jump–diffusion model simulation results and historical data, the model is able to represent the stochastic learning processes as seen from data of 34 North Sea fields. Although the model is developed with specific application to modeling reservoir subsurface uncertainty in mind, the generic reverse Wiener jump diffusion stochastic process is applicable to other types of epistemic uncertainty, in which uncertainty stems from limited and imperfect human knowledge of underlying natural or physical systems that can be considered to be in quasi steady state.

Future work includes refining and tuning the model parameters in $\boldsymbol{W}(t)$ with larger data sets from reservoirs in different basins. If larger data sets are available, a further multi-parameter calibration and validation can be done by segregating the data sets into calibration and validation sets, or applying the Bootstrap re-sampling technique and conducting a sensitivity study with respect to using different datasets for model tuning and validation. Another item for future work is to examine and classify the sources of jumps in EUR such as geological fault discoveries or the presence of aquifers.

Nomenclature

AF	appreciation factor
UR	ultimate recovery
EUR	estimated ultimate recovery
$\Delta\mu$	initial error for EUR (or reserve estimate)
dP	increment for a stochastic variable in GBM
α_P	drift rate for GBM
σ_P	volatility for GBM
dz	increment for Wiener process, a random sample from $N(0,1)$
$\boldsymbol{D}(t_0)$	initial distribution vector for EUR or reserve estimates
$\boldsymbol{W}(t)$	model parameter vector at time t
$\boldsymbol{E}(t)$	an ensemble of evolution trajectories at time t
$\boldsymbol{I}(t)$	new information at time t
t	time [years]
Δt	increment time step [months]
g	update function for model parameter vector $\boldsymbol{W}(t)$
f	update function for distribution vector $\boldsymbol{D}(t)$
x	probabilistic EUR [normalized value against P50]
$\mu(t)$	mean of $log(x)$ at time t [normalized value against P50]
$\sigma(t)$	standard deviation of $\log(x)$ at time t [normalized value against P50]
$\lambda(t_0)$	probability of having zero resource for a prospect field
Δ_t	a random walk for EUR estimate mean from time $(t-\Delta t)$ to t
$p(t)$	the probability of a sudden jump at time t
α	exponential decline rate for the standard deviation of log (x), default [0.15]
β	exponential decline rate for the variation of random walk, default [0.2]
γ	exponential decline rate for the probability of sudden jump, default [0.15]
Σ_0	the initial standard deviation for random walk of $log(x)$, default [0.2]
p_0	the initial probability of a sudden jump, default [0.05]
σ_0	the initial standard deviation of the uncertainty variable, default [0.3]
a	a random number drawn from a uniform distribution between 0 and 1
b	a random number drawn from a standard normal distribution
c	a random number drawn from a uniform distribution between 0.5 and 1.5 or -1.5 and -0.5 with equal probability
d	a random number drawn from a uniform distribution between 0.5 and 1

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